

Introduction to Model Checking (Summer Term 2018)

— Exercise Sheet 7 (due 18th June) —

General Remarks

- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 12:15 or by dropping them into the “Introduction to Model Checking” box at our chair *before 12:00*. Do *not* hand in your solutions via L2P or via e-mail.
- If a task asks you to justify your answer, an explanation of your reasoning is sufficient. If you are required to prove a statement, you need to give a *formal* proof.

Exercise 1

(3+3 Points)

Let φ and ψ be LTL formulae. Consider the following new operators.

- “At next”: $\varphi AX \psi \iff$ the next time at which ψ holds, φ also holds.
- “While”: $\varphi WH \psi \iff$ φ holds at least as long as ψ holds.
- “Before”: $\varphi B \psi \iff$ if ψ holds at some point, φ does so (strictly) before.

- (a) Formalize the semantics of these operators on infinite words $\sigma \in (2^{AP})^\omega$.
- (b) Show that these operators are LTL-definable by providing equivalent LTL formulae.
Hint: You may use both the until and weak until operator.

Exercise 2★

(1+2 Points)

Let $AP = \{a, b\}$. Let $\varphi = (a \rightarrow \bigcirc \neg b) W (a \wedge b)$.

- (a) Transform $\neg\varphi$ into an equivalent LTL formula in PNF using the until operator U and the weak until operator W .
- (b) Determine which of the properties $P = \text{Words}(\varphi)$ and $P' = \text{Words}(\neg\varphi)$ are safety properties. Justify your answer.

Exercise 3

(2+2+2+2 Points)

Let φ, ψ, π be arbitrary LTL formulae. For each of the following pairs of LTL formulae, determine in which relation they are. More specifically, determine whether they are equivalent, one of them subsumes the other or they are incomparable. Prove your claims.

- (a) $\diamond \square \varphi$ and $\square \diamond \varphi$
- (b) $\diamond \square \varphi \wedge \diamond \square \psi$ and $\diamond (\square \varphi \wedge \square \psi)$
- (c) $\varphi \wedge \square (\varphi \rightarrow \bigcirc \diamond \varphi)$ and $\square \diamond \varphi$

(d) $(\varphi \text{ U } \psi) \text{ U } \pi$ and $\varphi \text{ U } (\psi \text{ U } \pi)$

Exercise 4

(2+1 Points)

We consider the release operator R which is defined by

$$\varphi \text{ R } \psi \stackrel{\text{def}}{=} \neg(\neg\varphi \text{ U } \neg\psi).$$

(a) Prove the expansion law

$$\varphi \text{ R } \psi \equiv \psi \wedge (\varphi \vee \text{O}(\varphi \text{ R } \psi)).$$

(b) Prove the equivalence

$$\varphi \text{ R } \psi \equiv (\neg\varphi \wedge \psi) \text{ W } (\varphi \wedge \psi).$$