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Introduction to Model Checking (Summer Term 2018)

— Exercise Sheet 3 (due 14th May) —

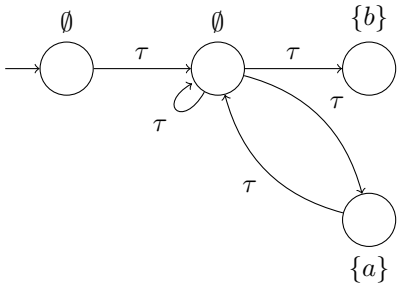
General Remarks

- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 12:15 or by dropping them into the "Introduction to Model Checking" box at our chair *before 12:00*. Do *not* hand in your solutions via L2P or via e-mail.

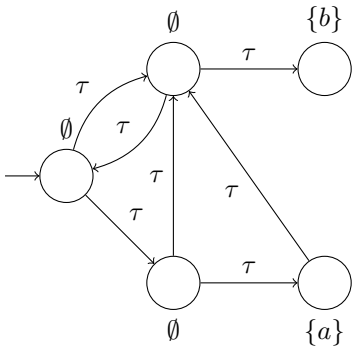
Exercise 1 (1+1 Points)

Consider the following transition systems. Note that the transition systems might contain terminal states.

TS_1 :



TS_2 :



(a) Give the traces of TS_1, i.e., $\mathit{Traces}(\mathrm{TS}_1)$.

(b) Are TS_1 and TS_2 trace equivalent?

Exercise 2★ (4+4 Points)

In the following we show that LT properties are not solely a theoretical concept but have a wide range of practical applications. As proof, we apply the concept of LT properties to movie/TV series quotes.

(a) We assume each following quote informally describes some property. Formulate these properties as LT properties over the given set AP of atomic propositions:

(i) "**Winter is coming.**"
AP = $\{\mathit{winter}\}$.
winter will eventually by reached.

(ii) "**Everything is awesome.**"
AP = $\{\mathit{awesome}\}$.
awesome always holds.

(iii) "**I'll be back.**"
AP = $\{\mathit{here}\}$.

I am currently *here* but at some point I will not be *here*. However, I will be *here* again at a later time.

(iv) "**You either die a hero, or you live long enough to see yourself become the villain.**"
AP = $\{live, hero\}$.
In the beginning, you *live* and are a *hero*. You either cease to *live* and die, still being a *hero*, or you *live* but become the villain, i.e., you are not a *hero* anymore.

(v) "**By night one way, by day another**
Thus shall be the norm
Till you receive true love's kiss
then, take love's true form."
AP = $\{day, form_1, form_2, true\_form, kiss\}$.
You start by having $form_1$ at night, i.e., not *day*. You alternate between $form_1$ at night and $form_2$ by *day*. This alternation goes on till at some point you receive true love's *kiss* and from there on have love's $true\_form$.

(vi) "**A Lannister always pays his debts.**"
AP = $\{in\_debt\}$.
Whenever a Lannister is in_debt, he will be in_debt as long as he has not payed back his debt. If he has payed back his debt, he is no longer in_debt. A Lannister can be in_debt arbitrarily (but finitely) many times.

(vii) "**Anything is possible [if you just believe].**"
AP = $\{ap_1, \ldots, ap_n\}$.
We do not consider the second part here and just concentrate on the fact, that everything is possible.

(viii) "**It's gonna be legen... wait for it... dary!**"
AP = $\{legen, wait\_for\_it, dary\}$.
In the beginning it is *legen*, then we have to $wait\_for\_it$ for some time, and then it is *dary* at some point.

(b) Determine for all LT properties of (a) whether they are

(i) safety properties *and/or*

(ii) liveness properties.

Justify your answers.

Exercise 3 (3+3 Points)

(a) Let P and P' be liveness properties over AP. Prove or disprove the following claims:

(i) $P \cup P'$ is a liveness property,

(ii) $P \cap P'$ is a liveness property.

(b) Answer the same questions for P and P' being safety properties.

Hint: you can use the distributivity of union over closure for LT properties P, P':

$$closure(P \cup P') = closure(P) \cup closure(P')$$

Exercise 4 (4 Points)

Let P be an LT property. Prove: $pref(closure(P)) = pref(P)$.