Globalübung

Einsicht:
- In den Tutorien
- Prozedere
- Nachholtermin:
  1. Klausurtermin

Ergebnisse d. PÜ:
voraussichtlich heute Abend

Logarithmen:

$$2^{?} = 8 \qquad \log_2(8)$$

$$? = 3$$

$$b^y = x \qquad \log_b(x)$$

$$1 \cdot 2 \cdot 2 \cdot 2 = 8$$

$$\frac{\frac{\frac{8}{2}}{2}}{2} = 1$$

```
while (n > 1) {
    n := n/2
}
```

$$\log_b(a) = \boxed{\frac{\log_c(a)}{\log_c(b)}} = \frac{\log(a)}{\log(b)}$$

$2^1 = 2$

3

$2^2 = 4$

Nein:

$$\Omega\left(n^{\frac{\log 3}{\log 2} + \varepsilon}\right) \ni n^2 \quad \text{für } \varepsilon = \frac{1}{2}$$

$$\frac{\cancel{\log} 3}{\cancel{\log} 2} = \frac{3}{2} = 1.5$$

$$1 < \frac{\log 3}{\log 2} = \log_2(3) < 2$$

Laufzeitanalyse:

Laufzeit: Fkt

Eingabe $\to$ konkrete Laufzeit

$\mathbb{R}_{\geq 0}$

$A(n)$: Integer / $\mathbb{N}$ $\rightsquigarrow$ void.

for i in 1...n: for $i \in \{1...n\}$

print(i)

Ausgabe:

1 2 3 4 .... n.

Analyse:

$$\sum_{i=1}^{n} 1 \rightsquigarrow n.$$

geschlossenen Form

for i in 1..n:

print(i)

print(i)

print(i)

konstante Anzahl Statements.

$\hat{c}$

Analyse:

$$\sum_{i=1}^{n} 3 \rightsquigarrow 3 \cdot n. \rightsquigarrow \Theta(n).$$

```
for i in 1...n:
    print(...)        } n mal.
    if i mod 2 == 0:
        print.        } n/2 mal.
```

In $O(n)$.

$n + \frac{n}{2} \rightsquigarrow \Theta(n)$.

```
for i in 1..n:
    if false:
        print(...).
```

In $\Theta(0)$.

und in $O(n)$.

```
for i in 1..n
    if i < 25:
        print(...).   } 24 mal
```

In $\Theta(24) = \Theta(24)$

$= \Theta(1)$.

```
for i in 1..n:
    for j in 1..n:
        print(i,j).
```

Ausgabe:

$$\text{n-mal}\left\{\underbrace{\begin{array}{ccccc}1,1 & 1,2 & 1,3 & \dots & 1,n\\ 2,1 & 2,2 & & \dots & \\ \vdots & & & & \\ n,1 & \dots & & & n,n.\end{array}}_{\text{n-mal.}}\right.$$

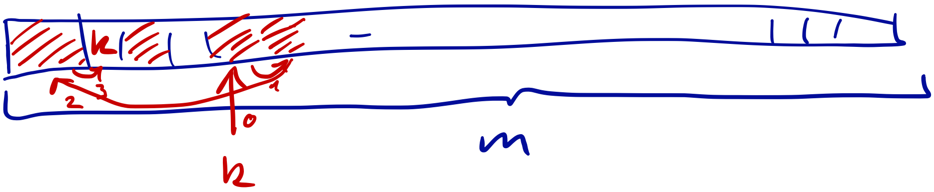
$$n \cdot n = n^2.$$

```
for i in 1..n:
    for j in 1..10 :      23!-47^2
        print(...).
```

$$\underbrace{\boxed{}}_{\text{n-mal}}\Big\}\ \text{10-mal}$$

$$n \cdot 10 \rightsquigarrow O(n)$$

Average-Case Analyse Erfolgreiche Suche beim offenen Hashing



- Anzahl an Slots: m
- Anzahl belegter Slots: n
- Schlüssel, die in Hashtabelle enthalten sind (in Einfügereihenfolge):

  $$k_1, k_2, \dots, k_n$$

- Wie oft muss man sondieren, um Schlüssel k_i, $1 \leq i \leq n \leq m$, zu finden?

- Beim Einfügen von k_i:
  - Anteil freier Slots:

    $$\frac{m-(i-1)}{m}$$

  - Anteil belegter Slots:

    $$\frac{i-1}{m}$$

  - Erwartete Anzahl an Sondierungsschritten:

    $$\sum_{j=0}^{\infty} \left(\frac{i-1}{m}\right)^j \cdot \left(\frac{m-(i-1)}{m}\right) \cdot (j+1)$$

$$\alpha = \frac{n}{m}$$

$$1 - \frac{n}{m} = \frac{m-n}{m}$$

$$1 - \frac{m-(i-1)}{m} = \frac{\cancel{m-m+} i+1}{m} = \frac{i-1}{m}$$

$$\sum_{j=0}^{\infty} \bar{p}^{j} p (j+1) = \sum_{j=0}^{\infty} \left(\bar{p}^{j} p \cdot j + \bar{p}^{j} p \cancel{\cdot 1} \right) \qquad \bar{p} := 1 - p$$

$$= \sum_{j=0}^{\infty} \bar{p}^{j} p j + \sum_{j=0}^{\infty} \bar{p}^{j} p$$

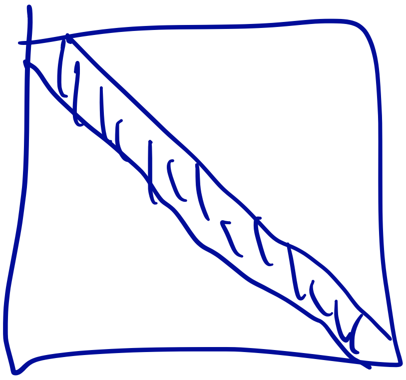
$$= p \left(\underbrace{\sum_{j=0}^{\infty} \bar{p}^{j} j}_{\frac{\bar{p}}{(\bar{p}-1)^2}} + \underbrace{\sum_{j=0}^{\infty} \bar{p}^{j}}_{\frac{1}{1-\bar{p}}} \right)$$

$$= p \left(\frac{\bar{p}}{(\bar{p}-1)^2} + \frac{1}{1-\bar{p}} \right) \qquad \Big| \; \bar{p} = 1 - p$$

$$= p \left(\frac{1-p}{(\cancel{1} - p \cancel{- 1})^2} + \frac{1}{\cancel{1-1}+p} \right)$$

$$= p \left(\frac{1-p}{p^2} + \frac{1}{p} \right) = p \left(\frac{1-p}{p^2} + \frac{p}{p^2} \right)$$

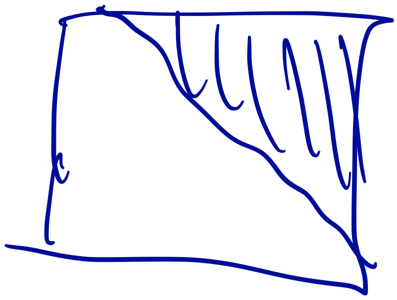
$$= \frac{p \cancel{- p^2 + p^2}}{p^2} = \frac{p}{p^2} = \frac{1}{p}$$



```
for i in 1..n:
    for j in 1..n:
        if i==j:
            print
```

n^2

n. mal.



$$n \cdot n \cdot \frac{1}{2} \rightsquigarrow \Theta(n^2).$$

```
c=0
while n > 1:
    n = n/2
    c = c+1
for i in 1..c:
    print
```

Wert von $c \in \Theta(\log n)$

```
j = 1
for i in 1..n:
    j = j · i
for i in 1..j:
    print(...)
```

Erwartete Anzahl an Sondierungs-schritten, um k_i zu suchen:

$$\boxed{\frac{m}{m-i+1}} =: \mathbb{E}(S_{k_i})$$

- W'keit, dass k_i, für ein i, gesucht wird:

$$\frac{1}{n}$$

- Erwartete Anzahl an Sondierungsschritten insgesamt

$$\frac{1}{n} \sum_{i=1}^{n} \mathbb{E}(S_{k_i})$$

$$\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}(S_{k_i})$$

$$\boxed{\boxed{\frac{m}{m-i+1}}} =: \mathbb{E}(S_{k_i})$$

$$H_x = \frac{1}{1}+\frac{1}{2}+\dots+\frac{1}{x} \in O(\ln(x))$$

$$= \frac{1}{n}\sum_{i=1}^{n}\frac{m}{m-i+1}$$

$$= \frac{1}{n}\Bigg(\underbrace{\frac{m}{m}+\frac{m}{m-1}+\frac{m}{m-2}+\frac{m}{m-3}+\dots+\frac{m}{m-(n-2)}+\frac{m}{m-(n-1)}}\ \underbrace{+\frac{m}{m-n}+\dots+\frac{m}{1}} - \Big(\underbrace{\frac{m}{m-n}+\dots+\frac{m}{1}}\Big)\Bigg)$$

$$= \frac{m}{n}\left(H_m - H_{m-n}\right)$$

$$\in O\left(\frac{m}{n}\left(\ln(m) - \ln(m-n)\right)\right)$$

$$\log(x) - \log(y) = \log\left(\frac{x}{y}\right)$$

$$= O\left(\frac{m}{n}\ln\left(\frac{m}{m-n}\right)\right)$$

$$\boxed{\alpha = \frac{n}{m}}$$

$$= O\left(\frac{1}{\alpha}\ln\left(\frac{1}{1-\alpha}\right)\right)$$