Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL ⟵
- automata-based LTL model checking
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

Linear Temporal Logic (LTL)

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$$\varphi \; ::= \; \mathit{true} \; \big| \; a \; \big| \; \varphi_1 \wedge \varphi_2 \; \big| \; \neg\varphi \; \big| \; \bigcirc \varphi \; \big| \; \varphi_1 \, \mathsf{U} \, \varphi_2$$

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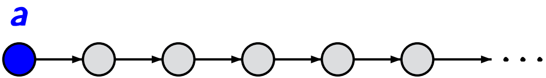

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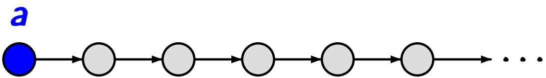
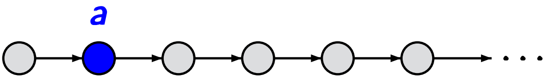


next operator $\bigcirc a$



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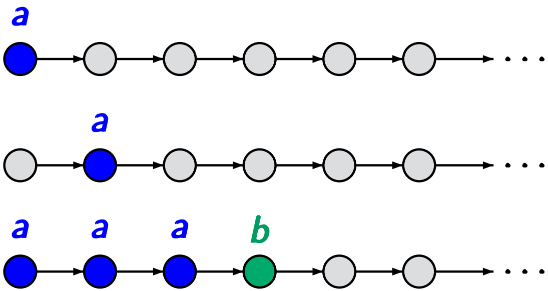

Derived operators in LTL

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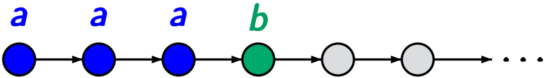

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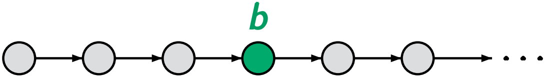

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$\Diamond\varphi$	$\stackrel{\text{def}}{=}$	$true \,\mathsf{U}\, \varphi$	eventually
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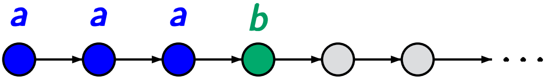

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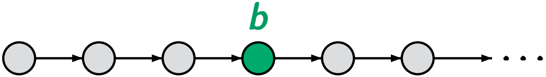

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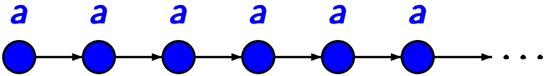
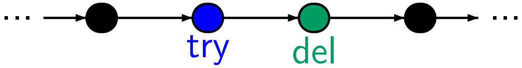


Next $\bigcirc$, until U and eventually $\Diamond$

$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\,\text{delivered})$



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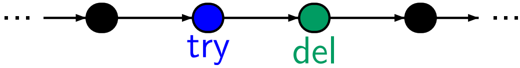

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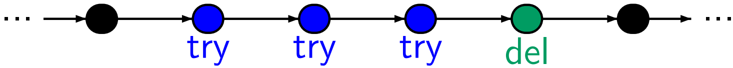

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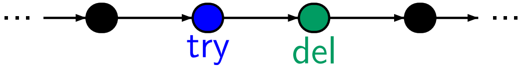

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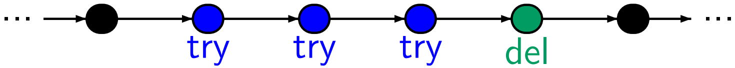

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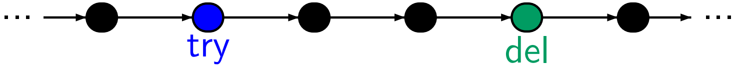

Examples for LTL formulas

$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \ \varphi_1 \, \mathsf{U} \, \varphi_2$$

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traffic light: $\Box(\mathit{yellow} \vee \bigcirc\neg \mathit{red})$

Infinitely often and eventually forever

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LTL-semantics

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$

Semantics of LTL over infinite words

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$$\sigma \models \bigcirc\varphi \quad \text{iff} \quad \mathit{suffix}(\sigma, 1) = A_1 \, A_2 \, A_3 \, \ldots \models \varphi$$

Semantics of LTL over infinite words

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$\sigma \models \bigcirc\varphi$	iff	$\mathit{suffix}(\sigma, 1) = A_1\, A_2\, A_3 \ldots \models \varphi$
$\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2$	iff	there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

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LT property of formula φ:

$$Words(\varphi) \overset{\text{def}}{=} \left\{\sigma \in \left(2^{AP}\right)^\omega : \sigma \models \varphi\right\}$$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$$\vdots$$

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LTL-semantics of derived operators $\Diamond$ and $\Box$

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$$
\begin{array}{lll}
\sigma \models \varphi_1 \,\mathsf{U}\, \varphi_2 & \text{iff} & \text{there exists } j \geq 0 \text{ such that} \\
 & & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2 \quad \text{and} \\
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 & & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi \\
\sigma \models \Box\varphi & \text{iff} & \text{for all } j \geq 0 \text{ we have:} \\
 & & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi
\end{array}
$$

LTL semantics over TS

given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

assumption: $\mathcal{T}$ has no terminal states, i.e., all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
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LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

LTL semantics over paths of TS

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$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

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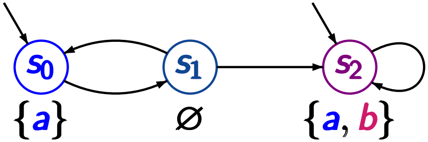
remind: LT property of an LTL formula:

$$Words(\varphi) = \{\sigma \in (2^{AP})^{\omega} : \sigma \models \varphi\}$$

Example: LTL-semantics over paths



$AP = \{a, b\}$

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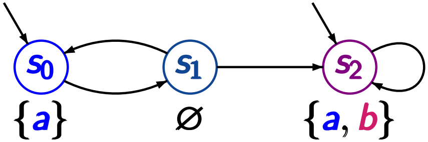

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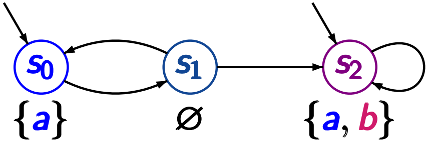

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$\pi \models a$

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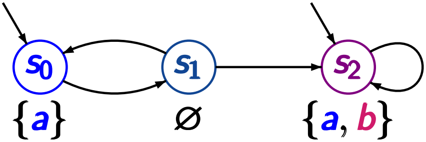

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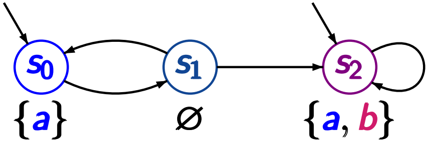

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$\pi \models a$, but $\pi \not\models b$ as $L(s_0) = \{a\}$

$\pi \models \bigcirc(\neg a \wedge \neg b)$

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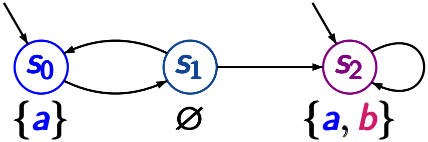

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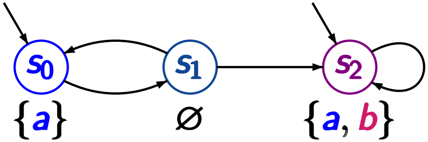
$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$

Example: LTL-semantics over paths



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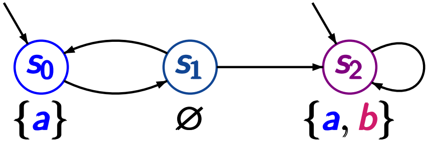

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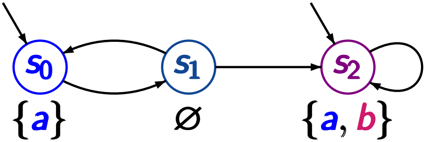

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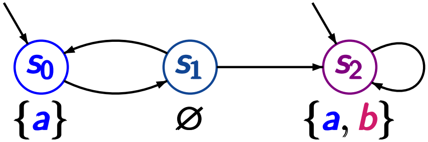

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$\pi \models (\neg b)\, \mathsf{U}\, (a \wedge b)$	as $s_0, s_1 \models \neg b$
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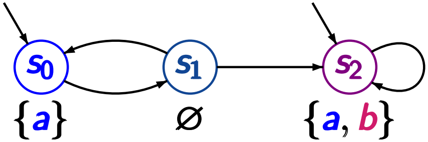

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$\pi \models (\neg b) \,\mathsf{U}\, (a \wedge b)$	as $s_0, s_1 \models \neg b$
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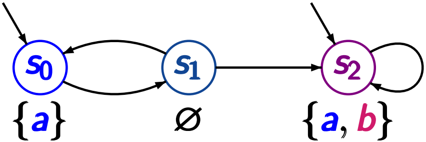
Correct or wrong ?



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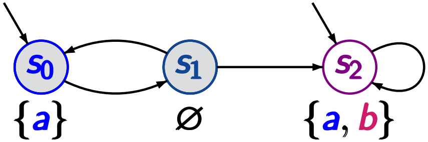

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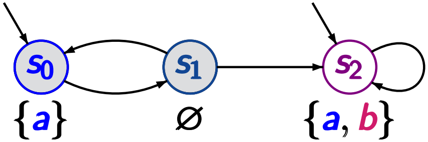

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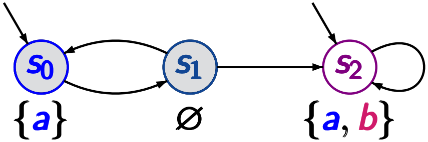

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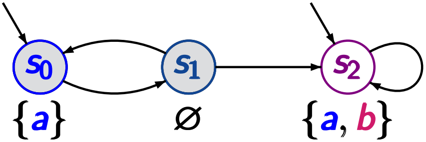

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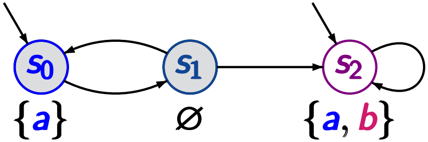
$\pi \models \Diamond b \rightarrow (a \, \mathsf{U} \, b)$?

Correct or wrong ?



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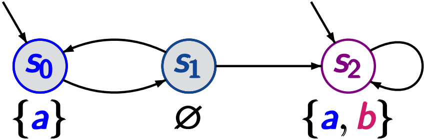

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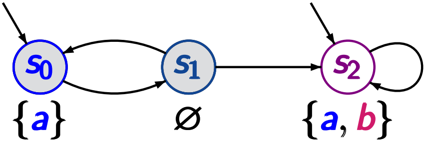

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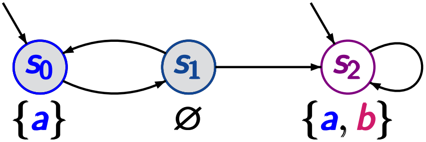

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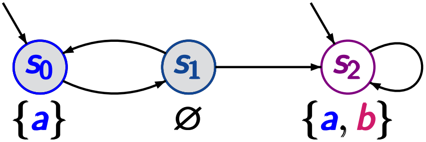

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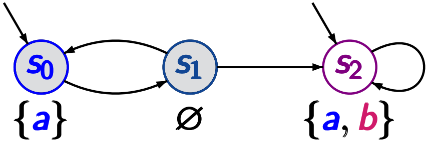

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$\pi \models \Box\Diamond a$?

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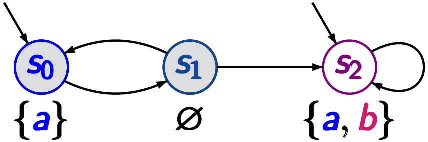

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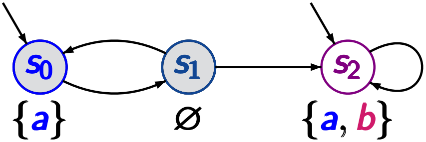

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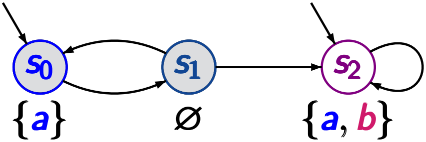

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path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$ $trace(\pi) = (\{a\}\, \varnothing)^\omega$

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$\pi \models \bigcirc \bigcirc \neg b$	as $s_0 \models \neg b$
$\pi \not\models \Box a$	as $s_1 \not\models a$
$\pi \models \Box\Diamond a$	as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often
$\pi \not\models \Diamond\Box a$	as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0 \, A_1 \, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$:

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$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \lozenge\varphi$	iff	there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \square\varphi$	iff	for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \square\lozenge\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

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$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box\varphi$	iff	for almost all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$

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↑

satisfaction relation for LT properties

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interpretation of φ over states:

$$
\begin{aligned}
s \models \varphi \quad & \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s) \\
& \text{iff} \quad s \models Words(\varphi) \\
& \text{iff} \quad Traces(s) \subseteq Words(\varphi)
\end{aligned}
$$

Interpretation of LTL formulas over TS

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Interpretation of LTL formulas over TS

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LTL formula φ over AP

$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

iff $trace(\pi) \models \varphi$ for all $\pi \in Paths(\mathcal{T})$

Interpretation of LTL formulas over TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
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LTL formula φ over AP

$$\begin{aligned}\mathcal{T} \models \varphi \quad &\text{iff} \quad s_0 \models \varphi \text{ for all } s_0 \in S_0 \\ &\text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(\mathcal{T}) \\ &\text{iff} \quad Traces(\mathcal{T}) \subseteq Words(\varphi)\end{aligned}$$

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LTL formula φ over AP

$$
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& \text{iff} \quad Traces(\mathcal{T}) \subseteq Words(\varphi) \\
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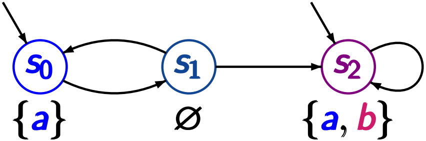
\end{aligned}
$$

↑

satisfaction relation for LT properties

Which formulas hold for $\mathcal{T}$?



$AP = \{a, b\}$

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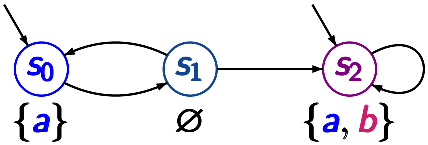

$AP = \{a, b\}$

$\mathcal{T} \models a$

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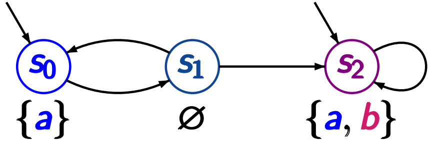

$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

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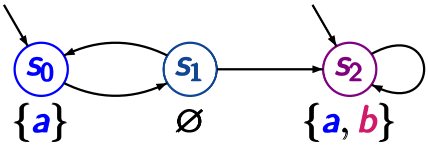
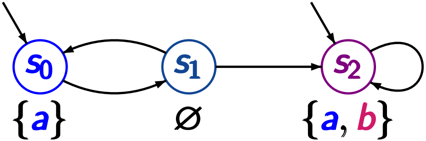


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

$\mathcal{T} \models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?



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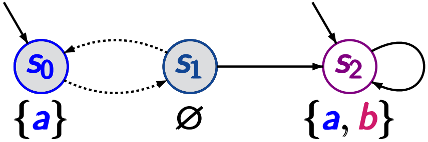

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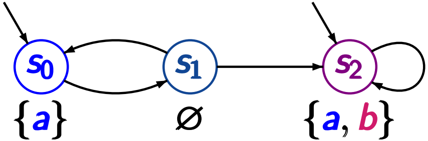

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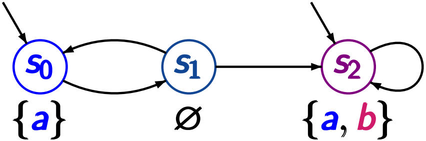

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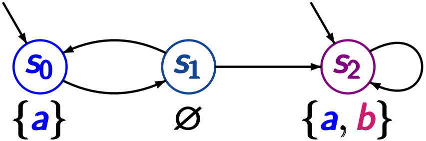

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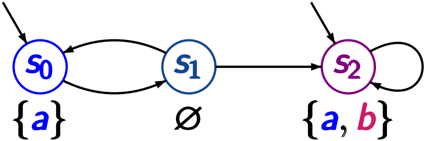

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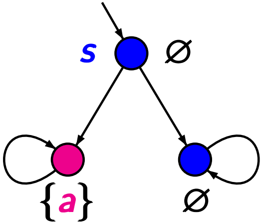

$s \not\models \Diamond a$ and $s \not\models \neg\Diamond a$

Provide an LTL formula over $AP = \{a, b\}$ for ...

- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^{\omega}$ such that:

$$\forall i \geq 0.\ (\ a \in A_i \implies i \geq 1 \wedge b \in A_{i-1}\ )$$

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where $n_1, n_2, n_3, \ldots \geq 0$

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Self-duality of the next operator

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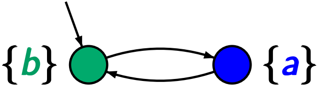

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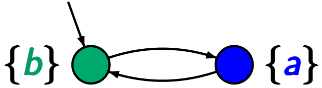

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similarly: $\Box(\varphi \wedge \psi) \;\equiv\; \Box\varphi \wedge \Box\psi$

$\Box(\varphi \vee \psi) \;\not\equiv\; \Box\varphi \vee \Box\psi$

Expansion laws

Expansion law for U

until: $$\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

Expansion laws for U and $\Diamond$

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always: $\Box\psi \equiv$ **?**

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&\equiv \psi \wedge \neg\bigcirc\Diamond\neg\psi \quad \longleftarrow \boxed{\text{double negation}}
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$\longleftarrow$ self duality of $\bigcirc$

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&\equiv \psi \wedge \bigcirc\Box\psi \qquad \longleftarrow \boxed{\text{definition of } \Box}
\end{aligned}$$

Expansion laws are fixed point equations

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...don't yield a complete characterization, e.g.,

$$\mathit{false} \equiv a \wedge \bigcirc \mathit{false}$$
$$\Box a \equiv a \wedge \bigcirc\Box a$$

consider $\psi = a$

Expansion laws are fixed point equations

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least fixed point

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greatest fixed point

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The LTL formula $\chi = \varphi \,\mathbf{U}\, \psi$ is the least solution of

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$$E = \mathit{Words}(\psi) \cup \left\{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E \right\}$$

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It even holds that $\mathit{Words}(\varphi \, \mathbf{U} \, \psi)$ least LT-property E s.t.

(1) $\mathit{Words}(\psi) \subseteq E$

(2) $\{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E \} \subseteq E$