# DSAL

- Aufgabenblatt 4
- platzkomplexität
- Master theorem.

Eingabe: Liste L mit Zahlen 1, .. $k$

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x = 0
for i in L:
    x = i
return x
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Ausgabe des letzte Element.

Platz $\in \Theta(1)$

$\in \Theta(\log k)$ $\Leftarrow$

$\in \Theta(k)$

$$T(n) = 2 \cdot T\left(\frac{n}{4}\right) + \sqrt{n}$$

$\log_4 n$

$T(n) + \sqrt{n}$ . . . . . . . . . . $\sqrt{n}$

$T\left(\frac{n}{4}\right) + \sqrt{\frac{n}{4}}$ $T\left(\frac{n}{4}\right) + \sqrt{\frac{n}{4}}$ . . . . . . $2 \cdot \sqrt{\frac{n}{4}} = \sqrt{n}$

$T\left(\frac{n}{16}\right) + \sqrt{\frac{n}{16}}$ $T\left(\frac{n}{16}\right) \sqrt{\frac{n}{16}}$ . . . . . . . . . . $\to \sqrt{n}.$

$$T(n) = \sum_{i=0}^{\log_4 n - 1} \sqrt{n} + 2^{\log_4 n}$$

$$= \sqrt{n} \cdot \underbrace{(1 + 1 + \ldots + 1)}_{\log_4(n) \text{ mal.}} + \overbrace{\left(4^{\frac{1}{2}}\right)^{\log_4 n}}^{\sqrt{n}} = \sqrt{n} \cdot \log_4 n + \sqrt{n}$$

$$\Rightarrow T(n) \in \Theta(\sqrt{n} \log_4 n)$$

$$T(n) = b \cdot T\left(\frac{n}{c}\right) + f(n) \qquad b \geq 1,\ c > 1$$

$$n^{E}, \quad E = \frac{\log(b)}{\log(c)}$$

1. $f(n) \in O(n^{E-\varepsilon}),\ \varepsilon > 0$ $\qquad T(n) \in \Theta(n^{E})$

2. $f(n) \in \Theta(n^{E})$ $\qquad T(n) \in \Theta(n^{E} \cdot \log n)$

3. $f(n) \in \underline{\Omega}(n^{E+\varepsilon}),\ \varepsilon > 0$ $\qquad T(n) \in \Theta(f(n))$

   und $b \cdot f\left(\frac{n}{c}\right) \leq d \cdot f(n),\ d < 1$

$$T(n) = 2\,T\left(\frac{n}{4}\right) + \sqrt{n}$$

$$\left.\begin{array}{l} b = 2 \\ c = 4 \end{array}\right\} \varepsilon = \frac{\log(b)}{\log(c)} = \frac{1}{2} \qquad n^{\varepsilon} = n^{\frac{1}{2}}$$

$$f(n) = n^{\frac{1}{2}} = \sqrt{n} \in \Theta(\sqrt{n})$$

$$\Rightarrow T(n) \in \Theta\left(n^{\frac{1}{2}} \log(n)\right)$$

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$$T(n) = 2 \cdot T(n)$$

$$b = 2$$

$$c = 1$$

$$T(n) = 4 \cdot T\left(\frac{n}{4}\right) + n \cdot \log_2(n)$$

$$\left.\begin{array}{l} b = 4 \\ c = 4 \end{array}\right\} \quad \mathcal{E} = \frac{\log 4}{\log 4} = 1 \qquad n^{\mathcal{E}} = n$$

$$\lim_{n \to \infty} \frac{f(n)}{n^{\mathcal{E}+\varepsilon}} = \lim_{n \to \infty} \frac{n \cdot \log_2 n}{n^{1+\varepsilon}}$$

$$= \lim_{n \to \infty} \frac{\ln(n)}{\ln(2) \; n^{\varepsilon}} \overset{\lim_{n\to\infty}}{\underset{\vee}{=}} \frac{1}{n \cdot \ln(2) \cdot n^{\varepsilon - 1}}$$

$$= \frac{1}{\ln(2) \cdot n^{\varepsilon}} = 0$$

$$\lim_{n \to \infty}$$

$$f(n) \notin \mathcal{O}(n^{\mathcal{E}+\varepsilon})$$

$\Rightarrow$ Master Theorem nicht anwendbar!

$$T(n) = 16 \cdot T\left(\frac{n}{4}\right) + \underbrace{\frac{n^3 + 2n}{n}}_{f(n)}$$

$b = 16 \geq 1$

$c = 4 > 1$

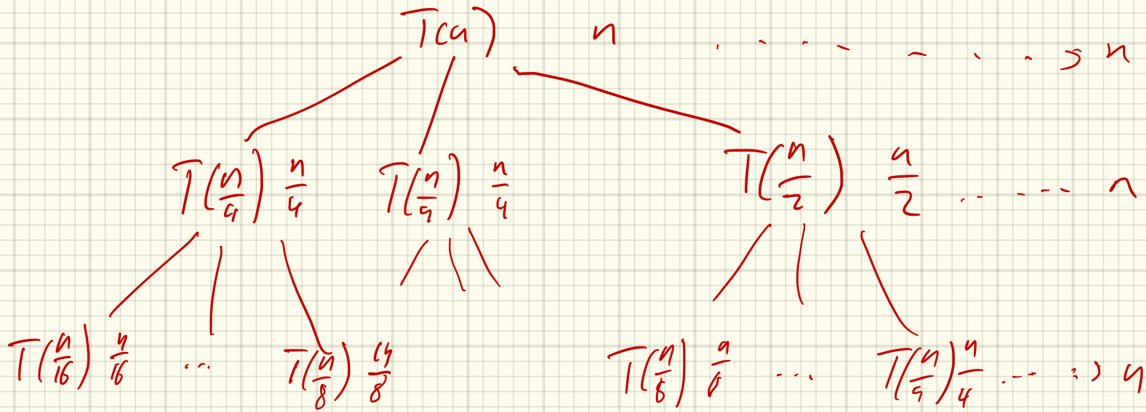
$$\varepsilon = \frac{\log(16)}{\log(4)} = \frac{4}{2} = 2$$

$$\lim_{n \to \infty} \frac{f(n)}{n^{\varepsilon}} = \lim_{n \to \infty} \frac{n^2 + 2}{n^2} = \lim_{n \to \infty} 1 + \frac{2}{n^2} = 1$$

$$\Rightarrow \underline{f(n) \in \Theta(n^{\varepsilon})}$$

$$T(n) \in \Theta\left(\underline{n^{\overset{\varepsilon}{2}} \log n}\right)$$

$$T(n) = 2 \cdot T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n.$$




Ziel: $T(n) \in O(\ldots)$

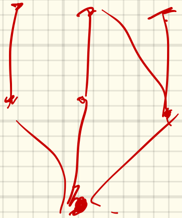
Idee: Worst-case Abschätzung.

längste Pfad: Länge $\log n - 1$ $\rightarrow$ Annahme

max. Länge = Höhe.

$n \longrightarrow n+1$

$\forall x \leq n \longrightarrow n.$



$$T(n) = \underbrace{\sum_{i=0}^{\log n - 1} n}_{\text{"= } n \log n \text{ ?"}} + \underbrace{3^{\log n}}_{3^{\log_2 n} = 3^{\frac{\log_3 n}{\log_3 2}} = n^{\frac{1}{\log_3 2}} < n^{1}}$$

Vermutung: $T(n) \in \mathcal{O}(n \log n)$

Beweis: Übungsblatt 5.

Erweitertes Master theorem:

Gegeben
$$T(n) = \begin{cases} g(n) & 1 \le n \le n_0 \\ b \cdot T\left(\frac{n}{c}\right) + f(n) & \text{für } n > n_0 \end{cases}$$

- $b > 0$, $c > 1$, $n_0 \ge c$ $\rightsquigarrow$ alles Konstanten.
- $n \ge 1$ und $n \in \mathbb{R}$! • $d_1 \le g(n) \le d_2$ $\forall 1 \le n \le n_0$

$f(n) \in O(n^d)$ für ein $d \in \mathbb{R}$, $f(n) \ge 0 \; \forall n$

Eindeutige Zahl $p$ bestimmen s.d. $\frac{b}{c^p} = 1$.

Und $T(n) \in \Theta\left( x^p \left( 1 + \int_1^n \frac{f(x)}{x^{p+1}} \, dx \right)\right)$

Bsp.: $3\,T\left(\frac{n}{2}\right) + n^2$

- $b = 3$, $c = 2$ $\Rightarrow p = \log_2 3$.
- Wähle $n_0 \geq C = 2$.

  also z.B. $2$.
- Beachte $f(n) \leq f(2)$ $\forall\, 1 \leq n \leq 2$.

  Wähle $d_1 = 0$, $d_2 = f(2)$
- $n \in O(n^d)$ $d = 2$

$$T(n) = \left(n^{\log 3}\left(1 + \int_1^n \frac{x^2}{x^{1+\log 3}}\,dx\right)\right)$$

$$\Rightarrow \quad n^{\log 3}\left(1 + n^{2-\log 3}\right) = \Theta(n^2).$$

# $(\text{Erweiterte})^2$ Mastertheorem.

$$T(n) \begin{cases} g(n) & 1 \leq n \leq n_0 \\ \sum_{i=1}^{k} b_i \cdot T\left(\frac{n}{c_i}\right) + f(n) \end{cases}$$

$b_i > 0, \quad c_i > 1, \quad n_0 \geq \max_{i \leq k} b_i$

$p$ Eindeutige Lösung $\sum b_i \left(\frac{1}{c_i^{\,p}}\right) = 1$

Bsp $\quad T(n) = 2 \cdot T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n$

$b_1 = 2 \qquad c_1 = 4$

$b_2 = 1 \qquad c_2 = 2$

$\Rightarrow \quad 2 \cdot \frac{1}{4^p} + \frac{1}{2}^p = 1$

$= 2^{1-2p} + 2^{-p} = 1.$

$\Rightarrow. \ p = 1.$

Überprüfe alle Bedingungen!

$$T(n) \in \Theta\left(n^1 \cdot \left(1 + \int_1^n \frac{\overset{= x}{f(x)}}{x^{p+1}}\, dx\right)\right)$$

$$= \Theta\left(n + n \int \frac{1}{x} dx\right) \to \Theta(n \cdot \log n).$$

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(Erweitertes)$^3$ MT.

$$T(n) \begin{cases} g(n) & 1 \leq n \leq n_0 \\ \sum b_i \cdot T\left(\frac{n}{c_i} + h_i(n)\right) & n \geq n_0 \end{cases}$$

sodass $\exists \varepsilon > 0 \quad |h_i(n)| \leq \frac{n}{\log^{1+\varepsilon} n}$

insbesondere 4 Bedingungen an $n_0$.

$$\left\lceil \frac{n}{4} \right\rceil = \frac{n}{4} + \underbrace{\left\lceil \frac{n}{4} \right\rceil - \frac{n}{4}}_{h(n)} \qquad |h(n)| \leq \frac{n}{\log^{1+\varepsilon} n}$$