Compiler Construction

Lecture 5: Syntax Analysis I (Introduction)

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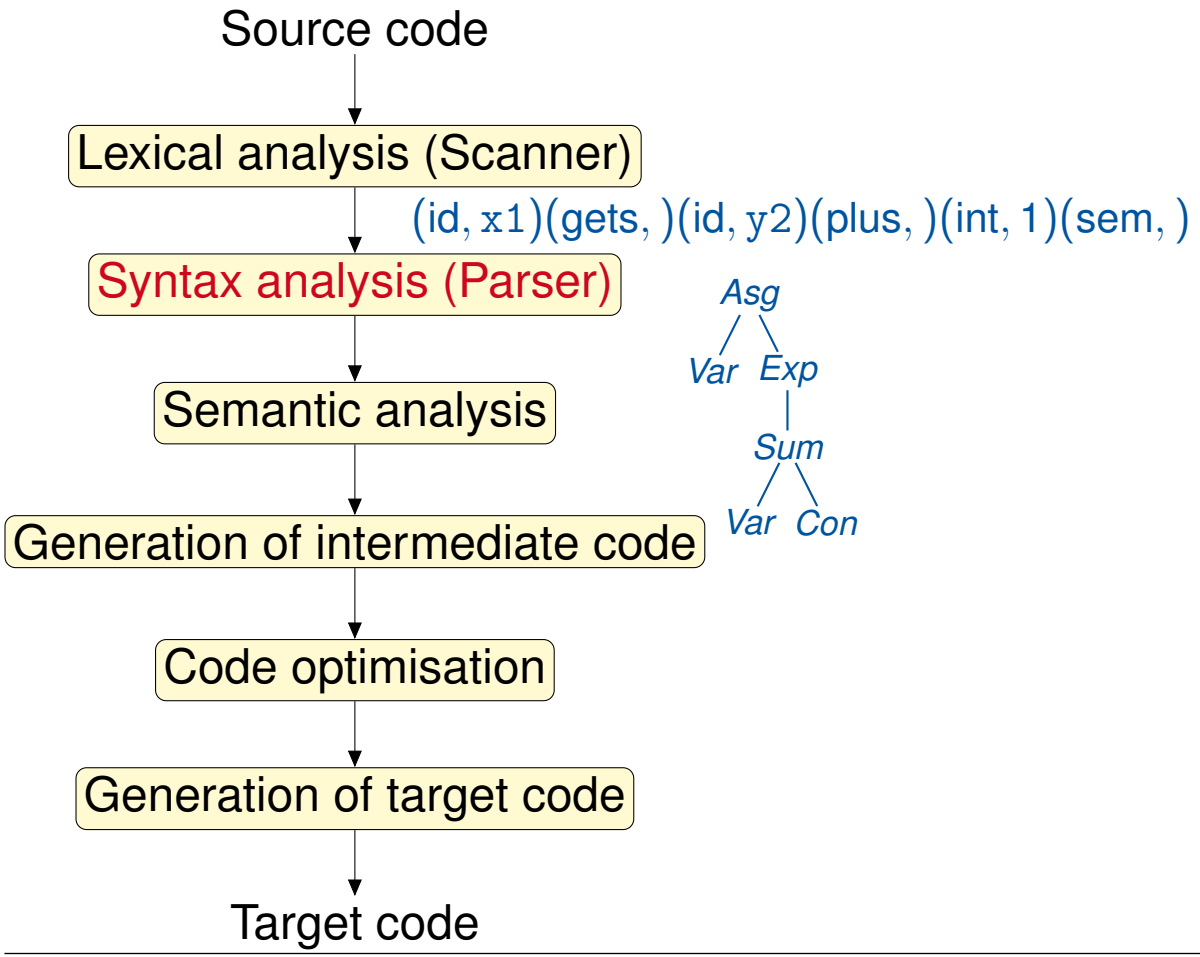
Conceptual Structure of a Compiler



context-free grammars/
pushdown automata

Problem Statement

Syntactic Structures

From Merriam-Webster's Online Dictionary

Syntax: the way in which linguistic elements (as words) are put together to form constituents (as phrases or clauses)

- **Starting point:** sequence of symbols as produced by the scanner
  - here: ignore attribute information
  - Σ (finite) set of tokens (= syntactic atoms/terminal symbols, (e.g., $\{\mathrm{id}, \mathrm{if}, \mathrm{int}, \ldots\}$)
  - $w \in \Sigma^*$ token sequence (obviously, not every $w \in \Sigma^*$ forms a valid program)
- **Syntactic units:**

  atomic: keywords, variable/type/procedure/... identifiers, numerals, arithmetic/Boolean operators, ...

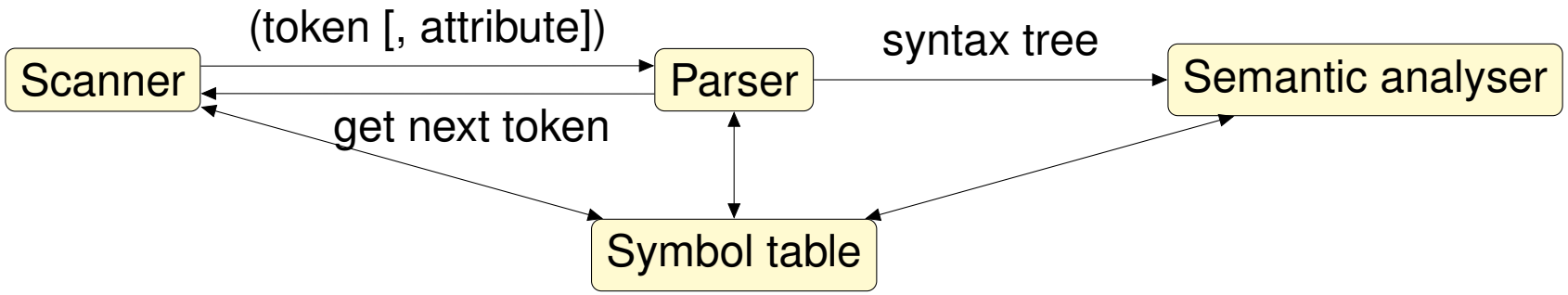
  complex: declarations, arithmetic/Boolean expressions, statements, ...
- **Observation:** the hierarchical structure of (complex) syntactic units can be described by context-free grammars

Syntax Analysis

Definition 5.1

The goal of syntax analysis is to determine the syntactic structure of a program, given by a token sequence, according to a context-free grammar.

The corresponding program is called a parser:



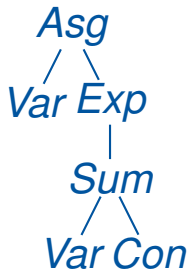
Example:

$\ldots\,␣\texttt{x1}␣\texttt{:=y2+}␣\texttt{1}␣\texttt{;}␣\ldots$

$\downarrow$ Scanner

$\ldots (\text{id}, p_1)(\text{gets},)(\text{id}, p_2)(\text{plus},)(\text{int}, 1)(\text{sem},)\ldots$

Parser $\longrightarrow$



Context-Free Grammars I

Definition 5.2 (Syntax of context-free grammars)

A context-free grammar (CFG) (over Σ) is a quadruple $G = \langle N, \Sigma, P, S \rangle$ where

- N is a finite set of nonterminal symbols,
- Σ is a (finite) alphabet of terminal symbols (disjoint from N),
- P is a finite set of production rules of the form $A \to \alpha$ where
  - $A \in N$ and
  - $\alpha \in X^*$ for $X := N \cup \Sigma$,
- $S \in N$ is a start symbol.

The set of all context-free grammars over Σ is denoted by CFG_Σ.

Remarks: as denotations we generally use

- $A, B, C, \ldots \in N$ for nonterminal symbols
- $a, b, c, \ldots \in \Sigma$ for terminal symbols
- $u, v, w, x, y, \ldots \in \Sigma^*$ for terminal words
- $\alpha, \beta, \gamma, \ldots \in X^*$ for sentences

Context-Free Grammars II

Context-free grammars generate context-free languages:

Definition 5.3 (Semantics of context-free grammars)

Let $G = \langle N, \Sigma, P, S \rangle$ be a context-free grammar.

- The derivation relation $\Rightarrow \subseteq X^+ \times X^*$ of G is defined by

$$\alpha \Rightarrow \beta \text{ iff there exist } \alpha_1, \alpha_2 \in X^*, A \to \gamma \in P \text{ such that } \alpha = \alpha_1 A \alpha_2 \text{ and } \beta = \alpha_1 \gamma \alpha_2.$$

- If additionally $\alpha_1 \in \Sigma^*$ or $\alpha_2 \in \Sigma^*$, then we respectively write $\alpha \Rightarrow_l \beta$ or $\alpha \Rightarrow_r \beta$ (leftmost/rightmost derivation).
- The language generated by G is given by

$$L(G) := \{w \in \Sigma^* \mid S \Rightarrow^* w\}.$$

- If a language $L \subseteq \Sigma^*$ is generated by some $G \in CFG_\Sigma$, then L is called context-free. The set of all context-free languages over Σ is denoted by CFL_Σ.

Remark: obviously, $L(G) = \{w \in \Sigma^* \mid S \Rightarrow_l^* w\} = \{w \in \Sigma^* \mid S \Rightarrow_r^* w\}$

Context-Free Languages

Example 5.4

The grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ over $\Sigma := \{a, b\}$, given by the productions

$$S \to aSb \mid \varepsilon,$$

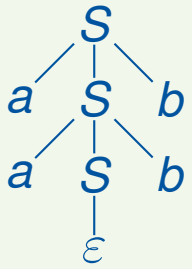
generates the context-free (and non-regular) language

$$L = \{a^n b^n \mid n \in \mathbb{N}\}.$$

The example derivation

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

can be represented by the following syntax tree for $aabb$:



Syntax Trees, Derivations, and Words

Observations

1. Every syntax tree yields exactly one word (= concatenation of terminal leaves).
2. Every syntax tree corresponds to exactly one leftmost derivation, and vice versa.
3. Every syntax tree corresponds to exactly one rightmost derivation, and vice versa.

Thus: syntax trees are uniquely representable by leftmost/rightmost derivations.

But: a word can have several syntax trees (see next slide)

Ambiguity of CFGs and CFLs I

Definition 5.5 (Ambiguity)

- A context-free grammar $G \in CFG_\Sigma$ is called unambiguous if every word $w \in L(G)$ has exactly one syntax tree. Otherwise it is called ambiguous.
- A context-free language $L \in CFL_\Sigma$ is called inherently ambiguous if every grammar $G \in CFG_\Sigma$ with $L(G) = L$ is ambiguous.

Example 5.6

on the board

Corollary 5.7

A grammar $G \in CFG_\Sigma$ is unambiguous
iff every word $w \in L(G)$ has exactly one leftmost derivation
iff every word $w \in L(G)$ has exactly one rightmost derivation.

Ambiguity of CFGs and CFLs II

Theorem 5.8

It is generally undecidable whether a given CFG is ambiguous or not.

Proof (idea).

Reduction from Post Correspondence Problem: given instance $(\vec{x}, \vec{y})$ of PCP, construct CFG G with two "branches" $S \to X \mid Y$ that respectively enumerate all $\vec{x}/\vec{y}$-concatenations (plus some index information).

Result: G is ambiguous iff $(\vec{x}, \vec{y})$ has a solution
(see [Hopcroft/Motwani/Ullman 2001, Section 9.5.2] for details) □

Remark: resolution of ambiguities by parser (later):

- `yacc`: operator precedences and associativities
- `ANTLR`: predicates

The Word Problem for Context-Free Languages

Problem 5.9 (Word problem for context-free languages)

Given $G \in CFG_\Sigma$ and $w \in \Sigma^*$, decide whether $w \in L(G)$ (and determine a corresponding syntax tree).

This problem is decidable for arbitrary CFGs:

- (for CFGs in Chomsky Normal Form)
  Using the tabular method by Cocke, Younger, and Kasami
  ("CYK Algorithm"; time/space complexity $\mathcal{O}(|w|^3)/\mathcal{O}(|w|^2)$)
- Using the predecessor method:

$$w \in L(G) \iff S \in \mathrm{pre}^*(\{w\})$$

  where $\mathrm{pre}^*(M) := \{\alpha \in X^* \mid \alpha \Rightarrow^* \beta \text{ for some } \beta \in M\}$
  (polynomial [non-linear] time complexity)

Parsing Context-Free Languages

Goal: exploit the special syntactic structures as present in programming languages (usually: no ambiguities) to devise parsing methods which are based on deterministic pushdown automata with linear space and time complexity

Two approaches:

Top-down parsing: construction of syntax tree from the root towards the leaves, representation as leftmost derivation

Bottom-up parsing: construction of syntax tree from the leaves towards the root, representation as (reversed) rightmost derivation

Leftmost/Rightmost Analysis I

Goal: compact representation of left-/rightmost derivations by index sequences

Definition 5.10 (Leftmost/rightmost analysis)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ where $P = \{\pi_1, \ldots, \pi_p\}$.

- If $i \in [p]$, $\pi_i = A \to \gamma$, $w \in \Sigma^*$, and $\alpha \in X^*$, then we write

$$wA\alpha \overset{i}{\Rightarrow}_l w\gamma\alpha \quad \text{and} \quad \alpha Aw \overset{i}{\Rightarrow}_r \alpha\gamma w.$$

- If $z = i_1 \ldots i_n \in [p]^*$, we write $\alpha \overset{z}{\Rightarrow}_l \beta$ if there exist $\alpha_0, \ldots, \alpha_n \in X^*$ such that $\alpha_0 = \alpha$, $\alpha_n = \beta$, and $\alpha_{j-1} \overset{i_j}{\Rightarrow}_l \alpha_j$ for every $j \in [n]$ (analogously for $\overset{z}{\Rightarrow}_r$).
- An index sequence $z \in [p]^*$ is called a leftmost analysis (rightmost analysis) of α if $S \overset{z}{\Rightarrow}_l \alpha$ ($S \overset{z}{\Rightarrow}_r \alpha$), respectively.

Leftmost/Rightmost Analysis II

Example 5.11

Grammar for arithmetic expressions: G_{AE} :

$$E \to E{+}T \mid T \quad (1,2)$$
$$T \to T{*}F \mid F \quad (3,4)$$
$$F \to (E) \mid \mathtt{a} \mid \mathtt{b} \quad (5,6,7)$$

Leftmost derivation of $(\mathtt{a})*\mathtt{b}$:

$$E \overset{2}{\Rightarrow}_l T \overset{3}{\Rightarrow}_l T{*}F \overset{4}{\Rightarrow}_l F{*}F \overset{5}{\Rightarrow}_l (E){*}F$$
$$\overset{2}{\Rightarrow}_l (T){*}F \overset{4}{\Rightarrow}_l (F){*}F \overset{6}{\Rightarrow}_l (\mathtt{a}){*}F \overset{7}{\Rightarrow}_l (\mathtt{a}){*}\mathtt{b}$$

$\Longrightarrow$ leftmost analysis: 23452467

Rightmost derivation of $(\mathtt{a})*\mathtt{b}$:

$$E \overset{2}{\Rightarrow}_r T \overset{3}{\Rightarrow}_r T{*}F \overset{7}{\Rightarrow}_r T{*}\mathtt{b} \overset{4}{\Rightarrow}_r F{*}\mathtt{b}$$
$$\overset{5}{\Rightarrow}_r (E){*}\mathtt{b} \overset{2}{\Rightarrow}_r (T){*}\mathtt{b} \overset{4}{\Rightarrow}_r (F){*}\mathtt{b} \overset{6}{\Rightarrow}_r (\mathtt{a}){*}\mathtt{b}$$

$\Longrightarrow$ rightmost analysis: 23745246

Reducedness of Context-Free Grammars

General assumption in the following: every grammar is reduced

Definition 5.12 (Reduced CFG)

A grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ is called reduced if for every $A \in N$ there exist $\alpha, \beta \in X^*$ and $w \in \Sigma^*$ such that

$$S \Rightarrow^* \alpha A \beta \quad (A \text{ reachable}) \text{ and}$$
$$A \Rightarrow^* w \quad (A \text{ productive}).$$

Top-Down Parsing

Approach:

1. Given $G \in CFG_\Sigma$, construct a nondeterministic pushdown automaton (PDA) which accepts $L(G)$ and which additionally computes corresponding leftmost derivations (similar to the proof of "$L(CFG_\Sigma) \subseteq L(PDA_\Sigma)$")
   - input alphabet: Σ
   - pushdown alphabet: X
   - output alphabet: $[p]$
   - state set: not required
2. Remove nondeterminism by supporting lookahead on the input:
   $G \in LL(k)$ iff $L(G)$ recognisable by deterministic PDA with lookahead of k symbols

The Nondeterministic Top-Down Automaton I

Definition 5.13 (Nondeterministic top-down parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$. The nondeterministic top-down parsing automaton of G, $NTA(G)$, is defined by the following components.

- Input alphabet: Σ
- Pushdown alphabet: X
- Output alphabet: $[p]$
- Configurations: $\Sigma^* \times X^* \times [p]^*$ (top of pushdown to the left)
- Transitions for $w \in \Sigma^*$, $\alpha \in X^*$, and $z \in [p]^*$:
  expansion steps: if $\pi_i = A \to \beta$, then $(w, A\alpha, z) \vdash (w, \beta\alpha, zi)$
  matching steps: for every $a \in \Sigma$, $(aw, a\alpha, z) \vdash (w, \alpha, z)$
- Initial configuration for $w \in \Sigma^*$: (w, S, ε)
- Final configurations: $\{\varepsilon\} \times \{\varepsilon\} \times [p]^*$

Remark: $NTA(G)$ is nondeterministic iff G contains $A \to \beta \mid \gamma$

The Nondeterministic Top-Down Automaton II

Example 5.14

Grammar for arithmetic expressions (cf. Example 5.11):

$$
\begin{array}{lll}
G_{AE}: & E \to E\texttt{+}T \mid T & (1,2) \\
& T \to T\texttt{*}F \mid F & (3,4) \\
& F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

Leftmost analysis of `(a)*b`:

$$
\begin{array}{lll}
 & (\texttt{(a)*b}, E & , \varepsilon) \\
\vdash & (\texttt{(a)*b}, T & , 2) \\
\vdash & (\texttt{(a)*b}, T\texttt{*}F & , 23) \\
\vdash & (\texttt{(a)*b}, F\texttt{*}F & , 234) \\
\vdash & (\texttt{(a)*b}, \texttt{(}E\texttt{)*}F & , 2345) \\
\vdash & (\texttt{ a)*b}, E\texttt{)*}F & , 2345) \\
\vdash & (\texttt{ a)*b}, T\texttt{)*}F & , 23452) \\
\vdash & (\texttt{ a)*b}, F\texttt{)*}F & , 234524) \\
\vdash & (\texttt{ a)*b}, \texttt{a)*}F & , 2345246) \\
\vdash & (\texttt{)*b}, \texttt{)*}F & , 2345246) \\
\vdash & (\texttt{ *b}, \texttt{*}F & , 2345246) \\
\vdash & (\texttt{ b}, F & , 2345246) \\
\vdash & (\texttt{ b}, \texttt{b} & , 23452467) \\
\vdash & (\varepsilon, \varepsilon & , 23452467)
\end{array}
$$