Compiler Construction

Lecture 3: Lexical Analysis II (Extended Matching Problem)

Summer Semester 2017

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https://moves.rwth-aachen.de/teaching/ss-17/cc/

Der RWTH Hackathon am 12./13.05.2017 erwartet euch!

Veranstaltungsinfo

- Von Freitag, **12.05.2017 15 Uhr**
- Bis Samstag **13.05.2017 14 Uhr**
- Prämierung der Gewinner: Samstag ab 13:00 Uhr
- In der **Couvenhalle** (Nähe Hauptgebäude)

Ablauf und Preise

- Bearbeitung der Challenge bis spät in den Abend
- Büffet, **Snacks und Drinks** stehen für euch bereit
- Abgabe des Ergebnisses am nächsten Morgen
- **Preise** für die Gewinnerteams und ein Sonderpreis

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Unlock the Blockchain Technology
Develop a feature management solution with Smart Contracts based on Blockchain for an IoT device to enable features on demand or managed according to usage patterns.

Improve traffic safety
Design and implement new use cases on city traffic (simulated cars and guided ambulances) to improve city safety using cloud-based Internet of Things

Discover e-commerce technology
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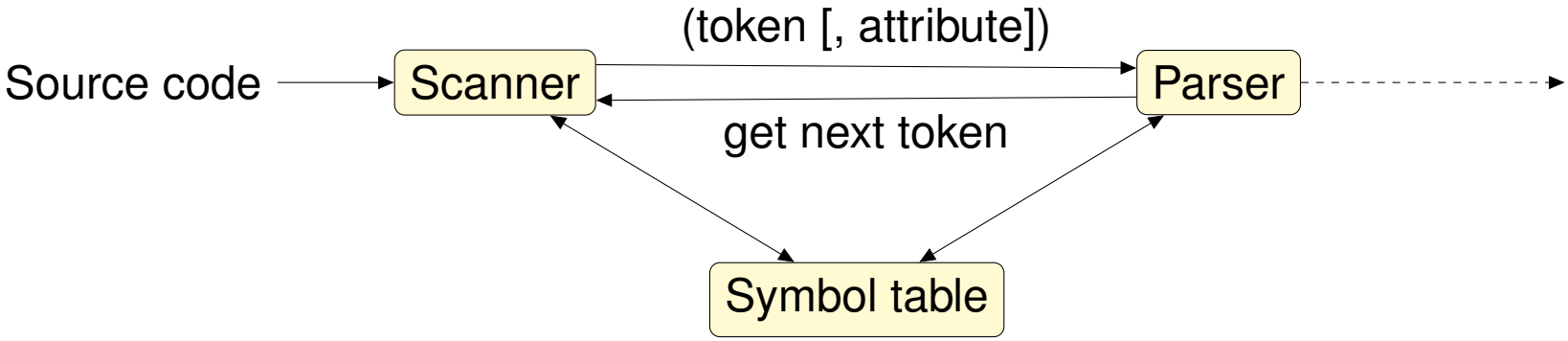
Lexical Analysis

Definition

The goal of lexical analysis is the decomposition a source program into a sequence of lexemes and their transformation into a sequence of symbols.

The corresponding program is called a scanner (or lexer):



Example:

$$\ldots\,␣\texttt{x1}␣\texttt{:=y2+}␣\texttt{1}␣\texttt{;}␣\ldots$$

$$\Downarrow$$

$$\ldots(\text{id}, p_1)(\text{gets},)(\text{id}, p_2)(\text{plus},)(\text{int}, 1)(\text{sem},)\ldots$$

Recap: Lexical Analysis

Regular Expressions

Definition (Syntax of regular expressions)

Given some alphabet Ω, the set of regular expressions over Ω, RE_Ω, is the least set with

- $\emptyset \in RE_\Omega$,
- $\Omega \subseteq RE_\Omega$, and
- whenever $\alpha, \beta \in RE_\Omega$, also $\alpha \mid \beta, \alpha \cdot \beta, \alpha^* \in RE_\Omega$.

Remarks:

- abbreviations: $\alpha^+ := \alpha \cdot \alpha^*$, $\varepsilon := \emptyset^*$
- $\alpha \cdot \beta$ often written as $\alpha\beta$
- Binding priority: $^* > \cdot > \mid$ (i.e., $a \mid b \cdot c^* := a \mid (b \cdot (c^*))$)

The DFA Method I

Known from *Formal Systems, Automata and Processes*:

Algorithm (DFA method)

Input: regular expression $\alpha \in RE_\Omega$, input string $w \in \Omega^*$

Procedure: 1. using Kleene's Theorem, construct $\mathfrak{A}_\alpha \in NFA_\Omega$ such that $L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$

2. apply powerset construction (cf. Definition 3.5) to obtain $\mathfrak{A}'_\alpha = \langle Q', \Omega, \delta', q'_0, F' \rangle \in DFA_\Omega$ with $L(\mathfrak{A}'_\alpha) = L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$
3. solve the matching problem by deciding whether $\delta'^*(q'_0, w) \in F'$

Output: "yes" or "no"

The DFA Method II

The powerset construction involves the following concept:

Definition (ε-closure)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in NFA_\Omega$. The ε-closure $\varepsilon(T) \subseteq Q$ of a subset $T \subseteq Q$ is the least set with (1) $T \subseteq \varepsilon(T)$ and (2) if $q \in \varepsilon(T)$, then $\delta(q, \varepsilon) \subseteq \varepsilon(T)$

Definition (Powerset construction)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in NFA_\Omega$. The powerset automaton $\mathfrak{A}' = \langle Q', \Omega, \delta', q_0', F' \rangle \in DFA_\Omega$ is defined by

- $Q' := 2^Q$
- $q_0' := \varepsilon(\{q_0\})$
- $\forall T \subseteq Q, a \in \Omega : \delta'(T, a) := \varepsilon\left(\bigcup_{q \in T} \delta(q, a)\right)$
- $F' := \{T \subseteq Q \mid T \cap F \neq \emptyset\}$

The Extended Matching Problem I

Definition 3.1

Let $n \geq 1$ and $\alpha_1, \ldots, \alpha_n \in RE_\Omega$ with $\varepsilon \notin [\![\alpha_i]\!] \neq \emptyset$ for every $i \in [n]$ (where $[n] := \{1, \ldots, n\}$). Let $\Sigma := \{T_1, \ldots, T_n\}$ be an alphabet of corresponding tokens and $w \in \Omega^+$. If $w_1, \ldots, w_k \in \Omega^+$ such that

- $w = w_1 \ldots w_k$ and
- for every $j \in [k]$ there exists $i_j \in [n]$ such that $w_j \in [\![\alpha_{i_j}]\!]$,

then

- $(w_1, \ldots, w_k)$ is called a decomposition and
- $(T_{i_1}, \ldots, T_{i_k})$ is called an analysis

of w w.r.t. $\alpha_1, \ldots, \alpha_n$.

Problem 3.2 (Extended matching problem)

Given $\alpha_1, \ldots, \alpha_n \in RE_\Omega$ and $w \in \Omega^+$, decide whether there exists a decomposition of w w.r.t. $\alpha_1, \ldots, \alpha_n$ and determine a corresponding analysis.

The Extended Matching Problem II

Observation: neither the decomposition nor the analysis are uniquely determined

Example 3.3

1. – $\alpha = a^+$
   – $w = aa$
   $\Rightarrow$ two decompositions (aa) and (a, a) with respective (unique) analyses (T_1) and (T_1, T_1)
2. – $\alpha_1 = a \mid b$, $\alpha_2 = a \mid c$
   – $w = a$
   $\Rightarrow$ unique decomposition (a) but two analyses (T_1) and (T_2)

Goal: make both unique $\Longrightarrow$ deterministic scanning

Ensuring Uniqueness

Two principles

1. Principle of the longest match (“maximal munch tokenisation”)
   - for uniqueness of decomposition
   - make lexemes as long as possible
   - motivated by practical considerations:
     - every proper prefix of an identifier is also an identifier
     - real-valued constants contain integer constants as prefixes
2. Principle of the first match
   - for uniqueness of analysis
   - choose first matching regular expression (in the given order)
   - therefore: arrange keywords before identifiers (if keywords protected)

Remark: uniqueness of analysis could also be achieved by requiring disjointness of symbol classes (i.e., $[\![\alpha_i]\!] \cap [\![\alpha_j]\!] = \emptyset$ for $i \neq j$)

- for example, $\mathit{Id} := (\mathtt{a} \mid \ldots)(\mathtt{a} \mid \ldots \mid \mathtt{0} \mid \ldots)^* \setminus \{\mathtt{if}, \mathtt{while}, \mathtt{begin}, \ldots\}$
- **but:** expensive implementation due to product construction ($A \setminus B = A \cap \overline{B}$; cf. Def. 3.10)

Principle of the Longest Match

Definition 3.4 (Longest-match decomposition)

A decomposition $(w_1, \ldots, w_k)$ of $w \in \Omega^+$ w.r.t. $\alpha_1, \ldots, \alpha_n \in RE_\Omega$ is called a longest-match (LM) decomposition if, for every $i \in [k]$, $x \in \Omega^+$, and $y \in \Omega^*$,

$$w = w_1 \ldots w_i xy \implies \text{there is no } j \in [n] \text{ such that } w_i x \in [\![\alpha_j]\!]$$

Corollary 3.5

Given w and $\alpha_1, \ldots, \alpha_n$,

- *at most one LM decomposition of w exists (clear by definition) and*
- *it is possible that w has a decomposition but no LM decomposition (see following example).*

Example 3.6

$w = aab$, $\alpha_1 = a^+$, $\alpha_2 = ab$

$\implies$ (a, ab) is a decomposition but no LM decomposition exists

First-Longest-Match Analysis

Principle of the First Match

Problem: a (unique) LM decomposition can have several associated analyses (since $[\![\alpha_i]\!] \cap [\![\alpha_j]\!] \neq \emptyset$ with $i \neq j$ is possible; cf. keyword/identifier problem)

Definition 3.7 (First-longest-match analysis)

Let $(w_1, \ldots, w_k)$ be the LM decomposition of $w \in \Omega^+$ w.r.t. $\alpha_1, \ldots, \alpha_n \in RE_\Omega$. Its first-longest-match analysis (FLM analysis) $(T_{i_1}, \ldots, T_{i_k})$ is determined by

$$i_j := \min\{l \in [n] \mid w_j \in [\![\alpha_l]\!]\}$$

for every $j \in [k]$.

Corollary 3.8

Given w and $\alpha_1, \ldots, \alpha_n$, there is at most one FLM analysis of w.
It exists iff the LM decomposition of w exists.

Implementation of FLM Analysis

Algorithm 3.9 (FLM analysis – overview)

Input: expressions $\alpha_1, \ldots, \alpha_n \in RE_\Omega$, tokens $\{T_1, \ldots, T_n\}$, input word $w \in \Omega^+$

Procedure:

1. for every $i \in [n]$, construct $\mathfrak{A}_i \in DFA_\Omega$ such that $L(\mathfrak{A}_i) = [\![\alpha_i]\!]$
   (see DFA method; Algorithm 3.3)
2. construct the product automaton $\mathfrak{A} \in DFA_\Omega$ such that $L(\mathfrak{A}) = \bigcup_{i=1}^{n} [\![\alpha_i]\!]$
3. partition the set of final states of $\mathfrak{A}$ to follow the first-match principle
4. extend the resulting DFA to a backtracking DFA which implements the longest-match principle
5. let the backtracking DFA run on w

Output: FLM analysis of w (if existing)

(2) The Product Automaton

Definition 3.10 (Product automaton)

Let $\mathfrak{A}_i = \langle Q_i, \Omega, \delta_i, q_0^{(i)}, F_i \rangle \in DFA_\Omega$ for every $i \in [n]$. The product automaton $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in DFA_\Omega$ is defined by

- $Q := Q_1 \times \ldots \times Q_n$
- $q_0 := (q_0^{(1)}, \ldots, q_0^{(n)})$
- $\delta((q^{(1)}, \ldots, q^{(n)}), a) := (\delta_1(q^{(1)}, a), \ldots, \delta_n(q^{(n)}, a))$
- $(q^{(1)}, \ldots, q^{(n)}) \in F$ iff there ex. $i \in [n]$ such that $q^{(i)} \in F_i$

Lemma 3.11

The above construction yields $L(\mathfrak{A}) = \bigcup_{i=1}^{n} L(\mathfrak{A}_i)$ $(= \bigcup_{i=1}^{n} [\![\alpha_i]\!])$.

Remark: similar construction for intersection ($F := F_1 \times \ldots \times F_n$)

(3) Partitioning the Final States

Definition 3.12 (Partitioning of final states)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in DFA_\Omega$ be the product automaton as constructed before. Its set of final states is partitioned into $F = \biguplus_{i=1}^{n} F^{(i)}$ by the requirement

$$(q^{(1)}, \ldots, q^{(n)}) \in F^{(i)} \iff q^{(i)} \in F_i \text{ and } \forall j \in [i-1] : q^{(j)} \notin F_j$$

(equivalently: $F^{(i)} := (Q_1 \setminus F_1) \times \ldots \times (Q_{i-1} \setminus F_{i-1}) \times F_i \times Q_{i+1} \times \ldots \times Q_n$)

Corollary 3.13

The above construction yields ($w \in \Omega^+$, $i \in [n]$):

$$\delta^*(q_0, w) \in F^{(i)} \text{ iff } w \in [\![\alpha_i]\!] \text{ and } w \notin \bigcup_{j=1}^{i-1} [\![\alpha_j]\!].$$

Definition 3.14 (Productive states)

Given $\mathfrak{A}$ as above, $q \in Q$ is called productive if there exists $w \in \Omega^*$ such that $\delta^*(q, w) \in F$. Notation: productive states $P \subseteq Q$ (and thus $F \subseteq P$).

(4) The Backtracking DFA I

Goal: extend $\mathfrak{A}$ to the backtracking DFA $\mathfrak{B}$ with output by equipping the input tape with two pointers: a backtracking head for marking the last encountered match, and a lookahead for determining the longest match.

A configuration of $\mathfrak{B}$ has three components
(remember: $\Sigma := \{T_1, \ldots, T_n\}$ denotes the set of tokens):

1. a mode $m \in \{N\} \uplus \Sigma$:
   - $m = N$ ("normal"): look for initial match (no final state reached yet)
   - $m = T \in \Sigma$: token T has been recognised, looking for possible longer match
2. an input tape $vqw \in \Omega^* \cdot Q \cdot \Omega^*$:
   - v: lookahead part of input ($v \neq \varepsilon \implies m \in \Sigma$)
   - q: current state of $\mathfrak{A}$
   - w: remaining input
3. an output tape $W \in \Sigma^* \cdot \{\varepsilon, \mathsf{lexerr}\}$:
   - Σ^*: sequence of tokens recognised so far
   - lexerr: a lexical error has occurred (i.e., a non-productive state was entered or the suffix of the input is not a valid lexeme)

(4) The Backtracking DFA II

Definition 3.15 (Backtracking DFA)

- The set of configurations of $\mathfrak{B}$ is given by
$$(\{N\} \uplus \Sigma) \times \Omega^* \cdot Q \cdot \Omega^* \times \Sigma^* \cdot \{\varepsilon, \text{lexerr}\}$$
- The initial configuration for an input word $w \in \Omega^+$ is $(N, q_0 w, \varepsilon)$.
- The transitions of $\mathfrak{B}$ are defined as follows (where $q' := \delta(q, a)$):
  - normal mode: look for initial match
$$(N, qaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} \quad (1) \\ (N, q'w, W) & \text{if } q' \in P \setminus F \quad (2) \\ \textbf{output: } W \cdot \text{lexerr} & \text{if } q' \notin P \quad (3) \end{cases}$$
  - backtrack mode: look for longest match
$$(T, vqaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} \quad (4) \\ (T, vaq'w, W) & \text{if } q' \in P \setminus F \quad (5) \\ (N, q_0 vaw, WT) & \text{if } q' \notin P \quad (6) \end{cases}$$
  - end of input:
$$\begin{aligned} (T, q, W) &\vdash \textbf{output: } WT && \text{if } q \in F && (7) \\ (N, q, W) &\vdash \textbf{output: } W \cdot \text{lexerr} && \text{if } q \in P \setminus F && (8) \\ (T, vaq, W) &\vdash (N, q_0 va, WT) && \text{if } q \in P \setminus F && (9) \end{aligned}$$

(4) The Backtracking DFA III

Lemma 3.16

Given the backtracking DFA $\mathfrak{B}$ as before and $w \in \Omega^+$,

$$(N, q_0 w, \varepsilon) \vdash^* \begin{cases} W \in \Sigma^* & \text{iff } W \text{ is the FLM analysis of } w \\ W \cdot \text{lexerr} & \text{iff no FLM analysis of } w \text{ exists} \end{cases}$$

Proof.

(omitted) □

Example 3.17

- $\Omega = \{a, b\}$, $w = baa$
- $n = 3$, $\Sigma = \{T_1, T_2, T_3\}$
- $\alpha_1 = a$ (“keyword”), $\alpha_2 = a^+ b$ (“identifier”), $\alpha_3 = b$ (“operator”)

(on the board)