



# Compiler Construction

**Lecture 17: Code Generation III (Jumping Code & Static Data Structures)**

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**Thomas Noll**

**Software Modeling and Verification Group**

**RWTH Aachen University**

<https://moves.rwth-aachen.de/teaching/ss-17/cc/>

# Recap: Syntax of EPL

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## Outline of Lecture 17

### Recap: Syntax of EPL

Boolean Expressions with Sequential Semantics

Implementation of Data Structures

Static Data Structures

Modifying the Abstract Machine

Modifying the Symbol Table

The Translation

A Translation Example

# Recap: Syntax of EPL

## Syntax of EPL

### Definition (Syntax of EPL)

The **syntax of EPL** is defined as follows:

$$\begin{aligned} \mathbb{Z} : & \quad z && \quad (* z \text{ is an integer} *) \\ Ide : & \quad I && \quad (* I \text{ is an identifier} *) \\ AExp : & \quad A ::= z \mid I \mid A_1 + A_2 \mid \dots \\ BExp : & \quad B ::= A_1 < A_2 \mid \text{not } B \mid B_1 \text{ and } B_2 \mid B_1 \text{ or } B_2 \\ Cmd : & \quad C ::= I := A \mid C_1 ; C_2 \mid \text{if } B \text{ then } C_1 \text{ else } C_2 \mid \text{while } B \text{ do } C \mid I() \\ Dcl : & \quad D ::= D_C D_V D_P \\ & \quad D_C ::= \varepsilon \mid \text{const } l_1 := z_1, \dots, l_n := z_n; \\ & \quad D_V ::= \varepsilon \mid \text{var } l_1, \dots, l_n; \\ & \quad D_P ::= \varepsilon \mid \text{proc } l_1 ; K_1 ; \dots ; \text{proc } l_n ; K_n; \\ Blk : & \quad K ::= D C \\ Pgm : & \quad P ::= \text{in/out } l_1, \dots, l_n ; K. \end{aligned}$$

# Recap: Syntax of EPL

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## Translation of Boolean Expressions

### Definition (Translation of Boolean expressions)

The mapping

$$bt : BExp \times Tab \times PC \times Lev \dashrightarrow AM$$

(“Boolean expression translation”) is defined by

$$bt(A_1 < A_2, st, a, lev) := at(A_1, st, a, lev); at(A_2, st, a', lev); a'' : LT;$$

$$bt(\text{not } B, st, a, lev) := bt(B, st, a, lev); a' : NOT;$$

$$bt(B_1 \text{ and } B_2, st, a, lev) := bt(B_1, st, a, lev); bt(B_2, st, a', lev); a'' : AND;$$

$$bt(B_1 \text{ or } B_2, st, a, lev) := bt(B_1, st, a, lev); bt(B_2, st, a', lev); a'' : OR;$$

# Boolean Expressions with Sequential Semantics

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## Boolean Expressions with Sequential Semantics

**So far:** Boolean expressions with **strict** semantics ( $\perp$  = nontermination/runtime error)

$$b_1 \diamond \perp = \perp$$

$$\perp \diamond b_2 = \perp$$

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$$b_1 \diamond \perp = \perp$$

$$\perp \diamond b_2 = \perp$$

**Now:** Boolean expressions with **sequential** semantics

$$\text{false} \wedge \perp = \text{false}$$

$$\text{true} \vee \perp = \text{true}$$

$$\perp \diamond b = \perp$$

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**Now:** Boolean expressions with **sequential** semantics

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$$\text{true} \vee \perp = \text{true}$$

$$\perp \diamond b = \perp$$

### Idea:

- employ jumping rather than Boolean instructions (“**jumping code**”)
- equip `bt` and `ct` with **two additional address parameters**:
  - $a_t$ : target address for **true**
  - $a_f$ : target address for **false**



# Boolean Expressions with Sequential Semantics

## Jumping Code for Boolean Expressions

### Definition 17.1 (Jumping code for Boolean expressions)

The mapping

$$\text{sbt} : BExp \times Tab \times PC^3 \times Lev \dashrightarrow AM$$

(“sequential Boolean expression translation”) is defined by

$$\begin{aligned} \text{sbt}(A_1 < A_2, st, a, a_t, a_f, l) &:= \text{at}(A_1, st, a, l) \\ &\quad \text{at}(A_2, st, a', l) \\ &\quad a'' : LT; \\ &\quad a'' + 1 : JFALSE(a_f); \\ &\quad a'' + 2 : JMP(a_t); \end{aligned}$$

$$\text{sbt}(\text{not } B, st, a, a_t, a_f, l) := \text{sbt}(B, st, a, a_f, a_t, l)$$

$$\begin{aligned} \text{sbt}(B_1 \text{ and } B_2, st, a, a_t, a_f, l) &:= \text{sbt}(B_1, st, a, a', a_f, l) \\ &\quad \text{sbt}(B_2, st, a', a_t, a_f, l) \end{aligned}$$

$$\begin{aligned} \text{sbt}(B_1 \text{ or } B_2, st, a, a_t, a_f, l) &:= \text{sbt}(B_1, st, a, a_t, a', l) \\ &\quad \text{sbt}(B_2, st, a', a_t, a_f, l) \end{aligned}$$

# Boolean Expressions with Sequential Semantics

## Jumping Code for Commands

### Definition 17.2 (Jumping code for commands)

The mapping

$$\text{sct} : \text{Cmd} \times \text{Tab} \times \text{PC} \times \text{Lev} \dashrightarrow \text{AM}$$

(“sequential command translation”) is defined by

$$\begin{aligned} \text{sct}(\text{if } B \text{ then } C_1 \text{ else } C_2, \text{st}, a, l) := & \text{sbt}(B, \text{st}, a, a_t, a_f, l) \\ & \text{sct}(C_1, \text{st}, a_t, l) \\ & a_f - 1 : \text{JMP}(a'); \\ & \text{sct}(C_2, \text{st}, a_f, l) \\ & a' : \end{aligned}$$

$$\begin{aligned} \text{sct}(\text{while } B \text{ do } C, \text{st}, a, l) := & \text{sbt}(B, \text{st}, a, a_t, a_f, l) \\ & \text{sct}(C, \text{st}, a_t, l) \\ & a_f - 1 : \text{JMP}(a); \\ & a_f : \end{aligned}$$

(remaining cases analogously)

# Boolean Expressions with Sequential Semantics

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## Example: Jumping Code

### Example 17.3

Translation of `while not (x < 1) and (x < y) do C:`

# Boolean Expressions with Sequential Semantics

## Example: Jumping Code

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Translation of `while not (x < 1) and (x < y) do C:`

**Strict:**

```
1 : LOAD(x);  
   LIT(1);  
   LT;  
   NOT;  
   LOAD(x);  
   LOAD(y);  
   LT;  
   AND;  
   JFALSE(a);  
   ct(C, ...)  
   JMP(1);  
a : ...
```

# Boolean Expressions with Sequential Semantics

## Example: Jumping Code

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Translation of `while not (x < 1) and (x < y) do C:`

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   ct(C,...)  
   JMP(1);  
a : ...
```

#### Sequential:

```
1 : LOAD(x);  
   LIT(1);  
   LT;  
   JFALSE(6);  
   JMP(a);  
6 : LOAD(x);  
   LOAD(y);  
   LT;  
   JFALSE(a);  
   JMP(11);  
11 : sct(C,...)  
     JMP(1);  
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# Boolean Expressions with Sequential Semantics

## Example: Jumping Code

### Example 17.3

Translation of `while not (x < 1) and (x < y) do C:`

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a : ...
```

If `x = 0`:  
9 instructions executed

#### Sequential:

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# Boolean Expressions with Sequential Semantics

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# Boolean Expressions with Sequential Semantics

## Example: Jumping Code

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If  $x = 0$ :  
5 instructions executed

⇒ generally: longer code, but shorter executions



# Implementation of Data Structures

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Source code: **data structures** = arrays, records, lists, trees, ...

⇒ **structured** state space, variables with components

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Abstract machine: **linear** memory structure, cells for storing atomic data

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Abstract machine: **linear** memory structure, cells for storing atomic data

Translation: **mapping** of structured state space to linear memory  
(**address computation**)

- **static** data structures: memory requirements known at compile time
- **dynamic** data structures: memory requirements runtime dependent
  - heap, pointers, garbage collection, ...

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Abstract machine: **linear** memory structure, cells for storing atomic data

Translation: **mapping** of structured state space to linear memory  
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- **static** data structures: memory requirements known at compile time
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  - heap, pointers, garbage collection, ...

### First step:

- static data structures (arrays and records)
- inductive type definitions
- no blocks or procedures (only for simplification; “orthogonal” extension)

# Static Data Structures

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# Static Data Structures

## Modified Syntax of EPL

### Definition 17.4 (Modified syntax of EPL)

The **modified syntax of EPL** is defined as follows (where  $n \geq 1$ ):

$\mathbb{Z}$  :  $z$  (\*  $z$  is an integer \*)  
 $\mathbb{B}$  :  $b ::= \text{true} \mid \text{false}$  (\*  $b$  is a Boolean \*)  
 $\mathbb{R}$  :  $r$  (\*  $r$  is a real number \*)  
*Con* :  $c ::= z \mid b \mid r$  (\*  $c$  is a constant \*)  
*Ide* :  $I, J$  (\*  $I, J$  are identifiers \*)  
*Type* :  $T ::= \text{bool} \mid \text{int} \mid \text{real} \mid I \mid \text{array}[z_1..z_2] \text{ of } T \mid$   
 $\text{record } l_1 : T_1; \dots; l_n : T_n \text{ end}$   
*Var* :  $V ::= I \mid V[E] \mid V.I$   
*Exp* :  $E ::= c \mid V \mid E_1 + E_2 \mid E_1 < E_2 \mid E_1 \text{ and } E_2 \mid \dots$   
*Cmd* :  $C ::= V := E \mid C_1; C_2 \mid \text{if } E \text{ then } C_1 \text{ else } C_2 \mid \text{while } E \text{ do } C$   
*Dcl* :  $D ::= D_C D_T D_V$   
 $D_C ::= \varepsilon \mid \text{const } l_1 := c_1; \dots; l_n := c_n;$   
 $D_T ::= \varepsilon \mid \text{type } l_1 := T_1; \dots; l_n := T_n;$   
 $D_V ::= \varepsilon \mid \text{var } l_1 : T_1; \dots; l_n : T_n;$   
*Pgm* :  $P ::= D C$

## Static Semantics I

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# Static Data Structures

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if  $D_T = \text{type } l_1 := T_1 ; \dots ; l_n := T_n$ ; and type identifier  $l$  occurs in  $T_j$ , then  $l \in \{l_1, \dots, l_{j-1}\}$ .

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- Every identifier used in a command  $C$  must be **declared** in  $D$  (as a constant or variable).
- Variables in expressions and assignments have a **base type** (`bool/int/real`; possibly via type identifiers).

## Static Semantics II

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- **Type compatibility**:  $\mathbb{Z} \subseteq \mathbb{R}$  in mathematics, but not on computers (different representation)  
⇒ **type casts**

**weak typing**: implicit casting by compiler (`2.5 + 1`, `1 + "42"`)

- risk of undetected errors; for **programming-in-the-small** (scripting languages)

**strong typing**: explicit casting by programmer

- enhanced software reliability; for **programming-in-the-large**

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**weak typing**: implicit casting by compiler (`2.5 + 1`, `1 + "42"`)
  - risk of undetected errors; for **programming-in-the-small** (scripting languages)**strong typing**: explicit casting by programmer
  - enhanced software reliability; for **programming-in-the-large**
- Instantiation of operators/functions/procedures/... for different parameter types:  
**polymorphism** or **overloading**

`+ : int × int → int`      `+ : real × real → real`

# Modifying the Abstract Machine

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# Modifying the Abstract Machine

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## The Modified Abstract Machine AM

- Additional **main storage** for keeping data values
- **Procedure stack** not required anymore (as procedures no longer supported)

### Definition 17.5 (Modified abstract machine for EPL)

The **modified abstract machine for EPL (AM)** is defined by the **state space**

$$S := PC \times DS \times MS$$

with

- the **program counter**  $PC := \mathbb{N}$ ,
- the **data stack**  $DS := \mathbb{R}^*$  ( $\text{true} \hat{=} 1$ ,  $\text{false} \hat{=} 0$ ; top to the right), and
- the **main storage**  $MS := \{\sigma \mid \sigma : \mathbb{N} \rightarrow \mathbb{R}\}$ .

# Modifying the Abstract Machine

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## New AM Instructions

### Definition 17.6 (New AM instructions)

- **Procedure instructions** are no longer needed.

# Modifying the Abstract Machine

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- **Procedure instructions** are no longer needed.
- **Transfer instructions** ( $\text{LOAD}(dif, off)$ ,  $\text{STORE}(dif, off)$ ) are replaced by the following instructions with the respective semantics  $\llbracket \cdot \rrbracket : S \dashrightarrow S$ :

$$\begin{aligned}\llbracket \text{LOAD} \rrbracket (pc, d : m, \sigma) &:= (pc + 1, d : \sigma(m), \sigma) \\ &\quad \text{if } m \in \mathbb{N} \\ \llbracket \text{STORE} \rrbracket (pc, d : m : r, \sigma) &:= (pc + 1, d, \sigma[m \mapsto r]) \\ &\quad \text{if } m \in \mathbb{N}\end{aligned}$$

# Modifying the Abstract Machine

## New AM Instructions

### Definition 17.6 (New AM instructions)

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$$\llbracket \text{LOAD} \rrbracket (pc, d : m, \sigma) := (pc + 1, d : \sigma(m), \sigma) \\ \text{if } m \in \mathbb{N}$$

$$\llbracket \text{STORE} \rrbracket (pc, d : m : r, \sigma) := (pc + 1, d, \sigma[m \mapsto r]) \\ \text{if } m \in \mathbb{N}$$

- Moreover the following **instruction for checking array bounds** is introduced:

$$\llbracket \text{CAB}(z_1, z_2) \rrbracket (pc, d : r, \sigma) := \begin{cases} (pc + 1, d : r, \sigma) & \text{if } r \in \{z_1, \dots, z_2\} \\ (0, d : \underbrace{\text{RTE}}_{\text{runtime error}}, \sigma) & \text{otherwise} \end{cases}$$



# Modifying the Symbol Table

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## Modifying the Symbol Table

$$\begin{aligned} \text{Tab} := \{ & \text{st} \mid \text{st} : \text{Ide} \dashrightarrow (\{\text{const}\} \times (\mathbb{B} \cup \mathbb{Z} \cup \mathbb{R})) \\ & \cup (\{\text{var}\} \times \text{Ide} \times \mathbb{N}) \\ & \cup (\{\text{type}\} \times \{\text{bool}, \text{int}, \text{real}\} \times \{1\}) \\ & \cup (\{\text{type}\} \times \{\text{array}\} \times \mathbb{Z}^2 \times \text{Ide} \times \mathbb{N}) \\ & \cup (\{\text{type}\} \times \{\text{record}\} \times (\text{Ide}^2 \times \mathbb{N})^* \times \mathbb{N}) \} \end{aligned}$$

### Remarks:

- **Variable descriptor** ( $\text{var}, l, m$ ): type  $l$ , memory address  $m$
- Last component  $m$  of **type** entry: **memory requirement** (base types: 1 “cell”)
- **Array descriptor** ( $\text{type}, \text{array}, z_1, z_2, l, m$ ):
  - bounds  $z_1, z_2$
  - component type identifier  $l$
- **Record descriptor** ( $\text{type}, \text{record}, l_1, J_1, o_1, \dots, l_n, J_n, o_n, m$ ):
  - selector  $l_k$
  - component type identifier  $J_k$
  - memory offset  $o_k$
- “Indexed” table lookup:  $\text{st}(l.l_k) := (J_k, o_k)$  if  $\text{st}(l) = (\text{type}, \text{record}, \dots, l_k, J_k, o_k, \dots, m)$

# Modifying the Symbol Table

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## Maintaining the Symbol Table I

The symbol table is again maintained by the function `update( $D$ ,  $st$ )` which specifies the update of symbol table `st` according to declaration  $D$ .

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## Maintaining the Symbol Table I

The symbol table is again maintained by the function  $\text{update}(D, \text{st})$  which specifies the update of symbol table  $\text{st}$  according to declaration  $D$ .

For the sake of simplicity we assume that  $D = D_C D_T D_V \in Dcl$  is **flattened**, i.e., that every subtype is named by an identifier:

- If  $D_T = \text{type } l_1 := T_1; \dots; l_n := T_n;$ , then for every  $k \in [n]$ 
  - $T_k \in \{\text{bool}, \text{int}, \text{real}\}$  or
  - $T_k \in \{l_1, \dots, l_{k-1}\}$  or
  - $T_k = \text{array}[z_1..z_2]$  of  $l_j$  where  $j \in [k-1]$  or
  - $T_k = \text{record } J_1 : l_{j_1}; \dots; J_l : l_{j_l}$  end where  $j_1, \dots, j_l \in [k-1]$
- For  $D_T$  as above,  $D_V$  must be of the form  $D_V = \text{var } J_1 : l_{j_1}; \dots; J_k : l_{j_k};$  where  $j_1, \dots, j_k \in [n]$

# Modifying the Symbol Table

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## Maintaining the Symbol Table II

### Definition 17.7 (Modified update function)

$\text{update} : Dcl \times Tab \dashrightarrow Tab$  is defined by

$$\begin{aligned} & \text{update}(D_C D_T D_V, st) \\ & := \text{update}(D_V, \text{update}(D_T, \text{update}(D_C, st))) \\ & \text{update}(\varepsilon, st) \\ & := st \end{aligned}$$

# Modifying the Symbol Table

---

## Maintaining the Symbol Table II

### Definition 17.7 (Modified update function)

$\text{update} : Dcl \times Tab \dashrightarrow Tab$  is defined by

$$\begin{aligned} & \text{update}(D_C D_T D_V, st) \\ & := \text{update}(D_V, \text{update}(D_T, \text{update}(D_C, st))) \\ & \text{update}(\varepsilon, st) \\ & := st \\ & \text{update}(\text{const } l_1 := c_1; \dots; l_n := c_n; , st) \\ & := st[l_1 \mapsto (\text{const}, c_1), \dots, l_n \mapsto (\text{const}, c_n)] \end{aligned}$$

# Modifying the Symbol Table

## Maintaining the Symbol Table II

### Definition 17.7 (Modified update function)

$\text{update} : Dcl \times Tab \dashrightarrow Tab$  is defined by

$$\begin{aligned} & \text{update}(D_C D_T D_V, st) \\ & := \text{update}(D_V, \text{update}(D_T, \text{update}(D_C, st))) \\ & \text{update}(\varepsilon, st) \\ & := st \\ & \text{update}(\text{const } l_1 := c_1; \dots; l_n := c_n; , st) \\ & := st[l_1 \mapsto (\text{const}, c_1), \dots, l_n \mapsto (\text{const}, c_n)] \\ & \text{update}(\text{var } l_1 : J_1; \dots; l_n : J_n; , st) \\ & := st[l_1 \mapsto (\text{var}, J_1, o_1), \dots, l_n \mapsto (\text{var}, J_n, o_n)] \\ & \quad \text{if } st(J_k) = (\text{type}, \dots, m_k) \text{ and } o_k := \sum_{i=1}^{k-1} m_i \text{ for } k \in [n] \end{aligned}$$

# Modifying the Symbol Table

## Maintaining the Symbol Table III

### Definition 17.7 (Modified update function; continued)

$$\begin{aligned} & \text{update}(\text{type } l := \text{bool}; D'_T, \text{st}) \\ & := \text{update}(\text{type } D'_T, \text{st}[l \mapsto (\text{type}, \text{bool}, 1)]) \\ & \text{update}(\text{type } l := \text{int}; D'_T, \text{st}) \\ & := \text{update}(\text{type } D'_T, \text{st}[l \mapsto (\text{type}, \text{int}, 1)]) \\ & \text{update}(\text{type } l := \text{real}; D'_T, \text{st}) \\ & := \text{update}(\text{type } D'_T, \text{st}[l \mapsto (\text{type}, \text{real}, 1)]) \\ & \text{update}(\text{type } l := J; D'_T, \text{st}) \\ & := \text{update}(\text{type } D'_T, \text{st}[l \mapsto \text{st}(J)]) \\ & \text{update}(\text{type } l := \text{array}[z_1..z_2] \text{ of } J; D'_T, \text{st}) \\ & := \text{update}(\text{type } D'_T, \text{st}[l \mapsto (\text{type}, \text{array}, z_1, z_2, J, n \cdot m)]) \\ & \quad \text{if } \text{st}(J) = (\text{type}, \dots, m) \text{ and } n := z_2 - z_1 + 1 \\ & \text{update}(\text{type } l := \text{record } l_1:J_1; \dots; l_n:J_n \text{ end}; D'_T, \text{st}) \\ & := \text{update}(\text{type } D'_T, \text{st}[l \mapsto (\text{type}, \text{record}, l_1, J_1, o_1, \dots, l_n, J_n, o_n, o_{n+1})]) \\ & \quad \text{if } \text{st}(J_k) = (\text{type}, \dots, m_k) \text{ and } o_k := \sum_{i=1}^{k-1} m_i \text{ for } k \in [n] \end{aligned}$$



# Modifying the Symbol Table

---

## Maintaining the Symbol Table IV

### Example 17.8 (Modified `update` function)

```
Let  $D :=$  type Bool = bool; Int = int;  
    Array = array [1..20] of Bool;  
    Record = record S: Array; T: Int end;  
var x: Int; y: Array; z: Record;
```

## Modifying the Symbol Table

---

### Maintaining the Symbol Table IV

#### Example 17.8 (Modified update function)

```
Let  $D :=$  type Bool = bool; Int = int;  
    Array = array [1..20] of Bool;  
    Record = record S: Array; T: Int end;  
var x: Int; y: Array; z: Record;
```

Then

$$\text{update}(D, \text{st}) = \text{st} [ \begin{array}{l} \text{Bool} \mapsto (\text{type}, \text{bool}, 1), \\ \text{Int} \mapsto (\text{type}, \text{int}, 1), \\ \text{Array} \mapsto (\text{type}, \text{array}, 1, 20, \text{Bool}, 20), \\ \text{Record} \mapsto (\text{type}, \text{record}, \text{S}, \text{Array}, 0, \text{T}, \text{Int}, 20, 21), \\ x \mapsto (\text{var}, \text{Int}, 0), \\ y \mapsto (\text{var}, \text{Array}, 1), \\ z \mapsto (\text{var}, \text{Record}, 21) \end{array} ]$$

# The Translation

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## Outline of Lecture 17

Recap: Syntax of EPL

Boolean Expressions with Sequential Semantics

Implementation of Data Structures

Static Data Structures

Modifying the Abstract Machine

Modifying the Symbol Table

## The Translation

A Translation Example

# The Translation

---

## Translation of Variables I

The translation employs the following auxiliary function to determine the type identifier of a given variable:

### Definition 17.9 (vtype function)

The mapping

$$\text{vtype} : \text{Var} \times \text{Tab} \dashrightarrow \text{Ide}$$

is given by

$$\begin{aligned} \text{vtype}(I, \text{st}) &:= J && \text{if } \text{st}(I) = (\text{var}, J, m) \\ \text{vtype}(V[E], \text{st}) &:= J && \text{if } \text{vtype}(V, \text{st}) = I \text{ and } \text{st}(I) = (\text{type}, \text{array}, z_1, z_2, J, m) \\ \text{vtype}(V.I, \text{st}) &:= J && \text{if } \text{vtype}(V, \text{st}) = I' \text{ and } \text{st}(I'.I) = (J, o) \end{aligned}$$

# The Translation

## Translation of Variables II

Function `vt` generates code for computing the memory address of a variable on the data stack:

### Definition 17.10 (Translation of variables)

The mapping  $vt : Var \times Tab \dashrightarrow AM$  is given by

```
vt(I, st) := LIT(m);           if st(I) = (var, J, m)
vt(V[E], st) := vt(V, st)      % base address of V
                  et(E, st)     % array index
                  CAB(z1, z2);  % bounds checking
                  LIT(z1); SUB; % index difference
                  LIT(n); MULT; % relative address
                  ADD;          % address of V[E]
vt(V.I, st) := vt(V, st)      % address of V
                  LIT(o);       % offset
                  ADD;          % address of V.I
                  if vtype(V, st) = I', st(I'.I) = (J, o)
```

# The Translation

## Translation of Expressions

### Definition 17.11 (Translation of expressions)

The mapping

$$et : Exp \times Tab \dashrightarrow AM$$

is given by

$$\begin{aligned} et(c, st) &:= LIT(c); \\ et(V, st) &:= \begin{cases} LIT(c); & \text{if } V \in Ide \text{ and } st(V) = (const, c) \\ vt(V, st) & \text{otherwise} \\ LOAD; \end{cases} \\ et(E_1 + E_2, st) &:= \begin{array}{l} et(E_1, st) \\ et(E_2, st) \\ ADD; \end{array} \\ &\vdots \end{aligned}$$

# The Translation

---

## Translation of Commands and Programs

### Definition 17.12 (Translation of commands)

For the mapping

$$ct : Cmd \times Tab \dashrightarrow AM$$

only the handling of assignments needs to be adapted:

```
ct( $V := E$ , st) := vt( $V$ , st)  % address of left-hand side
                  et( $E$ , st)  % value of right-hand side
STORE;
```

# The Translation

## Translation of Commands and Programs

### Definition 17.12 (Translation of commands)

For the mapping

$$ct : \mathit{Cmd} \times \mathit{Tab} \dashrightarrow \mathit{AM}$$

only the handling of assignments needs to be adapted:

$$\begin{aligned} ct(V := E, st) &:= vt(V, st) \quad \% \text{ address of left-hand side} \\ &\quad et(E, st) \quad \% \text{ value of right-hand side} \\ &\quad \text{STORE;} \end{aligned}$$

### Definition 17.13 (Translation of programs)

The mapping

$$\text{trans} : \mathit{Pgm} \dashrightarrow \mathit{AM}$$

is defined by

$$\text{trans}(D \ C) := ct(C, \text{update}(D, st_\emptyset)) \quad \text{where } st_\emptyset(l) \text{ is undefined for every } l \in \mathit{Ide}$$



# A Translation Example

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## Outline of Lecture 17

Recap: Syntax of EPL

Boolean Expressions with Sequential Semantics

Implementation of Data Structures

Static Data Structures

Modifying the Abstract Machine

Modifying the Symbol Table

The Translation

A Translation Example

# A Translation Example

---

## Translation Example I

### Example 17.14

```
 $P = \left. \begin{array}{l} \text{type Int=int; Array=array[1..10] of Int;} \\ \text{var a:Array; i:Int;} \\ \text{i:=1;} \\ \text{while i<=10 do} \\ \quad \text{a[i]:=i; i:=i+1;} \end{array} \right\} D$ 
```

  

```
 $\left. \begin{array}{l} \text{i:=1;} \\ \text{while i<=10 do} \\ \quad \text{a[i]:=i; i:=i+1;} \end{array} \right\} C$ 
```

# A Translation Example

## Translation Example I

### Example 17.14

```

$$P = \left. \begin{array}{l} \text{type Int=int; Array=array[1..10] of Int;} \\ \text{var a:Array; i:Int;} \\ \text{i:=1;} \\ \text{while i<=10 do} \\ \quad \text{a[i]:=i; i:=i+1;} \end{array} \right\} D$$
  

$$\left. \begin{array}{l} \text{i:=1;} \\ \text{while i<=10 do} \\ \quad \text{a[i]:=i; i:=i+1;} \end{array} \right\} C$$
  

$$\text{trans}(P) = \text{ct}(C, \text{update}(D, \text{st}_\emptyset))$$

```

# A Translation Example

## Translation Example I

### Example 17.14

$$\begin{aligned} P = & \left. \begin{array}{l} \text{type Int=int; Array=array[1..10] of Int;} \\ \text{var a:Array; i:Int;} \\ \text{i:=1;} \\ \text{while i<=10 do} \\ \quad \text{a[i]:=i; i:=i+1;} \end{array} \right\} D \\ & \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} C \\ \text{trans}(P) = & \text{ct}(C, \text{update}(D, \text{st}_\emptyset)) \\ & \text{st} := \text{update}(D, \text{st}_\emptyset) \\ & = \text{st}_\emptyset[ \quad \text{Int} \mapsto (\text{type}, \text{int}, 1), \\ & \quad \text{Array} \mapsto (\text{type}, \text{array}, 1, 10, \text{Int}, 10), \\ & \quad \text{a} \mapsto (\text{var}, \text{Array}, 0), \\ & \quad \text{i} \mapsto (\text{var}, \text{Int}, 10) ] \end{aligned}$$

# A Translation Example

## Translation Example I

### Example 17.14

$$\left. \begin{array}{l} P = \text{type Int=int; Array=array[1..10] of Int;} \\ \text{var a:Array; i:Int;} \end{array} \right\} D$$
$$\left. \begin{array}{l} \text{i:=1;} \\ \text{while i<=10 do} \\ \quad \text{a[i]:=i; i:=i+1;} \end{array} \right\} C$$
$$\begin{aligned} \text{trans}(P) &= \text{ct}(C, \text{update}(D, \text{st}_\emptyset)) \\ &= \text{st}_\emptyset[ \text{Int} \mapsto (\text{type}, \text{int}, 1), \\ &\quad \text{Array} \mapsto (\text{type}, \text{array}, 1, 10, \text{Int}, 10), \\ &\quad \text{a} \mapsto (\text{var}, \text{Array}, 0), \\ &\quad \text{i} \mapsto (\text{var}, \text{Int}, 10) ] \\ \text{ct}(C, \text{st}) &= \text{ct}(\text{i:=1}, \text{st}) \text{ct}(\text{while i<=10 do a[i]:=i; i:=i+1}, \text{st}) \end{aligned}$$

# A Translation Example

## Translation Example I

### Example 17.14

$$\left. \begin{array}{l} P = \text{type Int=int; Array=array[1..10] of Int;} \\ \text{var a:Array; i:Int;} \\ \text{i:=1;} \\ \text{while i<=10 do} \\ \quad \text{a[i]:=i; i:=i+1;} \end{array} \right\} D$$
$$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} C$$
$$\text{trans}(P) = \text{ct}(C, \text{update}(D, \text{st}_\emptyset))$$
$$\text{st} := \text{update}(D, \text{st}_\emptyset)$$
$$= \text{st}_\emptyset[ \quad \text{Int} \mapsto (\text{type}, \text{int}, 1),$$
$$\quad \text{Array} \mapsto (\text{type}, \text{array}, 1, 10, \text{Int}, 10),$$
$$\quad \text{a} \mapsto (\text{var}, \text{Array}, 0),$$
$$\quad \text{i} \mapsto (\text{var}, \text{Int}, 10) ]$$
$$\text{ct}(C, \text{st}) = \text{ct}(\text{i:=1}, \text{st}) \text{ct}(\text{while i<=10 do a[i]:=i; i:=i+1}, \text{st})$$
$$\text{ct}(\text{i:=1}, \text{st}) = \text{vt}(\text{i}, \text{st}) \quad \% \text{ address of i}$$
$$\text{et}(1, \text{st}) \quad \% \text{ value of 1}$$
$$\text{STORE};$$
$$= \text{LIT}(10); \text{LIT}(1); \text{STORE};$$

# A Translation Example

---

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1, st)
  = a' : et(i<=10, st) JFALSE(a'');
      ct(a[i]:=i; i:=i+1, st) JMP(a'); a' :
```

# A Translation Example

---

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1, st)
    = a' : et(i<=10, st) JFALSE(a'');
        ct(a[i]:=i; i:=i+1, st) JMP(a'); a' :
et(i<=10, st) = LIT(10); LOAD; LIT(10); LE;
```



# A Translation Example

---

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1,st)
    = a' : et(i<=10,st) JFALSE(a'');
        ct(a[i]:=i; i:=i+1,st) JMP(a'); a'' :
    et(i<=10,st) = LIT(10); LOAD; LIT(10); LE;
ct(a[i]:=i; i:=i+1,st) = ct(a[i]:=i,st) ct(i:=i+1,st)
```

# A Translation Example

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1, st)
    = a' : et(i<=10, st) JFALSE(a'');
        ct(a[i]:=i; i:=i+1, st) JMP(a'); a' :
    et(i<=10, st) = LIT(10); LOAD; LIT(10); LE;
ct(a[i]:=i; i:=i+1, st) = ct(a[i]:=i, st) ct(i:=i+1, st)
ct(a[i]:=i, st) = vt(a[i], st) % address of a[i]
                et(i, st)    % value of i
                STORE;
```

# A Translation Example

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1, st)
    = a' : et(i<=10, st) JFALSE(a'');
        ct(a[i]:=i; i:=i+1, st) JMP(a'); a' :
    et(i<=10, st) = LIT(10); LOAD; LIT(10); LE;
ct(a[i]:=i; i:=i+1, st) = ct(a[i]:=i, st) ct(i:=i+1, st)
ct(a[i]:=i, st) = vt(a[i], st) % address of a[i]
                et(i, st)      % value of i
                STORE;
vt(a[i], st) = vt(a, st)      % address of a
              et(i, st)      % value of i
              CAB(1, 10);    % bounds checking
              LIT(1); SUB;   % index difference
              LIT(1); MULT;  % relative address
              ADD;           % address of a[i]
```

# A Translation Example

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1,st)
    = a' : et(i<=10,st) JFALSE(a'');
        ct(a[i]:=i; i:=i+1,st) JMP(a'); a' :
    et(i<=10,st) = LIT(10); LOAD; LIT(10); LE;
ct(a[i]:=i; i:=i+1,st) = ct(a[i]:=i,st) ct(i:=i+1,st)
ct(a[i]:=i,st) = vt(a[i],st) % address of a[i]
                et(i,st)    % value of i
                STORE;
vt(a[i],st) = vt(a,st)      % address of a
              et(i,st)     % value of i
              CAB(1,10);   % bounds checking
              LIT(1); SUB; % index difference
              LIT(1); MULT;% relative address
              ADD;         % address of a[i]
vt(a,st) = LIT(0);
```

# A Translation Example

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1,st)
    = a' : et(i<=10,st) JFALSE(a'');
        ct(a[i]:=i; i:=i+1,st) JMP(a'); a' :
    et(i<=10,st) = LIT(10); LOAD; LIT(10); LE;
ct(a[i]:=i; i:=i+1,st) = ct(a[i]:=i,st) ct(i:=i+1,st)
ct(a[i]:=i,st) = vt(a[i],st) % address of a[i]
                et(i,st)    % value of i
                STORE;
vt(a[i],st) = vt(a,st)      % address of a
              et(i,st)     % value of i
              CAB(1,10);   % bounds checking
              LIT(1); SUB; % index difference
              LIT(1); MULT;% relative address
              ADD;         % address of a[i]
vt(a,st) = LIT(0);
et(i,st) = LIT(10); LOAD;
```

# A Translation Example

## Translation Example II

### Example 17.14 (continued)

```
ct(while i<=10 do a[i]:=i; i:=i+1,st)
    = a' : et(i<=10,st) JFALSE(a'');
        ct(a[i]:=i; i:=i+1,st) JMP(a'); a' :
    et(i<=10,st) = LIT(10); LOAD; LIT(10); LE;
ct(a[i]:=i; i:=i+1,st) = ct(a[i]:=i,st) ct(i:=i+1,st)
ct(a[i]:=i,st) = vt(a[i],st) % address of a[i]
                et(i,st) % value of i
                STORE;
vt(a[i],st) = vt(a,st) % address of a
              et(i,st) % value of i
              CAB(1,10); % bounds checking
              LIT(1); SUB; % index difference
              LIT(1); MULT; % relative address
              ADD; % address of a[i]
vt(a,st) = LIT(0);
et(i,st) = LIT(10); LOAD;
ct(i:=i+1,st) = LIT(10); LIT(10); LOAD; LIT(1); ADD; STORE;
```