Compiler Construction

Lecture 10: Syntax Analysis VI ($LR(1)$ Parsing & Handling of Ambiguities)

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Outline of Lecture 10

Recap: $LR(0)$ and $SLR(1)$ Parsing

$LR(1)$ Parsing

$LALR(1)$ Parsing

Bottom-Up Parsing of Ambiguous Grammars

Expressiveness of LL and LR Grammars

Computing $LR(0)$ Sets

Theorem (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and reduced.

1. $LR(0)(\varepsilon)$ *is the least set such that*
   - $[S' \to \cdot S] \in LR(0)(\varepsilon)$ *and*
   - *if* $[A \to \cdot B\gamma] \in LR(0)(\varepsilon)$ *and* $B \to \beta \in P$,
     then $[B \to \cdot\beta] \in LR(0)(\varepsilon)$.
2. $LR(0)(\alpha Y)$ *(*$\alpha \in X^*, Y \in X$*) is the least set such that*
   - *if* $[A \to \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$,
     then $[A \to \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ *and*
   - *if* $[A \to \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ *and* $B \to \beta \in P$,
     then $[B \to \cdot\beta] \in LR(0)(\alpha Y)$.

$LR(0)$ Conflicts

Definition ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ such that

$$[A \to \alpha_1 \cdot a\alpha_2], [B \to \beta\cdot] \in I.$$

- I has a reduce/reduce conflict if there exist $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \to \alpha\cdot], [B \to \beta\cdot] \in I.$$

Lemma

$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.

Proof.

omitted □

Recap: $LR(0)$ and $SLR(1)$ Parsing

The goto Function

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields deterministic shift/reduce decision for $NBA(G)$ in a configuration with pushdown γ
$\implies$ new pushdown alphabet: $LR(0)(G)$ in place of X

Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but independent from γ in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

Definition ($LR(0)$ goto function)

The function $\mathrm{goto} : LR(0)(G) \times X \to LR(0)(G)$ is determined by

$$\mathrm{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that } I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).$$

The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the action function, which determines the shift/reduce decision (reminder: $\pi_0 = S' \to S$).

Definition ($LR(0)$ action function)

The $LR(0)$ action function $\mathrm{act} : LR(0)(G) \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$ is defined by

$$\mathrm{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot] \in I \\ \text{shift} & \text{if } [A \to \alpha_1 \cdot a\alpha_2] \in I \\ \text{accept} & \text{if } [S' \to S\cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

Corollary

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act is well defined.

Together, act and goto form the $LR(0)$ parsing table of G.

The $LR(0)$ Parsing Automaton I

Definition ($LR(0)$ parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LR(0)$. The (deterministic) $LR(0)$ parsing automaton of G is defined by the following components:

- Input alphabet Σ
- Pushdown alphabet $\Gamma := LR(0)(G)$
- Output alphabet $\Delta := [p] \cup \{0, \text{error}\}$
- Configurations $\Sigma^* \times \Gamma^* \times \Delta^*$
- Initial configuration (w, I_0, ε) where $I_0 := LR(0)(\varepsilon)$
- Final configurations $\{\varepsilon\} \times \{\varepsilon\} \times \Delta^*$
- Transitions:

shift: $(aw, \alpha I, z) \vdash (w, \alpha I I', z)$ if $\mathrm{act}(I) = \text{shift}$ and $\mathrm{goto}(I, a) = I'$

reduce: $(w, \alpha I I_1 \ldots I_n, z) \vdash (w, \alpha I I', zi)$ if $\mathrm{act}(I_n) = \text{red } i$, $\pi_i = A \to Y_1 \ldots Y_n$, $\mathrm{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0)$ if $\mathrm{act}(I) = \text{accept}$

error: $(w, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error})$ if $\mathrm{act}(I) = \text{error}$

The $SLR(1)$ Action Function

Definition ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\mathrm{act} : LR(0)(G) \times \Sigma_\varepsilon \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \to \alpha, [A \to \alpha\cdot] \in I, \text{ and } x \in \mathrm{fo}(A) \\ \text{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \to S\cdot] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Definition ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

act and the $LR(0)$ goto function (Definition 9.1) form the $SLR(1)$ parsing table of G.

The $SLR(1)$ Parsing Automaton

Definition ($SLR(1)$ parsing automaton)

The $SLR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 9.6), except for the transition relation:

shift: $(aw, \alpha I, z) \vdash (w, \alpha I I', z)$ if $\mathrm{act}(I, a) = \text{shift}$ and $\mathrm{goto}(I, a) = I'$

reduce$_a$: $(aw, \alpha I I_1 \ldots I_n, z) \vdash (aw, \alpha I I', zi)$ if $\mathrm{act}(I_n, a) = \text{red } i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

reduce$_\varepsilon$: $(\varepsilon, \alpha I I_1 \ldots I_n, z) \vdash (\varepsilon, \alpha I I', zi)$ if $\mathrm{act}(I_n, \varepsilon) = \text{red } i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0)$ if $\mathrm{act}(I, \varepsilon) = \text{accept}$

error$_a$: $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error})$ if $\mathrm{act}(I, a) = \text{error}$

error$_\varepsilon$: $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error})$ if $\mathrm{act}(I, \varepsilon) = \text{error}$

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Observation: not every element of $\mathrm{fo}(A)$ can follow every occurrence of A
$\Longrightarrow$ refinement of $LR(0)$ items by adding possible lookahead symbols

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Definition 10.1 ($LR(1)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$.

- If $S' \Rightarrow_r^* \alpha A a w \Rightarrow_r \alpha\beta_1\beta_2 a w$, then $[A \to \beta_1 \cdot \beta_2, a]$ is called an $LR(1)$ item for $\alpha\beta_1$.

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- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha\beta_1\beta_2$, then $[A \to \beta_1 \cdot \beta_2, \varepsilon]$ is called an $LR(1)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for γ, called the $LR(1)$ set (or: $LR(1)$ information) of γ.

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- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for γ, called the $LR(1)$ set (or: $LR(1)$ information) of γ.
- $LR(1)(G) := \{LR(1)(\gamma) \mid \gamma \in X^*\}$.

$LR(1)$ Items and Sets II

Corollary 10.2

1. *For every* $\gamma \in X^*$, $LR(1)(\gamma)$ *is finite.*

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1. *For every* $\gamma \in X^*$, $LR(1)(\gamma)$ *is finite.*
2. $LR(1)(G)$ *is finite.*
3. *For every* $\gamma \in X^*$, $LR(1)(\gamma)$ *"contains"* $LR(0)(\gamma)$*, i.e.,*

$$\{[A \to \beta_1 \cdot \beta_2] \mid [A \to \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

$LR(1)$ Items and Sets II

Corollary 10.2

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$$\{[A \to \beta_1 \cdot \beta_2] \mid [A \to \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

4. $[A \to \beta_1 \cdot \beta_2, x] \in I \in LR(1)(G) \implies x \in \mathrm{fo}(A)$

$LR(1)$ Conflicts

Definition 10.3 ($LR(1)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(1)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ and $x \in \Sigma_\varepsilon$ such that

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- I has a reduce/reduce conflict if there exist $x \in \Sigma_\varepsilon$ and $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
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- I has a reduce/reduce conflict if there exist $x \in \Sigma_\varepsilon$ and $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
$$[A \to \alpha\cdot, x], [B \to \beta\cdot, x] \in I.$$

Lemma 10.4

$G \in LR(1)$ iff no $I \in LR(1)(G)$ contains conflicting items.

Computing $LR(1)$ Sets I

The computation of $LR(0)$ sets (cf. Theorem 8.15) can be extended to cover right contexts:

Theorem 10.5 (Computing $LR(1)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and reduced.

1. $LR(1)(\varepsilon)$ *is the least set such that*
   - $[S' \to \cdot S, \varepsilon] \in LR(1)(\varepsilon)$ *and*
   - *if* $[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon)$, $B \to \beta \in P$, *and* $y \in \mathrm{fi}(\gamma x)$, *then* $[B \to \cdot \beta, y] \in LR(1)(\varepsilon)$.

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2. $LR(1)(\alpha Y)$ ($\alpha \in X^*, Y \in X$) *is the least set such that*
   - *if* $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$, *then* $[A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$ *and*
   - *if* $[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y)$, $B \to \beta \in P$, *and* $y \in \mathrm{fi}(\gamma_2 x)$, *then* $[B \to \cdot\beta, y] \in LR(1)(\alpha Y)$.

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathtt{a} \quad R \to L$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathtt{a} \quad R \to L$

$[S' \to \cdot S, \varepsilon] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon)$: $[S' \to \cdot S, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

$[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon), B \to \beta \in P, y \in \mathrm{fi}(\gamma x) \implies [B \to \cdot\beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) : \quad [S' \to \cdot S, \varepsilon] \quad [S \to \cdot L{=}R, \varepsilon] \quad [S \to \cdot R, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for G_{LR} : $S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon), B \to \beta \in P, y \in \mathrm{fi}(\gamma x) \implies [B \to \cdot\beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon)$: $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, {=}]$
$[L \to \cdot \mathrm{a}, {=}]$

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Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon), B \to \beta \in P, y \in \mathrm{fi}(\gamma x) \implies [B \to \cdot\beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$
$[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$

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$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon), B \to \beta \in P, y \in \mathrm{fi}(\gamma x) \implies [B \to \cdot\beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, {=}]$
$[L \to \cdot\mathrm{a}, {=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot\mathrm{a}, \varepsilon]$

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$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$
$[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S\cdot, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$
$[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L):$ $[S \to L \cdot {=}R, \varepsilon]$

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$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot * R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot * R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S \cdot, \varepsilon]$

$I'_2 := LR(1)(L):$ $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L \cdot, \varepsilon]$

Computing $LR(1)$ Sets II

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$I'_0 := LR(1)(\varepsilon) :$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$
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$I'_1 := LR(1)(S) :$ $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L) :$ $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R) :$ $[S \to R\cdot, \varepsilon]$

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$I'_1 := LR(1)(S)$: $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$: $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R)$: $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*)$: $[L \to {*} \cdot R, {=}]$ $[L \to {*} \cdot R, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x) \implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$: $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$
$[L \to \cdot\mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot\mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S)$: $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$: $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R)$: $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*)$: $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, =]$ $[R \to \cdot L, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x) \implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot {*}R, {=}]$
	$[L \to \cdot \mathrm{a}, {=}]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot {*}R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)({*})$:	$[L \to {*} \cdot R, {=}]$	$[L \to {*} \cdot R, \varepsilon]$	$[R \to \cdot L, {=}]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot {*}R, {=}]$	$[L \to \cdot \mathrm{a}, {=}]$	$[L \to \cdot {*}R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$: $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S)$: $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$: $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R)$: $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*)$: $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_5 := LR(1)(\mathrm{a})$: $[L \to \mathrm{a}\cdot, =]$ $[L \to \mathrm{a}\cdot, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$: $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S)$: $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$: $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R)$: $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*)$: $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_5 := LR(1)(\mathrm{a})$: $[L \to \mathrm{a}\cdot, =]$ $[L \to \mathrm{a}\cdot, \varepsilon]$

$I'_6 := LR(1)(L{=})$: $[S \to L{=} \cdot R, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x) \implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot * R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot * R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot * R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot * R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x) \implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$

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Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for G_{LR} : $S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a} \cdot, =]$	$[L \to \mathrm{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R \cdot, =]$	$[L \to *R \cdot, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R\cdot, =]$	$[L \to *R\cdot, \varepsilon]$		
$I'_8 := LR(1)(*L)$:	$[R \to L\cdot, =]$	$[R \to L\cdot, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R\cdot, =]$	$[L \to *R\cdot, \varepsilon]$		
$I'_8 := LR(1)(*L)$:	$[R \to L\cdot, =]$	$[R \to L\cdot, \varepsilon]$		
$I'_9 := LR(1)(L{=}R)$:	$[S \to L{=}R\cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R\cdot, =]$	$[L \to *R\cdot, \varepsilon]$		
$I'_8 := LR(1)(*L)$:	$[R \to L\cdot, =]$	$[R \to L\cdot, \varepsilon]$		
$I'_9 := LR(1)(L{=}R)$:	$[S \to L{=}R\cdot, \varepsilon]$			
$I'_{10} := LR(1)(L{=}L)$:	$[R \to L\cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$: $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S)$: $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$: $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R)$: $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*)$: $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_5 := LR(1)(\mathrm{a})$: $[L \to \mathrm{a}\cdot, =]$ $[L \to \mathrm{a}\cdot, \varepsilon]$

$I'_6 := LR(1)(L{=})$: $[S \to L{=} \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_7 := LR(1)(*R)$: $[L \to *R\cdot, =]$ $[L \to *R\cdot, \varepsilon]$

$I'_8 := LR(1)(*L)$: $[R \to L\cdot, =]$ $[R \to L\cdot, \varepsilon]$

$I'_9 := LR(1)(L{=}R)$: $[S \to L{=}R\cdot, \varepsilon]$

$I'_{10} := LR(1)(L{=}L)$: $[R \to L\cdot, \varepsilon]$

$I'_{11} := LR(1)(L{=}*)$: $[L \to * \cdot R, \varepsilon]$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x) \implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R\cdot, =]$	$[L \to *R\cdot, \varepsilon]$		
$I'_8 := LR(1)(*L)$:	$[R \to L\cdot, =]$	$[R \to L\cdot, \varepsilon]$		
$I'_9 := LR(1)(L{=}R)$:	$[S \to L{=}R\cdot, \varepsilon]$			
$I'_{10} := LR(1)(L{=}L)$:	$[R \to L\cdot, \varepsilon]$			
$I'_{11} := LR(1)(L{=}*)$:	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$		

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x) \implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot * R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot * R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot * R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot * R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot * R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R\cdot, =]$	$[L \to *R\cdot, \varepsilon]$		
$I'_8 := LR(1)(*L)$:	$[R \to L\cdot, =]$	$[R \to L\cdot, \varepsilon]$		
$I'_9 := LR(1)(L{=}R)$:	$[S \to L{=}R\cdot, \varepsilon]$			
$I'_{10} := LR(1)(L{=}L)$:	$[R \to L\cdot, \varepsilon]$			
$I'_{11} := LR(1)(L{=}*)$:	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot * R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$

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Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a} \cdot, =]$	$[L \to \mathrm{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R \cdot, =]$	$[L \to *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L)$:	$[R \to L \cdot, =]$	$[R \to L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L{=}R)$:	$[S \to L{=}R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L{=}L)$:	$[R \to L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L{=}*)$:	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_{12} := LR(1)(L{=}\mathrm{a})$:	$[L \to \mathrm{a} \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for G_{LR} : $S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$:	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S)$:	$[S' \to S \cdot, \varepsilon]$			
$I'_2 := LR(1)(L)$:	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L \cdot, \varepsilon]$		
$I'_3 := LR(1)(R)$:	$[S \to R \cdot, \varepsilon]$			
$I'_4 := LR(1)(*)$:	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a})$:	$[L \to \mathrm{a} \cdot, =]$	$[L \to \mathrm{a} \cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=})$:	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R)$:	$[L \to *R \cdot, =]$	$[L \to *R \cdot, \varepsilon]$		
$I'_8 := LR(1)(*L)$:	$[R \to L \cdot, =]$	$[R \to L \cdot, \varepsilon]$		
$I'_9 := LR(1)(L{=}R)$:	$[S \to L{=}R \cdot, \varepsilon]$			
$I'_{10} := LR(1)(L{=}L)$:	$[R \to L \cdot, \varepsilon]$			
$I'_{11} := LR(1)(L{=}*)$:	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_{12} := LR(1)(L{=}\mathrm{a})$:	$[L \to \mathrm{a} \cdot, \varepsilon]$			
$I'_{13} := LR(1)(L{=}*R)$:	$[L \to *R \cdot, \varepsilon]$			

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$$
\begin{array}{lllll}
I'_0 := LR(1)(\varepsilon): & [S' \to \cdot S, \varepsilon] & [S \to \cdot L{=}R, \varepsilon] & [S \to \cdot R, \varepsilon] & [L \to \cdot *R, =] \\
 & [L \to \cdot \mathrm{a}, =] & [R \to \cdot L, \varepsilon] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_1 := LR(1)(S): & [S' \to S \cdot, \varepsilon] & & & \\
I'_2 := LR(1)(L): & [S \to L \cdot {=}R, \varepsilon] & [R \to L \cdot, \varepsilon] & & \\
I'_3 := LR(1)(R): & [S \to R \cdot, \varepsilon] & & & \\
I'_4 := LR(1)(*): & [L \to * \cdot R, =] & [L \to * \cdot R, \varepsilon] & [R \to \cdot L, =] & [R \to \cdot L, \varepsilon] \\
 & [L \to \cdot *R, =] & [L \to \cdot \mathrm{a}, =] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_5 := LR(1)(\mathrm{a}): & [L \to \mathrm{a} \cdot, =] & [L \to \mathrm{a} \cdot, \varepsilon] & & \\
I'_6 := LR(1)(L{=}): & [S \to L{=} \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_7 := LR(1)(*R): & [L \to *R \cdot, =] & [L \to *R \cdot, \varepsilon] & & \\
I'_8 := LR(1)(*L): & [R \to L \cdot, =] & [R \to L \cdot, \varepsilon] & & \\
I'_9 := LR(1)(L{=}R): & [S \to L{=}R \cdot, \varepsilon] & & & \\
I'_{10} := LR(1)(L{=}L): & [R \to L \cdot, \varepsilon] & & & \\
I'_{11} := LR(1)(L{=}*): & [L \to * \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_{12} := LR(1)(L{=}\mathrm{a}): & [L \to \mathrm{a} \cdot, \varepsilon] & & & \\
I'_{13} := LR(1)(L{=}*R): & [L \to *R \cdot, \varepsilon] & & & \\
I'_{14} := \emptyset & & & &
\end{array}
$$

Computing $LR(1)$ Sets II

Example 10.6 (cf. Example 9.15)

$LR(1)(G_{LR})$ for $G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$I'_0 := LR(1)(\varepsilon):$	$[S' \to \cdot S, \varepsilon]$	$[S \to \cdot L{=}R, \varepsilon]$	$[S \to \cdot R, \varepsilon]$	$[L \to \cdot *R, =]$
	$[L \to \cdot \mathrm{a}, =]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_1 := LR(1)(S):$	$[S' \to S\cdot, \varepsilon]$			
$I'_2 := LR(1)(L):$	$[S \to L \cdot {=}R, \varepsilon]$	$[R \to L\cdot, \varepsilon]$		
$I'_3 := LR(1)(R):$	$[S \to R\cdot, \varepsilon]$			
$I'_4 := LR(1)(*):$	$[L \to * \cdot R, =]$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, =]$	$[R \to \cdot L, \varepsilon]$
	$[L \to \cdot *R, =]$	$[L \to \cdot \mathrm{a}, =]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_5 := LR(1)(\mathrm{a}):$	$[L \to \mathrm{a}\cdot, =]$	$[L \to \mathrm{a}\cdot, \varepsilon]$		
$I'_6 := LR(1)(L{=}):$	$[S \to L{=} \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_7 := LR(1)(*R):$	$[L \to *R\cdot, =]$	$[L \to *R\cdot, \varepsilon]$		
$I'_8 := LR(1)(*L):$	$[R \to L\cdot, =]$	$[R \to L\cdot, \varepsilon]$		
$I'_9 := LR(1)(L{=}R):$	$[S \to L{=}R\cdot, \varepsilon]$			
$I'_{10} := LR(1)(L{=}L):$	$[R \to L\cdot, \varepsilon]$			
$I'_{11} := LR(1)(L{=}*):$	$[L \to * \cdot R, \varepsilon]$	$[R \to \cdot L, \varepsilon]$	$[L \to \cdot *R, \varepsilon]$	$[L \to \cdot \mathrm{a}, \varepsilon]$
$I'_{12} := LR(1)(L{=}\mathrm{a}):$	$[L \to \mathrm{a}\cdot, \varepsilon]$			
$I'_{13} := LR(1)(L{=}*R):$	$[L \to *R\cdot, \varepsilon]$			
$I'_{14} := \emptyset$				

In I'_2: shift on $=$/reduce on $\varepsilon \implies G_{LR} \in LR(1)$

The $LR(1)$ Action Function

Definition 10.7 ($LR(1)$ action function)

The $LR(1)$ action function

$$\mathrm{act} : LR(1)(G) \times \Sigma_\varepsilon \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot, x] \in I \\ \text{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \to S\cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

The $LR(1)$ Action Function

Definition 10.7 ($LR(1)$ action function)

The $LR(1)$ action function

$$\mathrm{act} : LR(1)(G) \times \Sigma_\varepsilon \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot, x] \in I \\ \text{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \to S\cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Corollary 10.8

For every $G \in CFG_\Sigma$, $G \in LR(1)$ iff its $LR(1)$ action function is well defined.

The $LR(1)$ goto Function

The goto function is defined in analogy to the $LR(0)$ case (Definition 9.1).

Definition 10.9 ($LR(1)$ goto function)

The function $\mathrm{goto} : LR(1)(G) \times X \to LR(1)(G)$ is determined by

$$\mathrm{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that } I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y).$$

The $LR(1)$ goto Function

The goto function is defined in analogy to the $LR(0)$ case (Definition 9.1).

Definition 10.9 ($LR(1)$ goto function)

The function $\mathrm{goto} : LR(1)(G) \times X \to LR(1)(G)$ is determined by

$$\mathrm{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that } I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y).$$

Again, act and goto form the $LR(1)$ parsing table of G.

The $LR(1)$ Parsing Table

Example 10.10 (cf. Example 10.6)

$I'_0 := LR(1)(\varepsilon) :$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot {*}R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot {*}R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S) :$ $[S' \to S \cdot, \varepsilon]$

$I'_2 := LR(1)(L) :$ $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L \cdot, \varepsilon]$

$I'_3 := LR(1)(R) :$ $[S \to R \cdot, \varepsilon]$

$I'_4 := LR(1)(*) :$ $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot {*}R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[L \to \cdot {*}R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_5 := LR(1)(\mathrm{a}) :$ $[L \to \mathrm{a} \cdot, =]$ $[L \to \mathrm{a} \cdot, \varepsilon]$

$I'_6 := LR(1)(L{=}) :$ $[S \to L{=} \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot {*}R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_7 := LR(1)({*}R) :$ $[L \to {*}R \cdot, =]$ $[L \to {*}R \cdot, \varepsilon]$

$I'_8 := LR(1)({*}L) :$ $[R \to L \cdot, =]$ $[R \to L \cdot, \varepsilon]$

$I'_9 := LR(1)(L{=}R) :$ $[S \to L{=}R \cdot, \varepsilon]$

$I'_{10} := LR(1)(L{=}L) :$ $[R \to L \cdot, \varepsilon]$

$I'_{11} := LR(1)(L{=}*) :$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot {*}R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_{12} := LR(1)(L{=}\mathrm{a}) :$ $[L \to \mathrm{a} \cdot, \varepsilon]$

$I'_{13} := LR(1)(L{=}{*}R) :$ $[L \to {*}R \cdot, \varepsilon]$

$I'_{14} := \emptyset$

	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5		red 5						
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

The $LR(1)$ Parsing Automaton I

Definition 10.11 ($LR(1)$ parsing automaton)

The $LR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 9.6), except for the transition relation:

shift: $(aw, \alpha I, z) \vdash (w, \alpha I I', z)$ if $\mathrm{act}(I, a) = \text{shift}$ and $\mathrm{goto}(I, a) = I'$

reduce$_a$: $(aw, \alpha I I_1 \ldots I_n, z) \vdash (aw, \alpha I I', zi)$ if $\mathrm{act}(I_n, a) = \text{red } i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

reduce$_\varepsilon$: $(\varepsilon, \alpha I I_1 \ldots I_n, z) \vdash (\varepsilon, \alpha I I', zi)$ if $\mathrm{act}(I_n, \varepsilon) = \text{red } i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0)$ if $\mathrm{act}(I, \varepsilon) = \text{accept}$

error$_a$: $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error})$ if $\mathrm{act}(I, a) = \text{error}$

error$_\varepsilon$: $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error})$ if $\mathrm{act}(I, \varepsilon) = \text{error}$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to *R \mid \mathrm{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR} : S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to *R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$(\mathtt{a{=}{*}a},\ I'_0 \qquad\qquad ,\ \varepsilon \qquad\qquad)$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to *R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$(\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon)$$
$$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon)$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=∗a:

$$(\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon)$$
$$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon)$$
$$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4)$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S \; (0) \quad S \to L{=}R \mid R \; (1,2) \quad L \to *R \mid \mathtt{a} \; (3,4) \quad R \to L \; (5)$

$LR(1)(G_{LR})$	act $*$	act $=$	act a	act ε	goto S	goto L	goto R	goto $*$	goto $=$	goto a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{aligned}
& (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash\ & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4)
\end{aligned}
$$

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The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{array}{ll}
 & (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4)
\end{array}
$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to *R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{array}{rl}
 & (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4)
\end{array}
$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR} : S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act $*$	act $=$	act a	act ε	goto S	goto L	goto R	goto $*$	goto $=$	goto a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{aligned}
& (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash\ & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash\ & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44)
\end{aligned}
$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR} : S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{aligned}
& (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash\ & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash\ & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445)
\end{aligned}
$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR} : S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=∗a:

$$
\begin{aligned}
& (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash\ & ({=}{*}\mathtt{a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash\ & ({=}{*}\mathtt{a},\ I'_0 I'_2,\ 4) \\
\vdash\ & ({*}\mathtt{a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash\ & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{10},\ 4453)
\end{aligned}
$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act $*$	act $=$	act a	act ε	goto S	goto L	goto R	goto $*$	goto $=$	goto a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{array}{ll}
 & (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{10},\ 4453) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_9,\ 44535)
\end{array}
$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of $\mathtt{a{=}{*}a}$:

$$
\begin{array}{rlll}
 & (\mathtt{a{=}{*}a}, & I'_0 & , \varepsilon\,) \\
\vdash & (\mathtt{{=}{*}a}, & I'_0 I'_5 & , \varepsilon\,) \\
\vdash & (\mathtt{{=}{*}a}, & I'_0 I'_2 & , 4\,) \\
\vdash & (\mathtt{{*}a}, & I'_0 I'_2 I'_6 & , 4\,) \\
\vdash & (\mathtt{a}, & I'_0 I'_2 I'_6 I'_{11} & , 4\,) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_{11} I'_{12} & , 4\,) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_{11} I'_{10} & , 44\,) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_{11} I'_{13} & , 445\,) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_{10} & , 4453\,) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_9 & , 44535\,) \\
\vdash & (\varepsilon, & I'_0 I'_1 & , 445351\,)
\end{array}
$$

The $LR(1)$ Parsing Automaton II

Example 10.12 (cf. Example 10.6)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to *R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

$LR(1)(G_{LR})$	act				goto					
	$*$	$=$	a	ε	S	L	R	$*$	$=$	a
I'_0	shift		shift		I'_1	I'_2	I'_3	I'_4		I'_5
I'_1				accept						
I'_2		shift		red 5					I'_6	
I'_3				red 2						
I'_4	shift		shift			I'_8	I'_7	I'_4		I'_5
I'_5		red 4		red 4						
I'_6	shift		shift			I'_{10}	I'_9	I'_{11}		I'_{12}
I'_7		red 3		red 3						
I'_8		red 5								
I'_9				red 1						
I'_{10}				red 5						
I'_{11}	shift		shift			I'_{10}	I'_{13}	I'_{11}		I'_{12}
I'_{12}				red 4						
I'_{13}				red 3						

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{aligned}
& (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash\ & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash\ & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash\ & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{10},\ 4453) \\
\vdash\ & (\varepsilon,\ I'_0 I'_2 I'_6 I'_9,\ 44535) \\
\vdash\ & (\varepsilon,\ I'_0 I'_1,\ 445351) \\
\vdash\ & (\varepsilon,\ \varepsilon,\ 4453510)
\end{aligned}
$$

Outline of Lecture 10

Recap: $LR(0)$ and $SLR(1)$ Parsing

$LR(1)$ Parsing

$LALR(1)$ Parsing

Bottom-Up Parsing of Ambiguous Grammars

Expressiveness of LL and LR Grammars

$LALR(1)$ Parsing

- **Motivation:** resolving conflicts using $LR(1)$ too expensive
- Example 9.15/10.6: $|LR(0)(G_{LR})| = 11$, $|LR(1)(G_{LR})| = 15$

$LALR(1)$ Parsing

- **Motivation:** resolving conflicts using $LR(1)$ too expensive
- Example 9.15/10.6: $|LR(0)(G_{LR})| = 11$, $|LR(1)(G_{LR})| = 15$
- Empirical evaluations:
  - A. Johnstone, E. Scott: *Generalised Reduction Modified LR Parsing for Domain Specific Language Prototyping*, HICSS '02, IEEE, 2002
  - X. Chen, D. Pager: *Full LR(1) Parser Generator Hyacc and Study on the Performance of LR(1) Algorithms*, C3S2E '11, ACM, 2011

Grammar	$\|LR(0)(G)\|$	$\|LR(1)(G)\|$
Pascal	368	1395
Ansi-C	381	1788
C++	1236	9723

$LR(0)$ Equivalence

Observation: potential redundancy by containment of $LR(0)$ sets in $LR(1)$ sets (cf. Corollary 10.2)

$LR(0)$ Equivalence

Observation: potential redundancy by containment of $LR(0)$ sets in $LR(1)$ sets (cf. Corollary 10.2)

Definition 10.13 ($LR(0)$ equivalence)

Let $\mathrm{lr}_0 : LR(1)(G) \to LR(0)(G)$ be defined by

$$\mathrm{lr}_0(I) := \{[A \to \beta_1 \cdot \beta_2] \mid [A \to \beta_1 \cdot \beta_2, x] \in I\}.$$

Two sets $I_1, I_2 \in LR(1)(G)$ are called $LR(0)$-equivalent (notation: $I_1 \sim_0 I_2$) if $\mathrm{lr}_0(I_1) = \mathrm{lr}_0(I_2)$.

$LALR(1)$ Parsing

$LALR(1)$ Sets

Corollary 10.14

For every $G \in CFG_\Sigma$, $|LR(1)(G)/\sim_0| = |LR(0)(G)|$.

$LALR(1)$ Parsing

$LALR(1)$ Sets

Corollary 10.14

For every $G \in CFG_\Sigma$, $|LR(1)(G)/\sim_0| = |LR(0)(G)|$.

Idea: merge $LR(0)$-equivalent $LR(1)$ sets
(maintaining the lookahead information, but possibly introducing conflicts)

$LALR(1)$ Sets

Corollary 10.14

For every $G \in CFG_\Sigma$, $|LR(1)(G)/\sim_0| = |LR(0)(G)|$.

Idea: merge $LR(0)$-equivalent $LR(1)$ sets
(maintaining the lookahead information, but possibly introducing conflicts)

Definition 10.15 ($LALR(1)$ sets)

Let $G \in CFG_\Sigma$.

- An information $I \in LR(1)(G)$ determines the $LALR(1)$ set

$$\bigcup [I]_{\sim_0} = \bigcup \{I' \in LR(1)(G) \mid I' \sim_0 I\}.$$

- The set of all $LALR(1)$ sets of G is denoted by $LALR(1)(G)$.

$LALR(1)$ Sets

Corollary 10.14

For every $G \in CFG_\Sigma$, $|LR(1)(G)/\sim_0| = |LR(0)(G)|$.

Idea: merge $LR(0)$-equivalent $LR(1)$ sets
(maintaining the lookahead information, but possibly introducing conflicts)

Definition 10.15 ($LALR(1)$ sets)

Let $G \in CFG_\Sigma$.

- An information $I \in LR(1)(G)$ determines the $LALR(1)$ set
$$\bigcup [I]_{\sim_0} = \bigcup \{I' \in LR(1)(G) \mid I' \sim_0 I\}.$$
- The set of all $LALR(1)$ sets of G is denoted by $LALR(1)(G)$.

Remark: by Corollary 10.14, $|LALR(1)(G)| = |LR(0)(G)|$
(but $LALR(1)$ sets provide additional lookahead information)

$LALR(1)$ Parsing

Sketch of $LALR(1)$ Parsing

- $LALR(1)$ action function

$$\mathrm{act} : LALR(1)(G) \times \Sigma_\varepsilon \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$$

  defined in analogy to the $LR(1)$ case (Definition 10.7)
- $G \in CFG_\Sigma$ has the $LALR(1)$ property ($G \in LALR(1)$) if its $LALR(1)$ action function is well defined
- Also $LR(1)$ goto function (Definition 10.9) carries over to the $LALR(1)$ case:

$$\mathrm{goto} : LALR(1)(G) \times X \to LALR(1)(G)$$

- act and goto form the $LALR(1)$ parsing table
- **But:** merging of $LR(1)$ sets can produce new conflicts
- $LALR(1)$ used by `yacc`/`bison` parser generator (later)

Outline of Lecture 10

Recap: $LR(0)$ and $SLR(1)$ Parsing

$LR(1)$ Parsing

$LALR(1)$ Parsing

Bottom-Up Parsing of Ambiguous Grammars

Expressiveness of LL and LR Grammars

Ambiguous Grammars

Reminder (Definition 5.5): $G \in CFG_\Sigma$ is called unambiguous if every word $w \in L(G)$ has exactly one syntax tree. Otherwise it is called ambiguous.

Ambiguous Grammars

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Lemma 10.16

If $G \in CFG_\Sigma$ is ambiguous, then $G \notin \bigcup_{k \in \mathbb{N}} LR(k)$.

Ambiguous Grammars

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Lemma 10.16

If $G \in CFG_\Sigma$ is ambiguous, then $G \notin \bigcup_{k \in \mathbb{N}} LR(k)$.

Proof.

Assume that there exist $k \in \mathbb{N}$ and $G \in LR(k)$ such that G is ambiguous.

Ambiguous Grammars

Reminder (Definition 5.5): $G \in CFG_\Sigma$ is called unambiguous if every word $w \in L(G)$ has exactly one syntax tree. Otherwise it is called ambiguous.

Lemma 10.16

If $G \in CFG_\Sigma$ is ambiguous, then $G \notin \bigcup_{k \in \mathbb{N}} LR(k)$.

Proof.

Assume that there exist $k \in \mathbb{N}$ and $G \in LR(k)$ such that G is ambiguous.
Hence there exists $w \in L(G)$ with different right derivations. Let $\alpha A v$ be the last common sentence of the two derivations (i.e., $\beta \neq \beta'$):

$$S \Rightarrow_r^* \alpha A v \begin{cases} \Rightarrow_r \alpha\beta v \Rightarrow_r^* w \\ \Rightarrow_r \alpha\beta' v \Rightarrow_r^* w \end{cases}$$

Bottom-Up Parsing of Ambiguous Grammars

Ambiguous Grammars

Reminder (Definition 5.5): $G \in CFG_\Sigma$ is called unambiguous if every word $w \in L(G)$ has exactly one syntax tree. Otherwise it is called ambiguous.

Lemma 10.16

If $G \in CFG_\Sigma$ is ambiguous, then $G \notin \bigcup_{k \in \mathbb{N}} LR(k)$.

Proof.

Assume that there exist $k \in \mathbb{N}$ and $G \in LR(k)$ such that G is ambiguous.
Hence there exists $w \in L(G)$ with different right derivations. Let $\alpha A v$ be the last common sentence of the two derivations (i.e., $\beta \neq \beta'$):

$$S \Rightarrow_r^* \alpha A v \begin{cases} \Rightarrow_r \alpha\beta v \Rightarrow_r^* w \\ \Rightarrow_r \alpha\beta' v \Rightarrow_r^* w \end{cases}$$

But since $\mathrm{first}_k(v) = \mathrm{first}_k(v)$ for every $v \in \Sigma^*$, Definition 8.8 yields $\beta = \beta'$. ↯ □

Ambiguous Grammars

Reminder (Definition 5.5): $G \in CFG_\Sigma$ is called unambiguous if every word $w \in L(G)$ has exactly one syntax tree. Otherwise it is called ambiguous.

Lemma 10.16

If $G \in CFG_\Sigma$ is ambiguous, then $G \notin \bigcup_{k \in \mathbb{N}} LR(k)$.

Proof.

Assume that there exist $k \in \mathbb{N}$ and $G \in LR(k)$ such that G is ambiguous.
Hence there exists $w \in L(G)$ with different right derivations. Let $\alpha A v$ be the last common sentence of the two derivations (i.e., $\beta \neq \beta'$):

$$S \Rightarrow_r^* \alpha A v \begin{cases} \Rightarrow_r \alpha\beta v \Rightarrow_r^* w \\ \Rightarrow_r \alpha\beta' v \Rightarrow_r^* w \end{cases}$$

But since $\mathrm{first}_k(v) = \mathrm{first}_k(v)$ for every $v \in \Sigma^*$, Definition 8.8 yields $\beta = \beta'$. ↯ □

However ambiguity is a natural specification method which generally avoids involved syntactic constructs.

Bottom-Up Parsing of Ambiguous Grammars I

Example 10.17 (Simple arithmetic expressions)

$G : E' \to E \quad (0) \qquad E \to E{+}E \mid E{*}E \mid \mathtt{a} \quad (1, 2, 3)$

Bottom-Up Parsing of Ambiguous Grammars I

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Precedence: $* > +$ Associativity: left (thus: $\mathtt{a+a*a+a} := \mathtt{(a+(a*a))+a}$)

Bottom-Up Parsing of Ambiguous Grammars I

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$LR(0)(G)$: $I_0 := LR(0)(\varepsilon) : \quad [E' \to \cdot E] \quad [E \to \cdot E{+}E] \; [E \to \cdot E{*}E] \; [E \to \cdot \mathtt{a}]$

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$I_2 := LR(0)(\mathtt{a}) : \quad [E \to \mathtt{a}\cdot]$

$I_3 := LR(0)(E{+}) : \quad [E \to E{+} \cdot E] \; [E \to \cdot E{+}E] \; [E \to \cdot E{*}E] \; [E \to \cdot \mathtt{a}]$

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Conflicts: I_1: $SLR(1)$-solvable (reduce on ε, shift on $+/*$)

Bottom-Up Parsing of Ambiguous Grammars I

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$I_2 := LR(0)(\mathtt{a})$: $[E \to \mathtt{a}\cdot]$

$I_3 := LR(0)(E+)$: $[E \to E{+} \cdot E]$ $[E \to \cdot E{+}E]$ $[E \to \cdot E{*}E]$ $[E \to \cdot \mathtt{a}]$

$I_4 := LR(0)(E*)$: $[E \to E{*} \cdot E]$ $[E \to \cdot E{+}E]$ $[E \to \cdot E{*}E]$ $[E \to \cdot \mathtt{a}]$

$I_5 := LR(0)(E{+}E)$: $[E \to E{+}E\cdot]$ $[E \to E \cdot {+}E]$ $[E \to E \cdot {*}E]$

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I_5, I_6: not $SLR(1)$-solvable ($+, * \in \mathrm{fo}(E)$)

Bottom-Up Parsing of Ambiguous Grammars I

Example 10.17 (Simple arithmetic expressions)

$G: E' \to E \quad (0) \qquad E \to E{+}E \mid E{*}E \mid \mathtt{a} \quad (1,2,3)$

Precedence: $* > +$ Associativity: left (thus: $\mathtt{a{+}a{*}a{+}a} := \mathtt{(a{+}(a{*}a)){+}a}$)

$LR(0)(G)$: $I_0 := LR(0)(\varepsilon): \quad [E' \to \cdot E] \quad [E \to \cdot E{+}E] \; [E \to \cdot E{*}E] \; [E \to \cdot \mathtt{a}]$

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$I_2 := LR(0)(\mathtt{a}): \quad [E \to \mathtt{a}\cdot]$

$I_3 := LR(0)(E{+}): \quad [E \to E{+} \cdot E] \; [E \to \cdot E{+}E] \; [E \to \cdot E{*}E] \; [E \to \cdot \mathtt{a}]$

$I_4 := LR(0)(E{*}): \quad [E \to E{*} \cdot E] \; [E \to \cdot E{+}E] \; [E \to \cdot E{*}E] \; [E \to \cdot \mathtt{a}]$

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Conflicts: I_1: $SLR(1)$-solvable (reduce on ε, shift on $+/*$)

I_5, I_6: not $SLR(1)$-solvable ($+, * \in \mathrm{fo}(E)$)

Solution: I_5: $* > + \implies \mathrm{act}(I_5, *) :=$ shift, $+$ left assoc. $\implies \mathrm{act}(I_5, +) :=$ red 1

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$I_6 := LR(0)(E{*}E)$: $[E \to E{*}E\cdot]$ $[E \to E \cdot {+}E]$ $[E \to E \cdot {*}E]$

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Solution: I_5: $* > + \implies \mathrm{act}(I_5, *) :=$ shift, $+$ left assoc. $\implies \mathrm{act}(I_5, +) :=$ red 1

I_6: $* > + \implies \mathrm{act}(I_6, +) :=$ red 2, $*$ left assoc. $\implies \mathrm{act}(I_6, *) :=$ red 2

Bottom-Up Parsing of Ambiguous Grammars II

Example 10.18 ("Dangling `else`")

$G : S' \to S \quad S \to \mathtt{i}S\mathtt{e}S \mid \mathtt{i}S \mid \mathtt{a}$

Bottom-Up Parsing of Ambiguous Grammars II

Example 10.18 (“Dangling `else`”)

$G : S' \to S \quad S \to \mathtt{i}S\mathtt{e}S \mid \mathtt{i}S \mid \mathtt{a}$

Ambiguity: `iiaea` := (1) `i(iaea)` (common) or (2) `i(ia)ea`

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$LR(0)(G)$: $I_0 := LR(0)(\varepsilon)$: $[S' \to \cdot S]\ [S \to \cdot \mathtt{i}S\mathtt{e}S]\ [S \to \cdot \mathtt{i}S]\ [S \to \cdot \mathtt{a}]$

$I_1 := LR(0)(S)$: $[S' \to S \cdot]$

$I_2 := LR(0)(\mathtt{i})$: $[S \to \mathtt{i} \cdot S\mathtt{e}S]\ [S \to \mathtt{i} \cdot S]\ [S \to \cdot \mathtt{i}S\mathtt{e}S]$ $[S \to \cdot \mathtt{i}S]\ [S \to \cdot \mathtt{a}]$

$I_3 := LR(0)(\mathtt{a})$: $[S \to \mathtt{a} \cdot]$

$I_4 := LR(0)(\mathtt{i}S)$: $[S \to \mathtt{i}S \cdot \mathtt{e}S]\ [S \to \mathtt{i}S \cdot]$

$I_5 := LR(0)(\mathtt{i}S\mathtt{e})$: $[S \to \mathtt{i}S\mathtt{e} \cdot S]\ [S \to \cdot \mathtt{i}S\mathtt{e}S]\ [S \to \cdot \mathtt{i}S]\ [S \to \cdot \mathtt{a}]$

$I_6 := LR(0)(\mathtt{i}S\mathtt{e}S)$: $[S \to \mathtt{i}S\mathtt{e}S \cdot]$

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$LR(0)(G)$: $I_0 := LR(0)(\varepsilon) : \quad [S' \to \cdot S]\ [S \to \cdot \mathtt{i}S\mathtt{e}S]\ [S \to \cdot \mathtt{i}S]\ [S \to \cdot \mathtt{a}]$

$I_1 := LR(0)(S) : \quad [S' \to S\cdot]$

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$I_6 := LR(0)(\mathtt{i}S\mathtt{e}S) : [S \to \mathtt{i}S\mathtt{e}S\cdot]$

Conflict in I_4: $\mathtt{e} \in \mathrm{fo}(S) \implies$ not $SLR(1)$-solvable

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$LR(0)(G)$:
$$
\begin{array}{ll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S]\ [S \to \cdot \mathtt{i}S\mathtt{e}S]\ [S \to \cdot \mathtt{i}S]\ [S \to \cdot \mathtt{a}] \\
I_1 := LR(0)(S): & [S' \to S\cdot] \\
I_2 := LR(0)(\mathtt{i}): & [S \to \mathtt{i} \cdot S\mathtt{e}S]\ [S \to \mathtt{i} \cdot S]\ [S \to \cdot \mathtt{i}S\mathtt{e}S] \\
 & [S \to \cdot \mathtt{i}S]\ [S \to \cdot \mathtt{a}] \\
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I_4 := LR(0)(\mathtt{i}S): & [S \to \mathtt{i}S \cdot \mathtt{e}S]\ [S \to \mathtt{i}S\cdot] \\
I_5 := LR(0)(\mathtt{i}S\mathtt{e}): & [S \to \mathtt{i}S\mathtt{e} \cdot S]\ [S \to \cdot \mathtt{i}S\mathtt{e}S]\ [S \to \cdot \mathtt{i}S]\ [S \to \cdot \mathtt{a}] \\
I_6 := LR(0)(\mathtt{i}S\mathtt{e}S): & [S \to \mathtt{i}S\mathtt{e}S\cdot]
\end{array}
$$

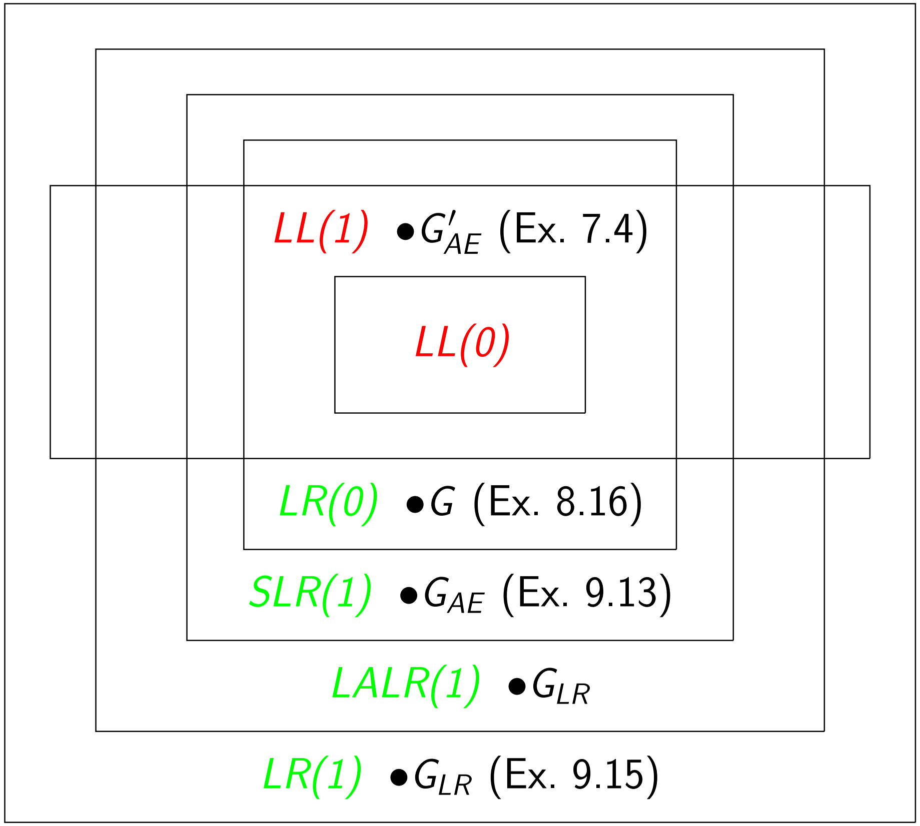
Conflict in I_4: $\mathtt{e} \in \mathrm{fo}(S) \implies$ not $SLR(1)$-solvable

Solution (1): $\mathrm{act}(I_4, \mathtt{e}) :=$ shift

Outline of Lecture 10

Expressiveness of LL and LR Grammars

Overview of Grammar Classes

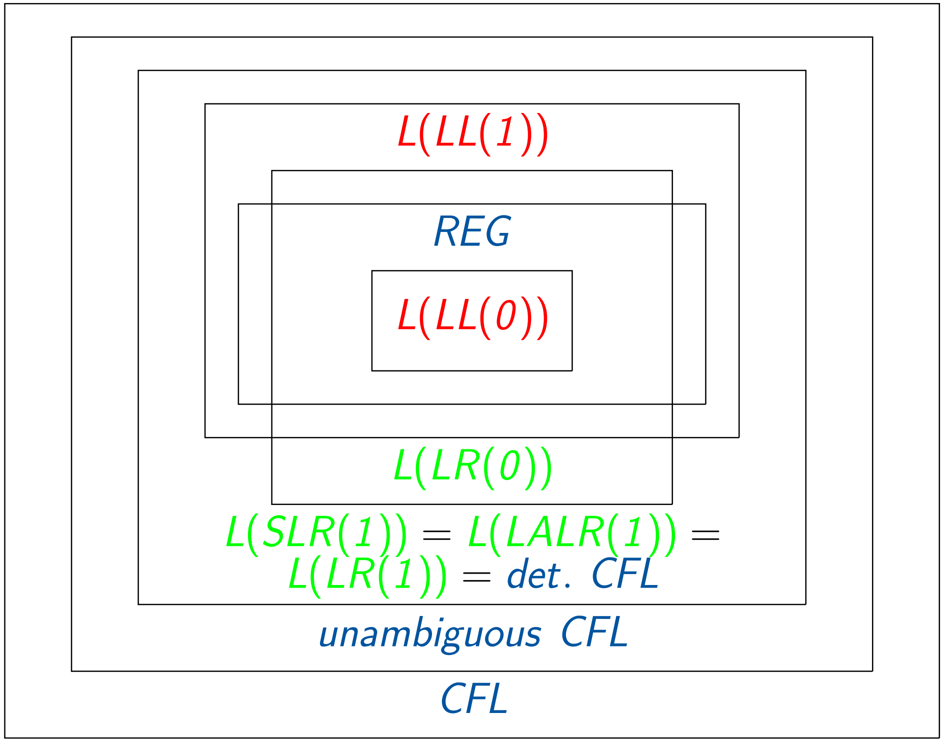


Moreover:

- $LL(k) \subsetneq LL(k+1)$ for every $k \in \mathbb{N}$
- $LR(k) \subsetneq LR(k+1)$ for every $k \in \mathbb{N}$
- $LL(k) \subseteq LR(k)$ for every $k \in \mathbb{N}$

Overview of Language Classes

(cf. O. Mayer: *Syntaxanalyse*, BI-Verlag, 1978, p. 409ff)



Moreover:

- $L(LL(k)) \subsetneq L(LL(k+1)) \subsetneq L(LR(1))$ for every $k \in \mathbb{N}$
- $L(LR(k)) = L(LR(1))$ for every $k \geq 1$

Expressiveness of LL and LR Grammars

Why $REG \not\subseteq L(LR(0))$?

Definition 10.19

A language $L \subseteq \Sigma^*$ is called prefix-free if $L \cap L \cdot \Sigma^+ = \emptyset$, i.e., if no proper prefix of an element of L is again in L.

Expressiveness of LL and LR Grammars

Why $REG \not\subseteq L(LR(0))$?

Definition 10.19

A language $L \subseteq \Sigma^*$ is called prefix-free if $L \cap L \cdot \Sigma^+ = \emptyset$, i.e., if no proper prefix of an element of L is again in L.

Lemma 10.20

$G \in LR(0) \implies L(G)$ *prefix-free*

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Proof.

on the board □

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Proof.

on the board □

Corollary 10.21

$\{a, aa\} \in REG \setminus L(LR(0))$

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$G \in LR(0) \implies L(G)$ *prefix-free*

Proof.

on the board □

Corollary 10.21

$\{a, aa\} \in REG \setminus L(LR(0))$

Conjecture: $L \in REG \setminus L(LR(0)) \implies L(G)$ not prefix-free?

Expressiveness of LL and LR Grammars

Why $REG \subseteq L(LL(1))$?

Lemma 10.22 (cf. Lecture 2)

Every $L \in REG$ can be recognized by a DFA.

Expressiveness of LL and LR Grammars

Why $REG \subseteq L(LL(1))$?

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Every DFA can be transformed into an equivalent $LL(1)$ grammar.

Expressiveness of LL and LR Grammars

Why $REG \subseteq L(LL(1))$?

Lemma 10.22 (cf. Lecture 2)

Every $L \in REG$ can be recognized by a DFA.

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Every DFA can be transformed into an equivalent $LL(1)$ grammar.

Proof.

see Exercise 4.1 □