apl. Prof. Dr. Thomas Noll

Sebastian Junges, Christoph Matheja, Matthias Volk

Compiler Construction 2017
— Exercise Sheet 3 —

Hand in until May 30th before the exercise class.

General Remarks

- There is *no* practical exercise this week.
- Please hand in a *single* solution of the theoretical exercises. We will ignore submissions consisting of several sheets that are not stapled.

Exercise 1 (2 Points)

Consider the context-free grammar G given by the following rules:

$$S \;\to\; S+S \mid S\,S \mid (\,S\,) \mid S^* \mid a$$

(a) Provide a leftmost analysis of the string $(a+a)^*a$.

(b) Provide a rightmost analysis of the string $(a+a)^*a$.

(c) Prove or disprove: G is unambiguous.

Exercise 2 (2 Points)

Complete the correctness proof of Theorem 6.1 by showing the direction omitted in the lecture. More precisely, let $G = \langle N, \Sigma, P, S\rangle \in \mathrm{CFG}_\Sigma$ and $NTA(G)$ as in the lecture (lecture 6, slide 4). Show that for each $w \in \Sigma^*$ and $z \in [p]^*$ it holds that

$$(w, S, \varepsilon) \;\vdash^*\; (\varepsilon, \varepsilon, z) \quad \text{implies} \quad z \text{ is a leftmost analysis of } w.$$

Exercise 3 (1 Points)

Show that for each context free grammar G it holds that $G \in LL(1)$ implies that G is unambiguous.

Exercise 4 (2 Points)

Two characterizations of $LL(1)$ have been given in the lecture.

First, by Lemma 6.5, a context free grammar $G = \langle N, \Sigma, P, S\rangle$ is in $LL(1)$ if and only if for all leftmost derivations of the form

$$S \;\Rightarrow_l^*\; wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \\ \Rightarrow_l w\gamma\alpha \end{cases}$$

such that $\beta \neq \gamma$, it follows that $first_1(\beta\alpha) \cap first_1(\gamma\alpha) = \emptyset$.

Second, by Theorem 6.10, G is in $LL(1)$ if and only if for all pairs of rules $A \to \beta|\gamma \in P$, where $\beta \neq \gamma$, we have

$$la(A \to \beta) \cap la(A \to \gamma) \ = \ \emptyset.$$

We now lift the latter statement to $LL(k)$ for arbitrary $k \in \mathbb{N}_{>0}$:

Definition: $G \in LL(k)$ iff for all pairs of rules $A \to \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$la_k(A \to \beta) \cap la_k(A \to \gamma) = \emptyset$$

where $la_k(A \to \beta) = \text{first}_k(\beta \ \text{follow}_k(A))$.

Show that the two characterizations are not equivalent for $k > 1$ by giving a grammar that is in $LL(2)$ according to the first characterization, but not according to the lifted version of the second one.

Exercise 5 (3 Points)

Consider the following grammar G:

$$\begin{aligned} S &\to (L) \mid a \\ L &\to L,S \mid L,SS \mid S \mid SS \end{aligned}$$

(a) Show that $G \notin LL(1)$.

(b) Transform G into an equivalent grammar in $LL(1)$, i.e. provide a grammar $G' \in LL(1)$ such that $L(G') = L(G)$.

(c) Prove that G' has the $LL(1)$ property.