Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

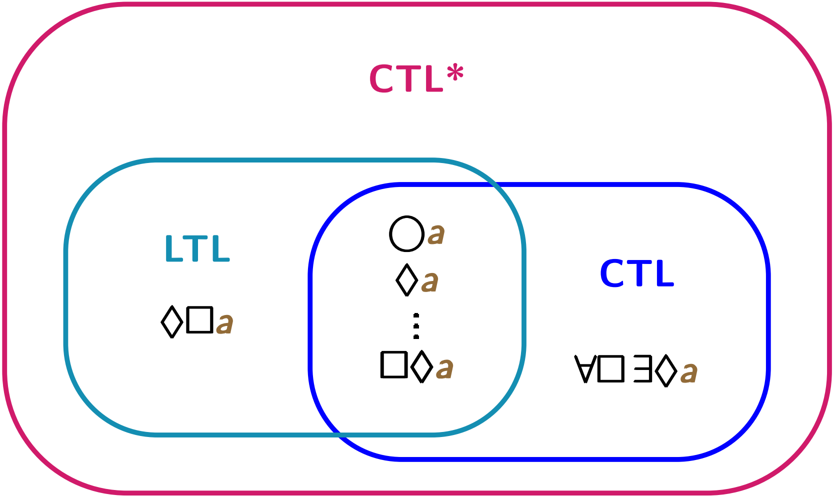
Linear Temporal Logic (LTL)

Computation Tree Logic

- syntax and semantics of CTL
- expressiveness of CTL and LTL
- CTL model checking
- fairness, counterexamples/witnesses
- CTL^+ and CTL^* ⟵

Equivalences and Abstraction

Combining LTL and CTL $\rightsquigarrow$ CTL*



Syntax of CTL*

state formulas:

$$\Phi \ ::= \ \textit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

path formulas:

$$\varphi \ ::= \ \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

derived operators:

- $\vee$, $\longrightarrow$, etc.
- eventually, always as in **LTL**:

$$\Diamond\varphi = \textit{true}\,\mathsf{U}\,\varphi, \quad \Box\varphi = \neg\Diamond\neg\varphi$$

- universal quantification: $\forall\varphi = \neg\exists\neg\varphi$

Semantics of CTL*

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system without terminal states.

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define by structural induction:

- a satisfaction relation $\models$ for states $s \in S$ and **CTL*** state formulas
- a satisfaction relation $\models$ for infinite path fragments π in $\mathcal{T}$ and **CTL*** path formulas

Semantics of CTL* state formulas

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$s \models true$		
$s \models a$	iff	$a \in L(s)$
$s \models \neg\Phi$	iff	$s \not\models \Phi$
$s \models \Phi_1 \wedge \Phi_2$	iff	$s \models \Phi_1$ and $s \models \Phi_2$
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↑ satisfaction relation $\models$ for **CTL*** path formulas

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$\pi \models \bigcirc\varphi$	iff	$suffix(\pi, 1) \models \varphi$
$\pi \models \varphi_1 \, \mathsf{U} \, \varphi_2$	iff	there exists $j \geq 0$ such that $suffix(\pi, j) \models \varphi_2$ $suffix(\pi, i) \models \varphi_1$ for $0 \leq i < j$

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Examples of CTL*-formulas

mutual exclusion:

safety $\forall\Box(\neg crit_1 \vee \neg crit_2)$

liveness $\forall\Box\Diamond crit_1 \;\wedge\; \forall\Box\Diamond crit_2$

progress property, e.g., $\forall\Box(request \rightarrow \Diamond response)$

persistence property, e.g., $\forall\Diamond\Box a$

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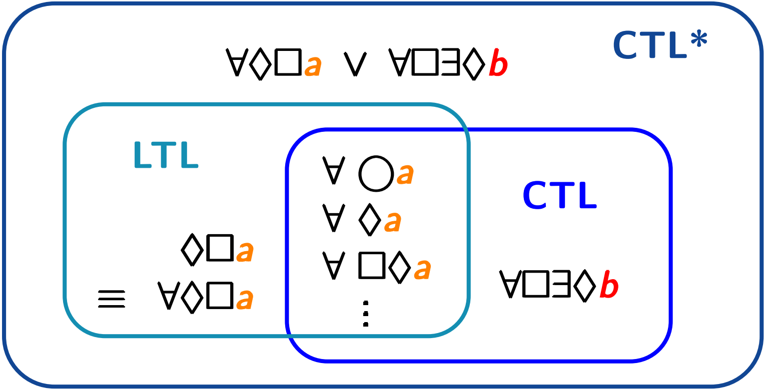
CTL* formulas with existential quantification, e.g.,
Hamilton path problem (for fixed initial state)

$$\exists\Big(\bigwedge_{v\in V}\big(\Diamond v \wedge \Box(v \rightarrow \bigcirc\Box\neg v)\big)\Big)$$

Expressiveness of CTL, LTL and CTL*

- **CTL** is a sublogic of **CTL***
- **LTL** is a sublogic of **CTL***
- **CTL*** is more expressive than **LTL** and **CTL**



Equivalence of CTL*-formulas

$$\Phi_1 \equiv \Phi_2 \quad \text{iff} \quad \text{for all transition systems } \mathcal{T}:$$
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Examples:

$$\neg\exists\Box\Diamond a \equiv \forall\Diamond\Box\neg a$$
$$\forall\Box\Diamond a \equiv \forall\Box\,\forall\Diamond a$$
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Correct or wrong?

CTLST4.6-14

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correct. Both formulas assert that an a-state is reachable from the current state within one or more steps.

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answer: **no**

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CTLST4.6-16

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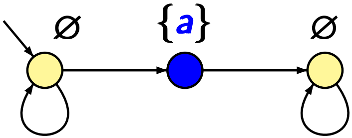

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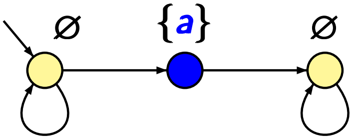

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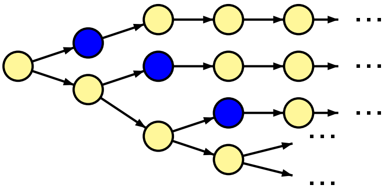

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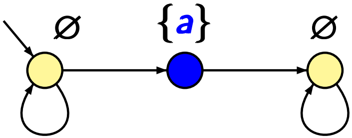

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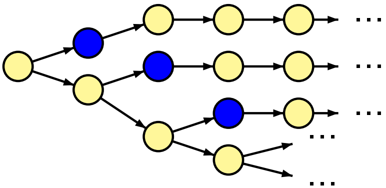

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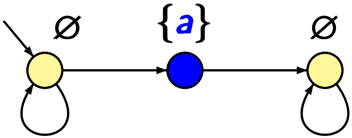

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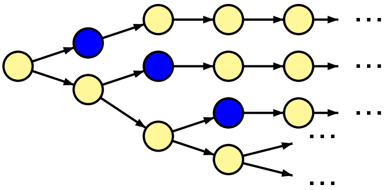

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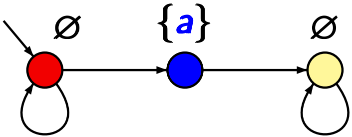

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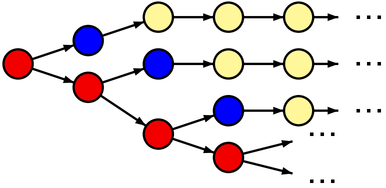

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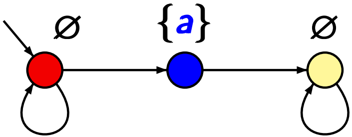

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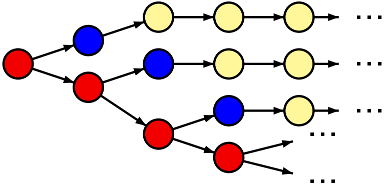

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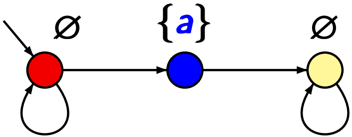

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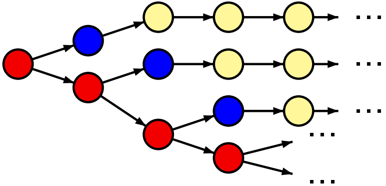

Equivalence of CTL*-formulas

$\neg\exists\varphi \;\equiv\; \forall\neg\varphi$ e.g., $\neg\exists\Box\Diamond a \;\equiv\; \forall\Diamond\Box\neg a$

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$\forall\Box\Diamond\varphi \equiv \forall\Box\forall\Diamond\varphi$ but: $\forall\Diamond\Box\varphi \not\equiv \forall\Diamond\forall\Box\varphi$

$\exists\Diamond\Box\varphi \equiv \exists\Diamond\exists\Box\varphi$ $\exists\Box\Diamond\varphi \not\equiv \exists\Box\exists\Diamond\varphi$

CTL* model checking

given: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
CTL* formula Φ

question: does $\mathcal{T} \models \Phi$ hold ?

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main procedure as for **CTL**:

```
FOR ALL subformulas Ψ of Φ DO
    compute Sat(Ψ) = {s ∈ S : s ⊨ Ψ}
OD
IF S₀ ⊆ Sat(Φ)
    THEN  return "yes"
    ELSE  return "no"
FI
```

Recursive computation of satisfaction sets

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$$\left.\begin{array}{lcl} Sat(true) & = & S \\ Sat(a) & = & \{s \in S : a \in L(s)\} \\ Sat(\Phi_1 \wedge \Phi_2) & = & Sat(\Phi_1) \cap Sat(\Phi_2) \\ Sat(\neg\Phi) & = & S \setminus Sat(\Phi) \end{array}\right\} \text{ as for } \textbf{CTL}$$

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$$\left.\begin{aligned} Sat(\forall\varphi) &= Sat_{LTL}(\varphi) \\ Sat(\exists\varphi) &= S \setminus Sat_{LTL}(\neg\varphi) \end{aligned}\right\} \text{ using an } \textbf{LTL} \text{ model checker}$$

Example: CTL* model checking

$$\Phi = \exists\Diamond\Box a \wedge \exists\Box\big(\bigcirc b \wedge \Diamond \neg\exists(a \,\mathsf{U}\, b)\big)$$

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1. calculate recursively the satisfaction sets $Sat(\Phi_i)$
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more precisely: existential **LTL** model checker

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$$\Phi \rightsquigarrow a_1 \wedge \exists\underbrace{\Box\big(\bigcirc b \wedge \Diamond a_2\big)}_{\textbf{LTL} \text{ formula } \varphi} = a_1 \wedge \exists\varphi$$

3. use an **LTL** model checker to compute $Sat(\exists\varphi)$

more precisely: existential **LTL** model checker

1. construct an **NBA** for φ
2. check via nested DFS whether $\mathcal{T} \otimes \mathcal{A} \models \exists\Box\Diamond F$

Example: CTL* model checking

$$\Phi = \underbrace{\exists\Diamond\Box a}_{\Phi_1} \wedge \exists\Box\big(\bigcirc b \wedge \Diamond \underbrace{\neg\exists(a \,\mathsf{U}\, b)}_{\Phi_2}\big)$$

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Fairness in CTL*

Correct or wrong?

Let $fair = \bigwedge_{1 \leq i \leq k} \Box\Diamond c_i$ be an unconditional **LTL** fairness assumption

$$s \models_{fair} \exists\Box a \quad \text{iff} \quad s \models \exists(fair \wedge \Box a)$$

Correct or wrong?

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↑ **CTL** with fairness

↑ **CTL*** semantic

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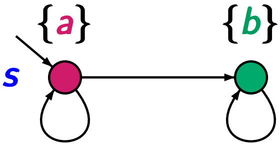

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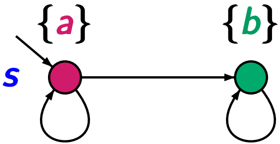

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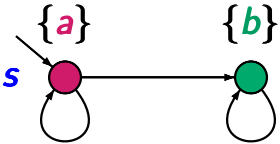

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$s \not\models \forall(fair \wedge \Box a)$

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correct.

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall(fair \wedge \Box a)$$

wrong. But we have:

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall(fair \rightarrow \Box a)$$

FairCTL*

FairCTL*

CTL* fairness assumptions are conjunctions of **CTL*** path formulas of the type

$\Box\Diamond\Phi$	unconditional fairness
$\Box\Diamond\Psi \rightarrow \Box\Diamond\Phi$	strong fairness
$\Diamond\Box\Psi \rightarrow \Box\Diamond\Phi$	weak fairness

where Φ and Ψ are **CTL*** state formulas

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where Φ and Ψ are **CTL*** state formulas

obvious definition of the satisfaction relation $\models_{fair}$

Satisfaction relation $\models_{fair}$ for CTL*

$s \models_{fair} \exists\varphi$ iff there exists $\pi \in \mathit{Paths}(s)$
with $\pi \models \mathit{fair}$ and $\pi \models_{fair} \varphi$

$\models$ standard **CTL*** satisfaction relation

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Complexity of CTL/LTL/CTL* model checking

Complexity of CTL/LTL model checking

	CTL	LTL	
		PSPACE-complete	
$\models$	$size(\mathcal{T}) \cdot \lvert\Phi\rvert$	$size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert)$	

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Complexity of CTL/LTL/CTL* model checking

	CTL	LTL	CTL*
	PTIME-complete	*PSPACE*-complete	?
$\models$	$size(\mathcal{T}) \cdot \lvert\Phi\rvert$	$size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert)$	?
$\models_{fair}$	$size(\mathcal{T}) \cdot \lvert\Phi\rvert \cdot \lvert fair\rvert$	$size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert) \cdot \lvert fair\rvert$	?

Complexity of CTL/LTL/CTL* model checking

	CTL	LTL and CTL*
	PTIME-complete	*PSPACE*-complete
$\models$	$\mathcal{O}(size(\mathcal{T}) \cdot \lvert\Phi\rvert)$	$\mathcal{O}(size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert))$
$\models_{fair}$	$\mathcal{O}(size(\mathcal{T}) \cdot \lvert\Phi\rvert \cdot \lvert fair\rvert)$	$\mathcal{O}(size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert) \cdot \lvert fair\rvert)$

	CTL	LTL and CTL*
	PTIME-complete	*PSPACE*-complete
$\models$	$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \lvert\Phi\rvert)$	$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert))$
$\models_{\mathit{fair}}$	$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \lvert\Phi\rvert \cdot \lvert \mathit{fair}\rvert)$	$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert) \cdot \lvert \mathit{fair}\rvert)$

model complexity, i.e., for fixed formula:

$\mathcal{O}(\,\mathit{size}(\mathcal{T})\,)$

correct or wrong?

$$\exists(\Diamond a \;\wedge\; \Diamond b) \;\equiv\; \exists\Diamond(a \wedge \exists\Diamond b) \vee \exists\Diamond(b \wedge \exists\Diamond a)$$

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correct.

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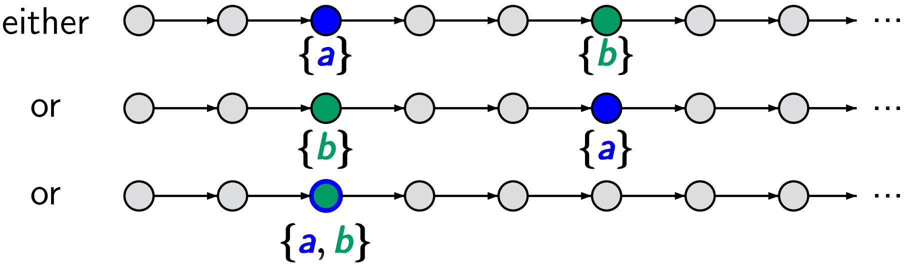
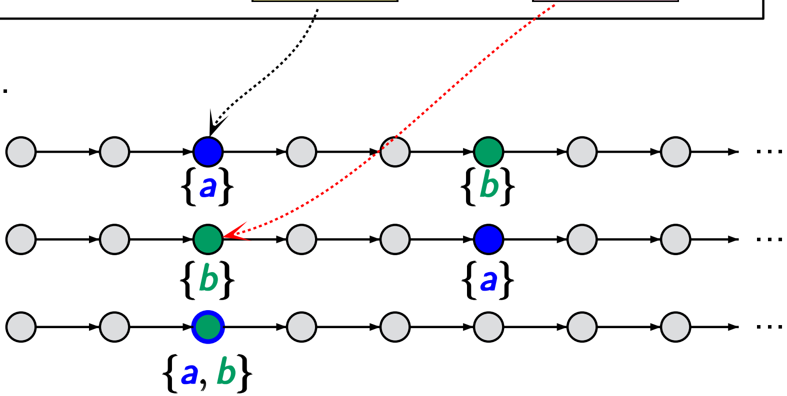


$$\exists(\Diamond a \wedge \Diamond b) \equiv \exists\Diamond(a \wedge \exists\Diamond b) \vee \exists\Diamond(b \wedge \exists\Diamond a)$$

correct.

either



or

or

The logic CTL^+

- **CTL** with Boolean operators for path formulas

The logic $\mathbf{CTL}^+$

- **CTL** with Boolean operators for path formulas
- sublogic of **CTL***

The logic CTL$^+$

- **CTL** with Boolean operators for path formulas
- sublogic of **CTL***

CTL$^+$ state formulas

$$\Phi ::= \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

CTL$^+$ path formulas

$$\varphi ::= \ldots$$

- **CTL** with Boolean operators for path formulas
- sublogic of **CTL***

CTL$^+$ state formulas

$$\Phi ::= true \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi \mid \forall\varphi$$

CTL$^+$ path formulas

$$\varphi ::= \ldots$$

universal quantification can be derived: $\forall\varphi \stackrel{\text{def}}{=} \neg\exists\neg\varphi$

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CTL$^+$ state formulas

$$\Phi ::= \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi \mid \forall\varphi$$

CTL$^+$ path formulas

$$\varphi ::= \bigcirc\Phi \mid \Phi_1 \, \mathsf{U} \, \Phi_2 \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi$$

- **CTL** with Boolean operators for path formulas
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e.g., $\exists(\Diamond b \wedge \Box a)$

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CTL^+ path formulas

$$\varphi ::= \bigcirc\Phi \mid \Phi_1 \,\mathsf{U}\, \Phi_2 \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi$$

e.g., $\exists(\Diamond b \wedge \Box a)$ and $\exists(\bigcirc b \rightarrow (a \,\mathsf{U}\, c))$
are CTL^+ formulas

Expressiveness of CTL^+

Expressiveness of $\mathbf{CTL}^+$

$\mathbf{CTL}^+$ is as expressive as **CTL**, i.e.,

For each $\mathbf{CTL}^+$-formula there exists an equivalent **CTL** formula.

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proof relies on a series of equivalence rules, e.g.:

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proof relies on a series of equivalence rules, e.g.:

$\exists(\neg\bigcirc\Phi) \rightsquigarrow \exists\bigcirc\neg\Phi$

CTL^{+} is as expressive as **CTL**, i.e.,

For each **CTL**$^{+}$-formula there exists an equivalent **CTL** formula.

proof relies on a series of equivalence rules, e.g.:

$$\exists(\neg\bigcirc\Phi) \rightsquigarrow \exists\bigcirc\neg\Phi$$

$$\exists(\neg(\Phi_1 \mathsf{U} \Phi_2)) \rightsquigarrow \exists\big((\Phi_1 \wedge \Phi_2) \mathsf{U} (\neg\Phi_1 \wedge \neg\Phi_2)\big) \vee \exists\Box\neg\Phi_2$$

$\mathbf{CTL}^+$ is as expressive as **CTL**, i.e.,

> For each $\mathbf{CTL}^+$-formula there exists an equivalent **CTL** formula.

proof relies on a series of equivalence rules, e.g.:

$$\exists(\neg\bigcirc\Phi) \rightsquigarrow \exists\bigcirc\neg\Phi$$

$$\exists(\neg(\Phi_1 \mathsf{U} \Phi_2)) \rightsquigarrow \exists((\Phi_1 \wedge \Phi_2) \mathsf{U} (\neg\Phi_1 \wedge \neg\Phi_2)) \vee \exists\Box\neg\Phi_2$$

$$\exists((\Psi_1 \mathsf{U} \Psi_2) \wedge (\Phi_1 \mathsf{U} \Phi_2)) \rightsquigarrow \ldots$$

$$\exists(\bigcirc\Psi \wedge (\Phi_1 \mathsf{U} \Phi_2)) \rightsquigarrow \ldots$$

From CTL$^+$ to CTL

$$\exists((a \,\mathsf{U}\, b) \wedge (c \,\mathsf{U}\, d)) \equiv \exists\big((a \wedge c) \,\mathsf{U}\, (b \wedge \exists(c \,\mathsf{U}\, d))\big) \vee \exists\big((c \wedge a) \,\mathsf{U}\, (d \wedge \exists(a \,\mathsf{U}\, b))\big)$$

CTL$^+$ formula **CTL** formula

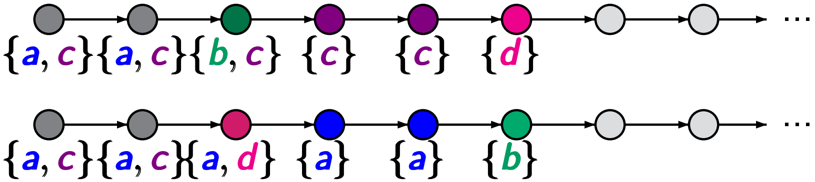
From CTL$^+$ to CTL

$$\exists((a \,\mathsf{U}\, b) \;\wedge\; (c \,\mathsf{U}\, d)) \;\equiv\; \exists((a \wedge c) \,\mathsf{U}\, (b \wedge \exists(c \,\mathsf{U}\, d)))$$
$$\vee\; \exists((c \wedge a) \,\mathsf{U}\, (d \wedge \exists(a \,\mathsf{U}\, b)))$$

CTL$^+$ formula   **CTL** formula

either

or



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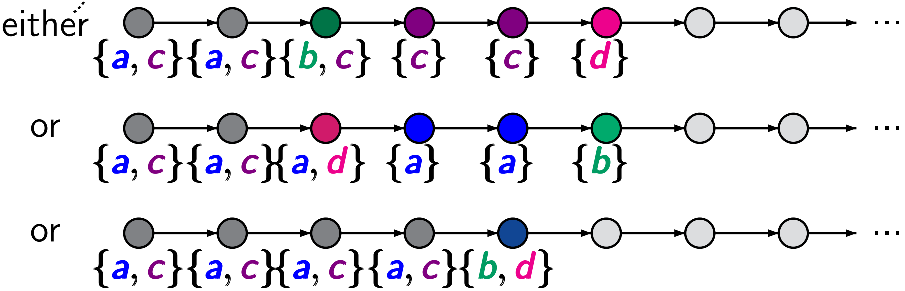
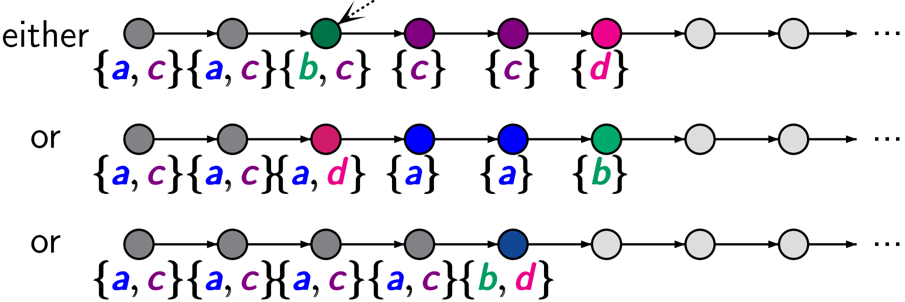
CTL$^+$ formula **CTL** formula



From CTL$^+$ to CTL

$$\exists((a \cup b) \;\wedge\; (c \cup d)) \;\equiv\; \exists\big((a \wedge c) \cup \boxed{b \wedge \exists(c \cup d)}\big) \;\vee\; \exists\big((c \wedge a) \cup (d \wedge \exists(a \cup b))\big)$$

CTL$^+$ formula **CTL** formula



From CTL$^+$ to CTL

$$\exists(\bigcirc a \ \wedge \ (b \, \mathsf{U} \, c))$$

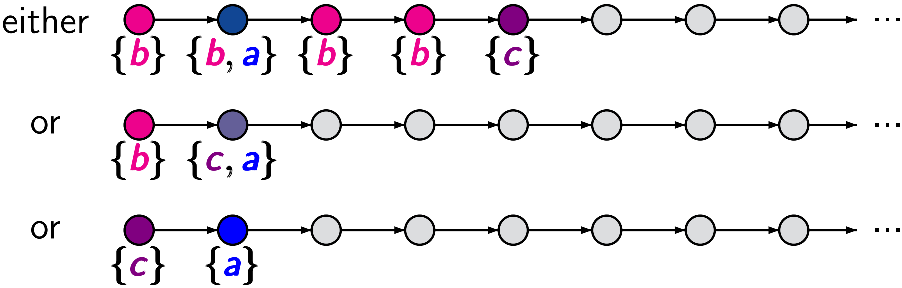
From CTL$^+$ to CTL

$$\exists(\bigcirc a \;\wedge\; (b \,\mathsf{U}\, c))$$

$$\equiv \quad (c \wedge \exists\bigcirc a) \vee \big(b \wedge \exists\bigcirc(a \wedge \exists(b \,\mathsf{U}\, c))\big)$$

$$\exists(\bigcirc a \;\wedge\; (b \,\mathsf{U}\, c))$$

$$\equiv \quad (c \wedge \exists\bigcirc a) \vee \big(b \wedge \exists\bigcirc(a \wedge \exists(b \,\mathsf{U}\, c))\big)$$



From CTL$^+$ to CTL

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