

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation Tree Logic

 syntax and semantics of CTL

 expressiveness of CTL and LTL



 CTL model checking

 fairness, counterexamples/witnesses

 CTL⁺ and CTL*

Equivalences and Abstraction

Equivalence of CTL and LTL formulas

COMPARISON4.2-1

Let ϕ be a **CTL** formula and φ an **LTL** formula.

Let ϕ be a **CTL** formula and φ an **LTL** formula.

$\phi \equiv \varphi$ iff for all transition systems \mathcal{T} and
all states s in \mathcal{T} :

$$s \models_{\text{CTL}} \phi \iff s \models_{\text{LTL}} \varphi$$

Equivalence of CTL and LTL formulas

Let Φ be a **CTL** formula and φ an **LTL** formula.

$\Phi \equiv \varphi$ iff for all transition systems \mathcal{T} and all states s in \mathcal{T} :

$$s \models_{\text{CTL}} \Phi \iff s \models_{\text{LTL}} \varphi$$

e.g.,

CTL formula Φ

LTL formula φ

a

a

$\forall \bigcirc a$

$\bigcirc a$

$\forall (a \cup b)$

$a \cup b$

$a, b \in AP$

More examples

CTL formula Φ	LTL formula φ
a	a
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$

More examples

CTL formula Φ	LTL formula φ
a	a
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{W} b)$	$a \text{W} b$

More examples

CTL formula Φ	LTL formula φ
a	a
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{W} b)$	$a \text{W} b$
$\forall \square \forall \diamond a$	$\square \diamond a$

More examples

CTL formula Φ	LTL formula φ
a	a
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{W} b)$	$a \text{W} b$
$\forall \square \forall \diamond a$	$\square \diamond a$

infinately often a

More examples

CTL formula Φ	LTL formula φ
a	a
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{ W } b)$	$a \text{ W } b$
$\forall \square \forall \diamond a$	$\square \diamond a$

infinately often a

but: $\forall \diamond \forall \square a \not\equiv \diamond \square a$

The CTL formula $\forall \diamond \forall \square a$

COMPARISON4.2-2

$s \models \forall\Diamond\forall\Box a$ iff on each path π from s
there is a state t with $t \models \forall\Box a$

The CTL formula $\forall\Diamond\forall\Box a$

COMPARISON4.2-2

$s \models \forall\Diamond\forall\Box a$ iff on each path π from s
there is a state t with $t \models \forall\Box a$

i.e., all states in the computation tree of t fulfill a

The CTL formula $\forall \diamond \forall \square a$

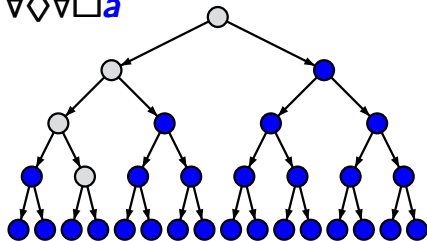
COMPARISON4.2-2

$s \models \forall \diamond \forall \square a$ iff on each path π from s
there is a state t with $t \models \forall \square a$



i.e., all states in the computation tree of t fulfill a

$\forall \diamond \forall \square a$



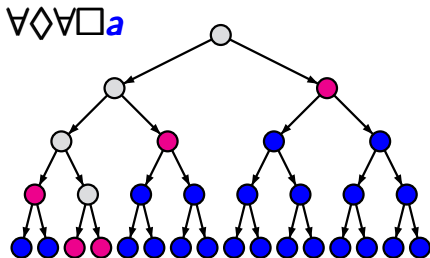
The CTL formula $\forall \diamond \forall \square a$

COMPARISON4.2-2

$s \models \forall \diamond \forall \square a$ iff on each path π from s
there is a state t with $t \models \forall \square a$



i.e., all states in the computation tree of t fulfill a



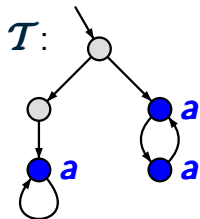
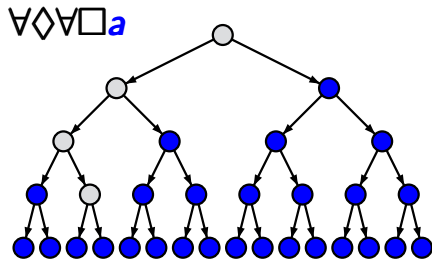
The CTL formula $\forall \diamond \forall \square a$

COMPARISON4.2-2

$s \models \forall \diamond \forall \square a$ iff on each path π from s
there is a state t with $t \models \forall \square a$



i.e., all states in the computation tree of t fulfill a



$\mathcal{T} \models \forall \diamond \forall \square a$

$\exists \square a \neq \forall \diamond \forall \square a$

COMPARISON 4.2-3

$$\Diamond \Box a \neq \forall \Diamond \forall \Box a$$

To prove that

$$\forall \Diamond \forall \Box a \neq \Diamond \Box a$$

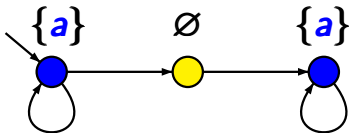
we provide an example for a TS \mathcal{T} s.t.

$$\mathcal{T} \models_{\text{LTL}} \Diamond \Box a$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \Diamond \forall \Box a$$

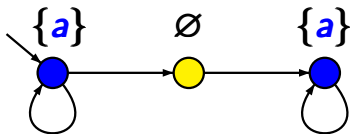
$$\diamond \square a \not\equiv \forall \diamond \forall \square a$$

transition system \mathcal{T}

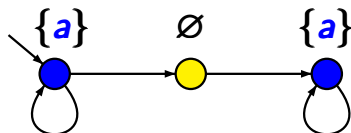


$$\Diamond \Box a \neq \forall \Diamond \forall \Box a$$

transition system \mathcal{T}



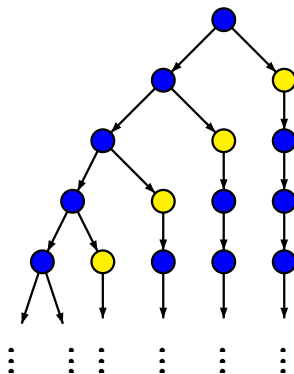
$$\mathcal{T} \models_{\text{LTL}} \Diamond \Box a$$

transition system \mathcal{T} 

$$\mathcal{T} \models_{\text{LTL}} \Diamond \Box a$$

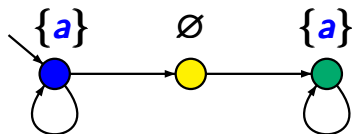
$$\mathcal{T} \not\models_{\text{CTL}} \forall \Diamond \forall \Box a$$

computation tree



$$\Diamond \Box a \neq \forall \Diamond \forall \Box a$$

transition system \mathcal{T}

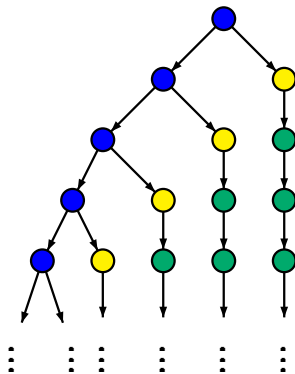


$$\mathcal{T} \models_{\text{LTL}} \Diamond \Box a$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \Diamond \forall \Box a$$

$$\text{Sat}(\forall \Box a) = \{\bullet\}$$

computation tree



From CTL to LTL, if possible

COMPARISON4.2-4

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or ...

without proof

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ
by removing of all path quantifiers \exists and \forall

without proof

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \Diamond \forall \Box a$$

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \diamond \forall \square a$$

↓

$$\varphi = \diamond \square a$$

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \Diamond \forall \Box a$$

↓

$$\varphi = \Diamond \Box a \not\equiv \Phi$$

For each **CTL** formula Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL** formula obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \Diamond \forall \Box a$$

↓

$$\varphi = \Diamond \Box a \not\equiv \Phi$$

hence: there is no LTL formula equivalent to Φ

For each CTL formula Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the LTL formula obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \square \forall \diamond a$$

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \square \forall \diamond a$$

↓

$$\varphi = \square \diamond a$$

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \square \forall \diamond a$$

↓

$$\varphi = \square \diamond a \equiv \Phi$$

“infinitely often a ”

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \Diamond (a \wedge \forall \bigcirc a)$$

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \Diamond (\forall \Diamond (a \wedge \forall \bigcirc a))$$

↓

$$\varphi = \Diamond (a \wedge \bigcirc a)$$

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\begin{aligned}\Phi &= \forall \Diamond (a \wedge \forall \bigcirc a) \\ \downarrow \\ \varphi &= \Diamond (a \wedge \bigcirc a) \neq \Phi\end{aligned}$$

For each **CTL formula** Φ the following holds:

- either there is **no** equivalent LTL formula
- or $\Phi \equiv \varphi$

where φ is the **LTL formula** obtained from Φ by removing of all path quantifiers \exists and \forall

without proof

$$\Phi = \forall \Diamond (a \wedge \forall \bigcirc a)$$

↓

$$\varphi = \Diamond (a \wedge \bigcirc a) \not\equiv \Phi$$

hence: there is no LTL formula equivalent to Φ

$$\Diamond(a \wedge \bigcirc a) \not\equiv \forall \Diamond(a \wedge \forall \bigcirc a)$$

COMPARISON4.2-4A

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$

To prove that

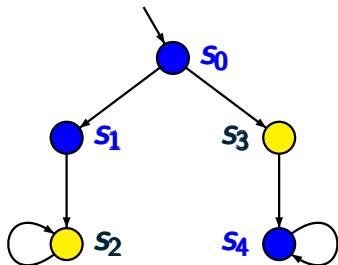
$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$

we provide an example for a TS \mathcal{T} s.t.

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond(a \wedge \forall \bigcirc a)$$

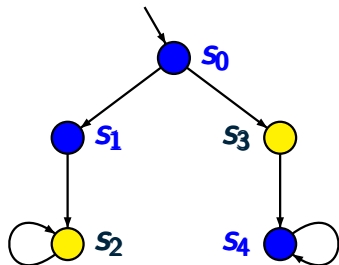
$$\Diamond(a \wedge \bigcirc a) \not\equiv \forall \Diamond(a \wedge \forall \bigcirc a)$$



$$\text{Yellow circle} = \emptyset$$

$$\text{Blue circle} = \{a\}$$

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$

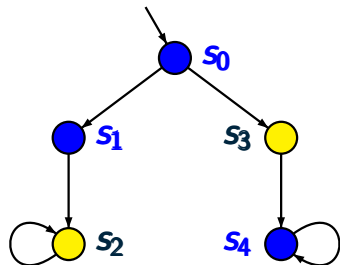


$$\text{Yellow Circle} = \emptyset$$

$$\text{Blue Circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$



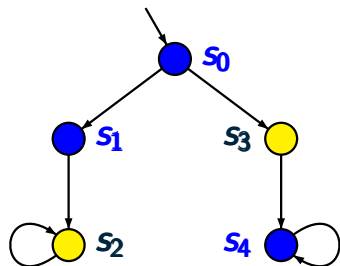
$$\text{yellow circle} = \emptyset$$

$$\text{blue circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$



$$\text{yellow circle} = \emptyset$$

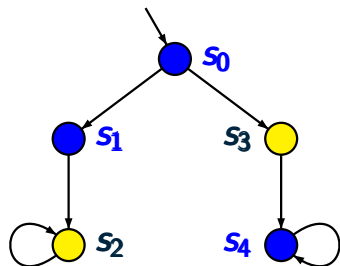
$$\text{blue circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond(a \wedge \forall \bigcirc a)$$

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$



$$\text{Yellow circle} = \emptyset$$

$$\text{Blue circle} = \{a\}$$

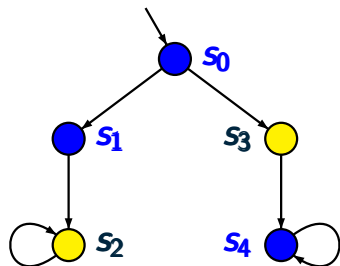
$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond(a \wedge \forall \bigcirc a)$$

$$\text{Sat}(a \wedge \forall \bigcirc a) = \{s_4\}$$

$$\Diamond(a \wedge \bigcirc a) \not\equiv \forall \Diamond(a \wedge \forall \bigcirc a)$$



$$\text{Yellow circle} = \emptyset$$

$$\text{Blue circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \Diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \Diamond(a \wedge \forall \bigcirc a)$$

$$\begin{aligned} \text{Sat}(a \wedge \forall \bigcirc a) &= \{s_4\} \\ s_0 s_1 s_2^\omega &\not\models_{\text{CTL}} \Diamond(a \wedge \forall \bigcirc a) \end{aligned}$$

The expressive powers of **LTL** and **CTL** are incomparable

The expressive powers of **LTL** and **CTL** are incomparable

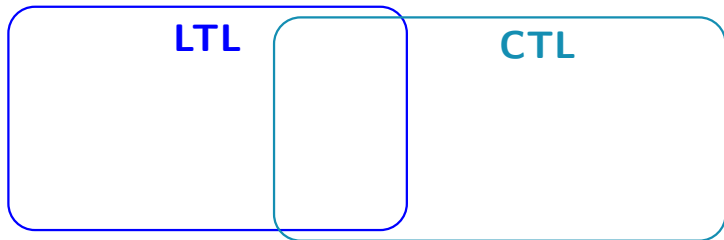
- The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent LTL formula

The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent LTL formula
- The **LTL** formula $\Diamond\Box a$ has no equivalent CTL formula

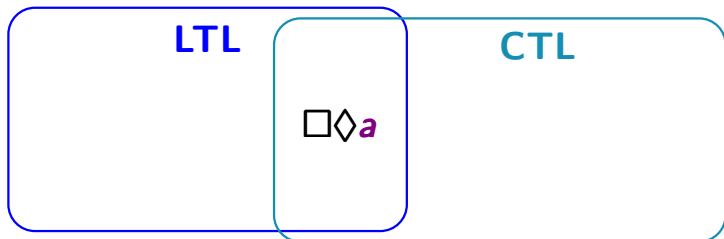
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent LTL formula
- The **LTL** formula $\Diamond\Box a$ has no equivalent CTL formula



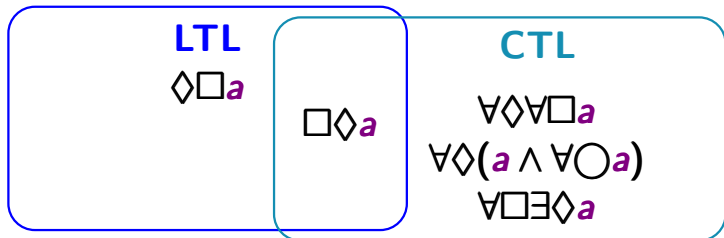
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent LTL formula
- The **LTL** formula $\Diamond\Box a$ has no equivalent CTL formula



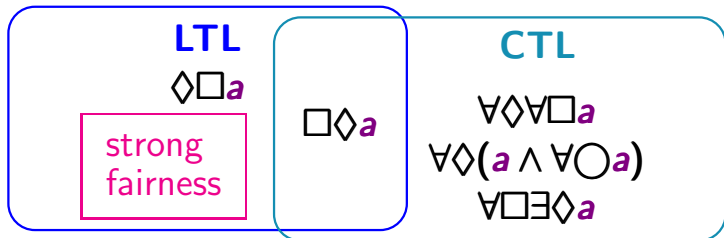
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent LTL formula
- The **LTL** formula $\Diamond\Box a$ has no equivalent CTL formula



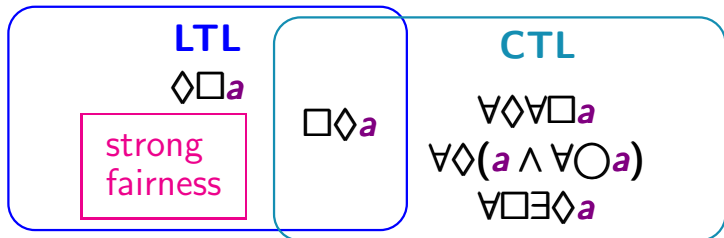
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent LTL formula
- The **LTL** formula $\Diamond\Box a$ has no equivalent CTL formula



The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent LTL formula
- The **LTL** formula $\Diamond\Box a$ has no equivalent CTL formula



The **CTL** formulas

$$\forall \Diamond (a \wedge \forall \bigcirc a)$$

$$\forall \Diamond \forall \square a$$

$$\forall \square \exists \Diamond a$$

have no equivalent **LTL** formula

The **CTL** formulas

$$\forall \Diamond (a \wedge \forall \bigcirc a)$$

$$\forall \Diamond \forall \Box a$$

$$\forall \Box \exists \Diamond a$$

have no equivalent **LTL** formula

Proof uses the fact that for each **CTL** formula Φ :

- either there is **no** equivalent **LTL** formula
- or $\Phi \equiv \varphi$ where φ is the **LTL** formula obtained from Φ by removing of all path quantifiers

The **CTL** formulas

$\forall \Diamond (a \wedge \forall \bigcirc a)$ ← already considered

$\forall \Diamond \forall \square a$ ← already considered

$\forall \square \exists \Diamond a$

have no equivalent **LTL** formula

Proof uses the fact that for each **CTL** formula Φ :

- either there is **no** equivalent **LTL** formula
- or $\Phi \equiv \varphi$ where φ is the **LTL** formula obtained from Φ by removing of all path quantifiers

The **CTL** formulas

$$\forall \Diamond (a \wedge \forall \bigcirc a)$$

$$\forall \Diamond \forall \Box a$$

$$\forall \Box \exists \Diamond a \leftarrow \text{alternative (direct) proof}$$

have no equivalent **LTL** formula

Proof uses the fact that for each **CTL** formula Φ :

- either there is **no** equivalent **LTL** formula
- or $\Phi \equiv \varphi$ where φ is the **LTL** formula obtained from Φ by removing of all path quantifiers

There is no LTL formula equivalent to $\forall \square \exists \diamond a$ COMPARISON4.2-5D

There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

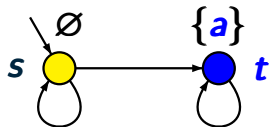
suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :

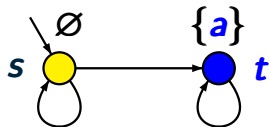


There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



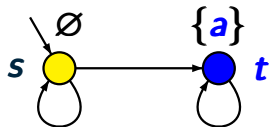
$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

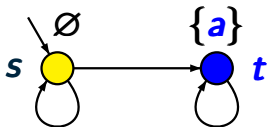
$$\mathcal{T}_1 \models \forall \square \exists \diamond a$$

There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

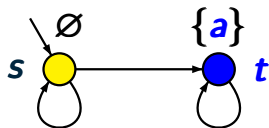
$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

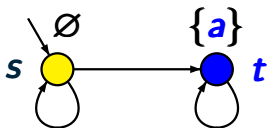
$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS \mathcal{T}_2 :



suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

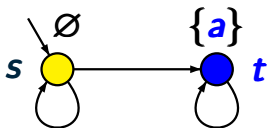
consider the following TS \mathcal{T}_2 :



$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\}$$

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS \mathcal{T}_2 :

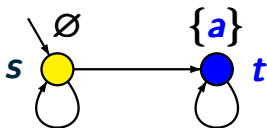


$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1)$$

There is no LTL formula equivalent to $\forall \square \exists \diamond a$ COMPARISON4.2-5D

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

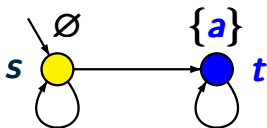
consider the following TS \mathcal{T}_2 :



$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS \mathcal{T}_2 :

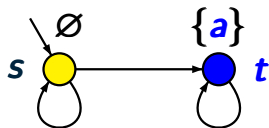


$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

$$\text{Hence: } \mathcal{T}_2 \models \varphi$$

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS \mathcal{T}_2 :



$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

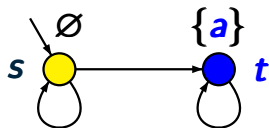
$$\text{Hence: } \mathcal{T}_2 \models \varphi$$

$$\implies \mathcal{T}_2 \models \forall \square \exists \diamond a$$

There is no LTL formula equivalent to $\forall \square \exists \diamond a$ COMPARISON4.2-5D

suppose φ is an **LTL** formula s.t. $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS \mathcal{T}_1 :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS \mathcal{T}_2 :



$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

$$\text{Hence: } \mathcal{T}_2 \models \varphi$$

$$\implies \mathcal{T}_2 \models \forall \square \exists \diamond a \quad \text{contradiction !!}$$

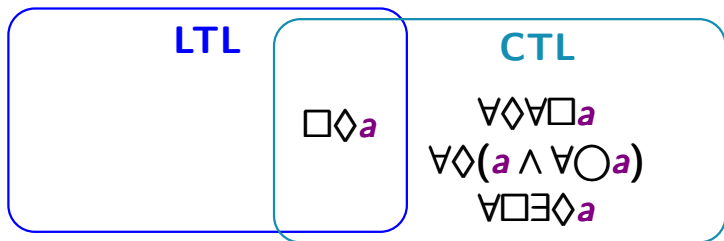
Expressiveness of LTL and CTL

COMPARISON4.2-5E

The expressive powers of **LTL** and **CTL** are incomparable

The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent **LTL** formula

The **LTL** formula $\Diamond\Box a$ has no equivalent **CTL** formula

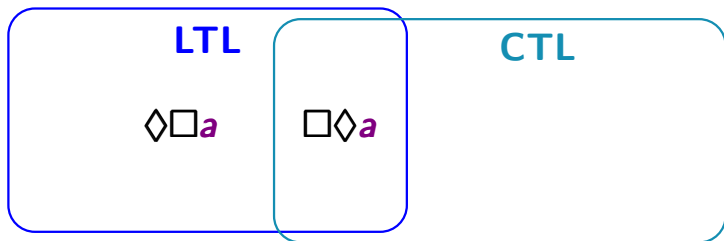


Expressiveness of LTL and CTL

The expressive powers of **LTL** and **CTL** are incomparable

The **CTL** formulas $\forall\Diamond(a \wedge \forall\bigcirc a)$, $\forall\Diamond\forall\Box a$ and $\forall\Box\exists\Diamond a$ have no equivalent **LTL** formula

The **LTL** formula $\Diamond\Box a$ has no equivalent **CTL** formula



There is no **CTL** formula which is equivalent to the **LTL** formula $\diamond\Box a$

There is no **CTL** formula which is equivalent to the **LTL** formula $\diamond\Box a$

Proof (sketch): provide sequences $(\mathcal{T}_n)_{n \geq 0}$, $(\mathcal{T}'_n)_{n \geq 0}$ of transition systems such that for all $n \geq 0$:

- (1) $\mathcal{T}_n \not\models \diamond\Box a$
- (2) $\mathcal{T}'_n \models \diamond\Box a$

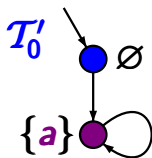
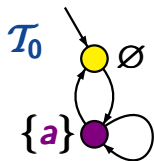
There is no **CTL** formula which is equivalent to the **LTL** formula $\diamond\Box a$

Proof (sketch): provide sequences $(\mathcal{T}_n)_{n \geq 0}$, $(\mathcal{T}'_n)_{n \geq 0}$ of transition systems such that for all $n \geq 0$:

- (1) $\mathcal{T}_n \not\models \diamond\Box a$
- (2) $\mathcal{T}'_n \models \diamond\Box a$
- (3) \mathcal{T}_n and \mathcal{T}'_n satisfy the same **CTL** formulas length $\leq n$

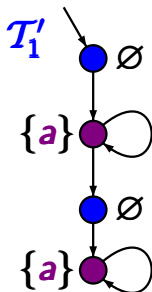
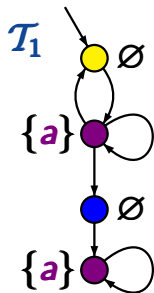
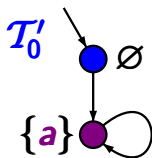
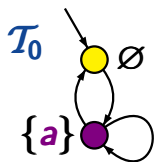
Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-6



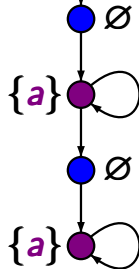
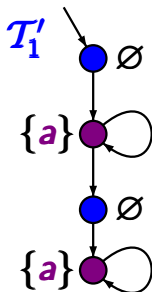
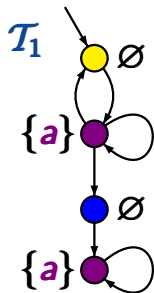
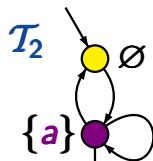
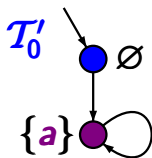
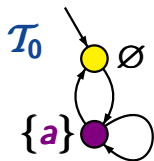
Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-6



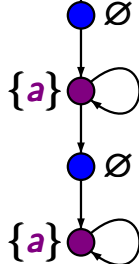
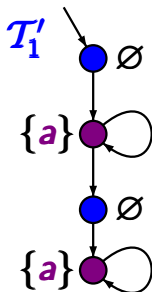
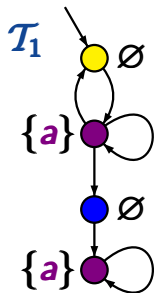
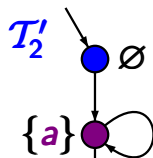
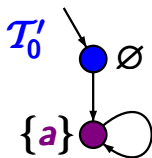
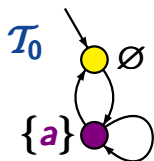
Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-6



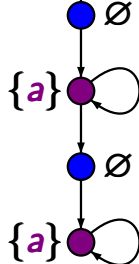
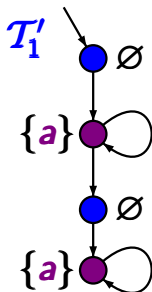
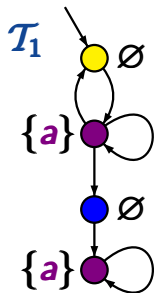
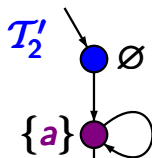
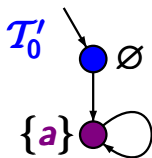
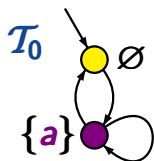
Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-6



Transition systems \mathcal{T}_n and \mathcal{T}'_n

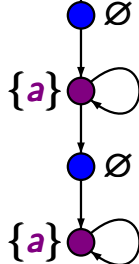
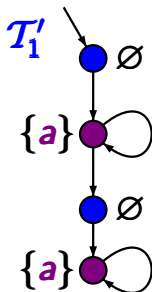
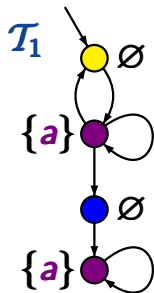
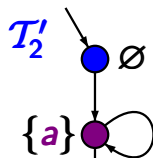
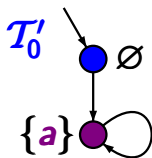
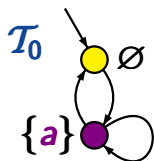
COMPARISON4.2-6



$\mathcal{T}_n \not\models \diamond \square a$

Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-6

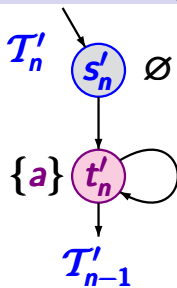
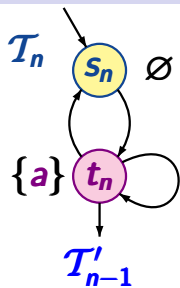


$\mathcal{T}_n \not\models \diamond \square a$

$\mathcal{T}'_n \models \diamond \square a$

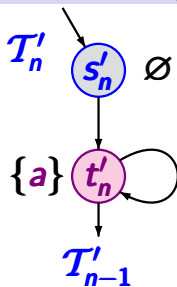
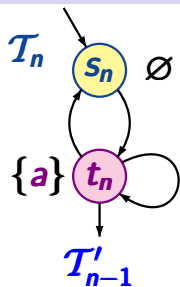
Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-7



Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-7

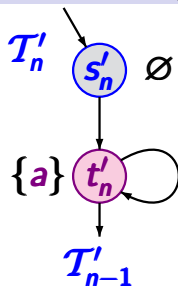
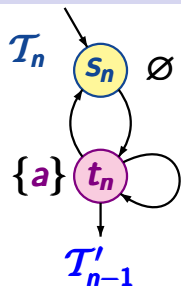


$$\mathcal{T}_n \not\models \diamond \square a$$

$$\mathcal{T}'_n \models \diamond \square a$$

Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-7



$$\mathcal{T}_n \not\models \diamond \Box a$$

$$\mathcal{T}'_n \models \diamond \Box a$$

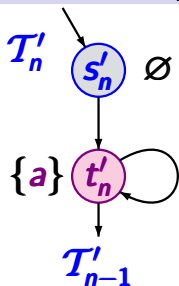
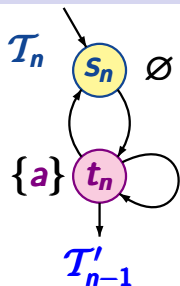
For all **CTL** formulas Φ of length $|\Phi| \leq n$:

$$s_n \models \Phi \quad \text{iff} \quad s'_n \models \Phi$$

$$t_n \models \Phi \quad \text{iff} \quad t'_n \models \Phi$$

Transition systems \mathcal{T}_n and \mathcal{T}'_n

COMPARISON4.2-7



$$\mathcal{T}_n \not\models \diamond \square a$$

$$\mathcal{T}'_n \models \diamond \square a$$

For all **CTL** formulas Φ of length $|\Phi| \leq n$:

$$s_n \models \Phi \quad \text{iff} \quad s'_n \models \Phi$$

$$t_n \models \Phi \quad \text{iff} \quad t'_n \models \Phi$$

Hence: \mathcal{T}_n and \mathcal{T}'_n fulfill the same **CTL** formulas of length $\leq n$

Does $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$ hold ?

Does $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$ hold ?

answer: **no.**

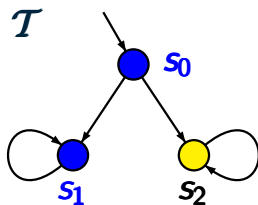
Does $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$ hold ?

answer: **no.**



Does $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$ hold ?

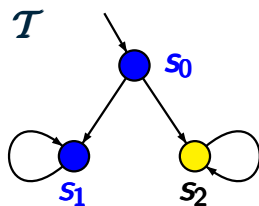
answer: **no.**



$$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$$

Does $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$ hold ?

answer: **no.**



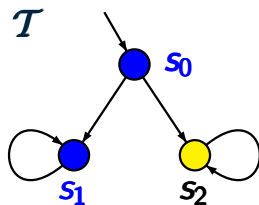
$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$

note: $\pi = s_0 s_2 s_2 s_2 \dots$ is a path in \mathcal{T} with

$trace(\pi) = \{a\} \emptyset \emptyset \emptyset \dots \notin Words(\Diamond(a \wedge \bigcirc a))$

Does $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$ hold ?

answer: **no.**

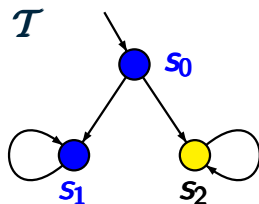


$$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$$

$$\mathcal{T} \models \forall \Diamond(a \wedge \exists \bigcirc a)$$

Does $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$ hold ?

answer: **no.**



$$\mathcal{T} \not\models \diamond (a \wedge \bigcirc a)$$

$$\mathcal{T} \models \forall \diamond (a \wedge \exists \bigcirc a)$$

$$\text{Sat}(\exists \bigcirc a) = \{s_0, s_1\}$$

$$\text{Sat}(\forall \diamond (a \wedge \exists \bigcirc a)) = \{s_0, s_1\}$$

For each **NBA** \mathcal{A} there is a **CTL** formula Φ
such that for all transition systems \mathcal{T} :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

For each **NBA** \mathcal{A} there is a **CTL** formula Φ
such that for all transition systems \mathcal{T} :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

wrong.

For each **NBA** \mathcal{A} there is a **CTL** formula Φ such that for all transition systems \mathcal{T} :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

wrong. consider, e.g., an NBA \mathcal{A} with

$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\diamond \square a)$$

For each **NBA** \mathcal{A} there is a **CTL** formula Φ such that for all transition systems \mathcal{T} :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

wrong. consider, e.g., an NBA \mathcal{A} with

$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\diamond \square a)$$

But there is no CTL formula Φ such that $\Phi \equiv \diamond \square a$

Correct or wrong?

If ϕ is **CTL** formula and ψ an **LTL** formula such that $\phi \equiv \psi$ then $\neg\phi \equiv \neg\psi$

Correct or wrong?

If ϕ is **CTL** formula and ψ an **LTL** formula such that $\phi \equiv \psi$ then $\neg\phi \equiv \neg\psi$

wrong.

Correct or wrong?

If ϕ is **CTL** formula and φ an **LTL** formula such that $\phi \equiv \varphi$ then $\neg\phi \equiv \neg\varphi$

wrong. E.g.,

$$\phi = \forall\Box\forall\Diamond a, \quad \varphi = \Box\Diamond a$$

Correct or wrong?

If ϕ is **CTL** formula and ψ an **LTL** formula such that $\phi \equiv \psi$ then $\neg\phi \equiv \neg\psi$

wrong. E.g.,

$$\phi = \forall\Box\forall\Diamond a, \quad \psi = \Box\Diamond a$$

- $\phi \equiv \psi$

If ϕ is **CTL** formula and ψ an **LTL** formula such that $\phi \equiv \psi$ then $\neg\phi \equiv \neg\psi$

wrong. E.g.,

$$\phi = \forall\Box\forall\Diamond a, \quad \psi = \Box\Diamond a$$

- $\phi \equiv \psi$
- there is no CTL formula that is equivalent to

$$\neg\psi \equiv \Diamond\Box\neg a$$

Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

Correct or wrong?

COMPARISON4.2-10

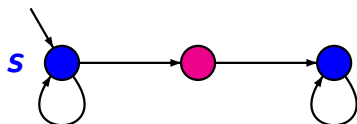
$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

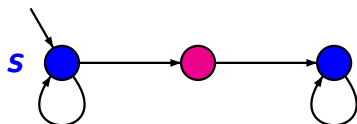


Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with $\pi \models \square \diamond a$

wrong.

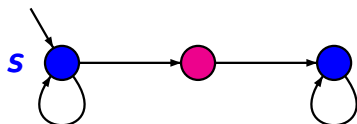


$s \models \exists \square \exists \diamond a$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with $\pi \models \square \diamond a$

wrong.



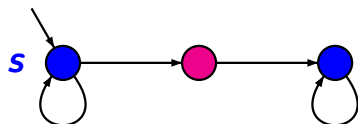
$s \models \exists \square \exists \diamond a$

note that: $s \models \exists \diamond a$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with $\pi \models \square \diamond a$

wrong.



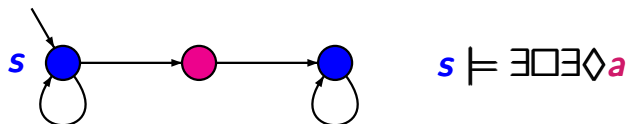
$s \models \exists \square \exists \diamond a$

note that: $s \models \exists \diamond a$

thus: $s s s \dots \models \square \exists \diamond a$

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.



note that: $s \models \exists \diamond a$

thus: $s s s \dots \models \square \exists \diamond a$

but there is no path where $\square \diamond a$ holds

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \diamond \square a$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \diamond \square a$

correct.

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

$$s \models \exists \diamond \exists \square a$$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

$$s \models \exists \diamond \exists \square a \text{ iff } s \not\models \forall \square \forall \diamond \neg a$$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in Paths(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in Paths(s)$ with
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

$$s \models \exists \diamond \exists \square a \text{ iff } s \not\models \forall \square \forall \diamond \neg a$$

$$\text{iff } s \not\models \square \diamond \neg a$$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \diamond \square a$

correct.

$$\begin{aligned} \exists \diamond \exists \square a &\equiv \neg \forall \square \forall \diamond \neg a \\ s \models \exists \diamond \exists \square a &\text{ iff } s \not\models \forall \square \forall \diamond \neg a \\ &\text{ iff } s \not\models \square \diamond \neg a \equiv \neg \diamond \square a \end{aligned}$$

Correct or wrong?

$s \models \exists \square \exists \diamond a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$ iff there is a path $\pi \in \text{Paths}(s)$ with
 $\pi \models \diamond \square a$

correct.

$$\begin{aligned} \exists \diamond \exists \square a &\equiv \neg \forall \square \forall \diamond \neg a \\ s \models \exists \diamond \exists \square a &\text{ iff } s \not\models \forall \square \forall \diamond \neg a \\ &\text{ iff } s \not\models \square \diamond \neg a \equiv \neg \diamond \square a \\ &\text{ iff there is a path } \pi \dots \end{aligned}$$

Correct or wrong?

COMPARISON4.2-11

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with
 $\pi \models \Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with
 $\pi \models \Box a$

correct

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with
 $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with
 $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state s with $s \not\models \neg\exists\Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with
 $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state s with $s \not\models \neg\exists\Box a$

iff there is an initial state s with $s \models \exists\Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$ iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with
 $\pi \models \Box a$

correct $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state s with $s \not\models \neg\exists\Box a$

iff there is an initial state s with $s \models \exists\Box a$

iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with $\pi \models \Box a$

Correct or wrong?

There is an **LTL** formula φ with $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\varphi$ iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with
 $\pi \models \varphi$

correct $\mathcal{T} \not\models \neg\exists\varphi$

iff there is an initial state s with $s \not\models \neg\exists\varphi$

iff there is an initial state s with $s \models \exists\varphi$

iff there is a path $\pi \in \text{Paths}(\mathcal{T})$ with $\pi \models \varphi$

Correct or wrong?

COMPARISON4.2-11A

$\mathcal{T} \not\models \neg \forall \Box a$ iff for all paths $\pi \in \text{Paths}(\mathcal{T})$:
 $\pi \models \Box a$

Correct or wrong?

$\mathcal{T} \not\models \neg \forall \Box a$ iff for all paths $\pi \in \text{Paths}(\mathcal{T})$:
 $\pi \models \Box a$

wrong.

Correct or wrong?

$\mathcal{T} \not\models \neg \forall \Box a$ iff for all paths $\pi \in \text{Paths}(\mathcal{T})$:
 $\pi \models \Box a$

wrong.

$\mathcal{T} \not\models \neg \forall \Box a$

Correct or wrong?

$\mathcal{T} \not\models \neg \forall \square a$ iff for all paths $\pi \in \text{Paths}(\mathcal{T})$:
 $\pi \models \square a$

wrong.

$\mathcal{T} \not\models \neg \forall \square a$

iff there is an initial state s with $s \not\models \neg \forall \square a$

Correct or wrong?

$\mathcal{T} \not\models \neg \forall \square a$ iff for all paths $\pi \in \text{Paths}(\mathcal{T})$:
 $\pi \models \square a$

wrong.

$\mathcal{T} \not\models \neg \forall \square a$

iff there is an initial state s with $s \not\models \neg \forall \square a$

iff there is an initial state s with $s \models \forall \square a$

Correct or wrong?

$\mathcal{T} \not\models \neg \forall \square a$ iff for all paths $\pi \in \text{Paths}(\mathcal{T})$:
 $\pi \models \square a$

wrong.

$\mathcal{T} \not\models \neg \forall \square a$

iff there is an initial state s with $s \not\models \neg \forall \square a$

iff there is an initial state s with $s \models \forall \square a$

but there might be another initial state t
s.t. $t \not\models \forall \square a$

Correct or wrong?

If \mathcal{T}_1 and \mathcal{T}_2 are trace equivalent TS then for all CTL formulas ϕ : $\mathcal{T}_1 \models \phi$ iff $\mathcal{T}_2 \models \phi$

Correct or wrong?

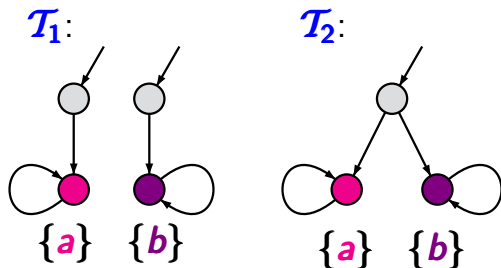
If \mathcal{T}_1 and \mathcal{T}_2 are trace equivalent TS then for all CTL formulas ϕ : $\mathcal{T}_1 \models \phi$ iff $\mathcal{T}_2 \models \phi$

wrong.

Correct or wrong?

If \mathcal{T}_1 and \mathcal{T}_2 are trace equivalent TS then for all CTL formulas ϕ : $\mathcal{T}_1 \models \phi$ iff $\mathcal{T}_2 \models \phi$

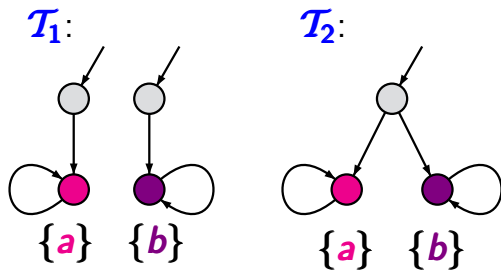
wrong.



Correct or wrong?

If \mathcal{T}_1 and \mathcal{T}_2 are trace equivalent TS then for all CTL formulas ϕ : $\mathcal{T}_1 \models \phi$ iff $\mathcal{T}_2 \models \phi$

wrong.

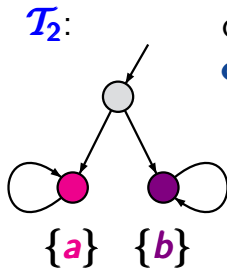
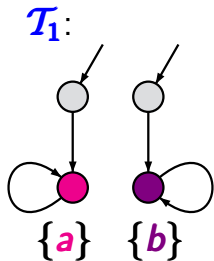


\mathcal{T}_1 and \mathcal{T}_2 are trace equivalent

Correct or wrong?

If \mathcal{T}_1 and \mathcal{T}_2 are trace equivalent TS then for all CTL formulas ϕ : $\mathcal{T}_1 \models \phi$ iff $\mathcal{T}_2 \models \phi$

wrong.



consider the CTL formula
 $\phi = \exists \bigcirc a \wedge \exists \bigcirc b$

$$\mathcal{T}_1 \not\models \phi$$

$$\mathcal{T}_2 \models \phi$$

\mathcal{T}_1 and \mathcal{T}_2 are trace equivalent