Exercise 1 (LTL Operators): (2 points)

Let φ and ψ be LTL formulae. Consider the following new operators:

a) "At next" $\varphi \,\mathsf{AX}\, \psi$:

$$A_1A_2\ldots \models \varphi \,\mathsf{AX}\, \psi \iff \text{for all } i \geq 0 \text{ where } A_iA_{i+1}\ldots \models \psi, \text{ for which there exists no } 0 \leq j < i \text{ where } A_jA_{j+1}\ldots \models \psi, \; A_iA_{i+1}\ldots \models \varphi \text{ holds}$$

b) "While" $\varphi \,\mathsf{WH}\, \psi$:

$$A_1A_2\ldots \models \varphi \,\mathsf{WH}\, \psi \iff \text{for all } i \geq 0 \text{ where } A_jA_{j+1}\ldots \models \psi \text{ for all } 0 \leq j < i, \; A_kA_{k+1}\ldots \models \varphi \text{ for all } 0 \leq k < i$$

c) "Before" $\varphi \,\mathsf{B}\, \psi$:

$$A_1A_2\ldots \models \varphi \,\mathsf{B}\, \psi \iff \text{for all } i \geq 0 \text{ where } A_iA_{i+1}\ldots \models \psi, \text{ there exists some } 0 \leq j < i \text{ where } A_jA_{j+1}\ldots \models \varphi$$

Show that these operators are LTL-definable by providing equivalent LTL formulae. You may use both the until and weak until operator.

Exercise 2 (LTL to Büchi): (3 points)

Let $\varphi = (a \wedge \bigcirc a)\mathsf{U}(a \wedge \neg \bigcirc a)$ be an LTL–formula over $AP = \{a\}$.

1. Compute all elementary sets with respect to φ.
2. Construct the GNBA $\mathcal{G}_\varphi$ according to the algorithm from the lecture such that $\mathcal{L}_\omega(\mathcal{G}_\varphi) = Words(\varphi)$.
3. Give an ω-regular expression E such that $\mathcal{L}_\omega(\mathcal{G}_\varphi) = \mathcal{L}_\omega(E)$.

Exercise 3 (CTL Equivalences): (3 points)

Prove or disprove the following implications:

(a) Let $\Phi_1 = \forall\Diamond a \vee \forall\Diamond b$ and $\Phi_2 = \forall\Diamond(a \vee b)$.
Prove or disprove the following implications: $\Phi_1 \implies \Phi_2$ and $\Phi_2 \implies \Phi_1$.

(b) Now consider $\Psi_1 = \exists(a\mathsf{U}\exists(b\mathsf{U}c))$ and $\Psi_2 = \exists(\exists(a\mathsf{U}b)\mathsf{U}c)$.
Again, prove or disprove $\Psi_1 \implies \Psi_2$ and $\Psi_2 \implies \Psi_1$.

Exercise 4 (CTL Normal Forms): (2 points)

Transform the CTL-formula $\Phi = \neg\forall\Diamond\big(\forall\,(\forall\Box b)\,\mathsf{U}\,(\forall\bigcirc a)\big)$ into an equivalent CTL-formula in

(a) existential normal form and

(b) positive normal form.