Lehrstuhl für Informatik 2
Software Modeling and Verification

Introduction to Model Checking SS16
Exercise Sheet 7 (due 22.06.2016)

Prof. Dr. Ir. Joost-Pieter Katoen

Christian Dehnert, Sebastian Junges

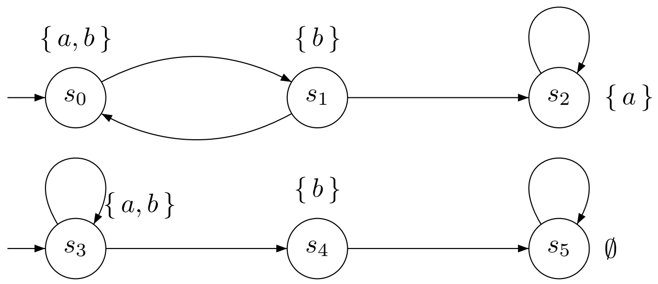
Exercise 1 (GNBA): (2 points)

a) Provide NBA $\mathcal{A}_1$ and $\mathcal{A}_2$ for the languages given by the expressions $(AC + B)^* B^\omega$ and $(B^* AC)^\omega$.

b) Apply the product construction to obtain an GNBA $\mathcal{G}$ and an NBA $\mathcal{A}$ with $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{A}_1) \cap \mathcal{L}_\omega(\mathcal{A}_2)$.
Hint: Do not apply simplifications in these steps

c) Justify, why $\mathcal{L}_\omega(\mathcal{G}) = \emptyset$ where $\mathcal{G}$ denotes the GNBA accepting the intersection.

Exercise 2 (LTL and Fairness): (3 points)



Consider the transition system TS above with the set AP $= \{a, b, c\}$ of atomic propositions. Note that this is a single transition system with two initial states. Consider the LTL fairness assumption

$$fair \;=\; \big(\Box\Diamond(a \wedge b) \;\rightarrow\; \Box\Diamond\neg c\big) \;\;\wedge\;\; \big(\Diamond\Box(a \wedge b) \rightarrow \Box\Diamond\neg b\big).$$

a) Determine the fair paths in TS, i.e., the initial, infinite paths satisfying *fair*

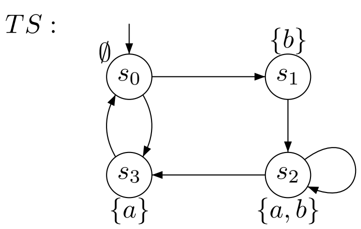
b) For each of the following LTL formulae:

$$\begin{aligned} \varphi_1 &= b \mathsf{U} \Box\neg b \\ \varphi_2 &= b \mathsf{W} \Box\neg b \\ \varphi_3 &= (\bigcirc\bigcirc\, b)\mathsf{U}(\Box\neg b) \end{aligned}$$

determine whether TS $\models_{fair} \varphi_i$. In case TS $\not\models_{fair} \varphi_i$, indicate a path $\pi \in Paths(\text{TS})$ for which $\pi \not\models \varphi_i$.

c) Redo the previous task but ignore the fairness assumption.

Exercise 3 (Model Checking): (3 points)



We consider the LTL formula $\varphi = \Box(a \rightarrow ((\neg b)\mathsf{U}(a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions and want to check $TS \models \varphi$ wrt. the transition system outlined above.

a) To check $TS \models \varphi$, convert $\neg\varphi$ into an equivalent LTL-formula ψ which is constructed according to the following grammar:

$$\varphi ::= true \mid false \mid a \mid b \mid \varphi \wedge \varphi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi \mathsf{U} \varphi.$$

Then construct $closure(\psi)$.

b) Give the elementary sets wrt. $closure(\psi)$!

c) Construct the GNBA $\mathcal{G}_\psi$ by providing its initial states, its acceptance set and its transition relation. Use the algorithm given in the lecture.
Hint: It suffices to provide the transition relation as a table.

d) Now, construct an NBA $\mathcal{A}_{\neg\varphi}$ **directly** from $\neg\varphi$, i.e. without relying on $\mathcal{G}_\psi$.
Hint: Four states suffice!

e) Construct $TS \otimes \mathcal{A}_{\neg\varphi}$.

f) Use the Nested DFS algorithm from the lecture to check $TS \models \varphi$. Therefore sketch the algorithm's main steps and interpret its outcome!