Exercise 1 (Equivalences): (4 points)

Let φ, ψ, π be arbitrary LTL formulae. For each of the following pairs of LTL formulae, determine in which relation they are. More specifically, determine whether they are equivalent, one of them subsumes the other or they are incomparable. Prove your claims.

a) $\Diamond\Box\varphi$ and $\Box\Diamond\varphi$

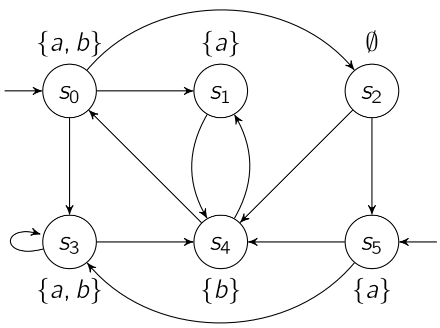
b) $\Diamond\Box\varphi \wedge \Diamond\Box\psi$ and $\Diamond(\Box\varphi \wedge \Box\psi)$

c) $\varphi \wedge \Box(\varphi \rightarrow \bigcirc\Diamond\varphi)$ and $\Box\Diamond\varphi$

d) $(\varphi \mathsf{U} \psi)\mathsf{U}\pi$ and $\varphi\mathsf{U}(\psi\mathsf{U}\pi)$

Exercise 2 (LTL satisfaction on TS): (2 points)

Consider the following transition system TS:



Determine whether TS $\models \varphi_i$ for each of the following properties. Justify your answers.

a) $\varphi_1 = \Box\Diamond a$

b) $\varphi_2 = \Diamond\Box a$

c) $\varphi_3 = a \rightarrow \bigcirc\bigcirc\, a$

d) $\varphi_4 = b\mathsf{R}a$ where $\varphi\mathsf{R}\psi \stackrel{\text{def}}{=} \neg(\neg\varphi\mathsf{U}\neg\psi)$

Exercise 3 (New LTL Operators): (2 points)

Let φ and ψ be LTL formulae and $i \in \mathbb{N}$. Consider the following new operators:

(i) "surround": $\varphi\, \mathcal{S}\, \psi$ holds if for any position i at which ψ holds, φ holds sometime before and sometime after i (every occurrence of ψ is "surrounded" by occurrences of φ).

(ii) "jump": $\mathcal{J}^i\varphi$ is true if, starting with the current position, φ is true at every i-th step (for example $\mathcal{J}^2 a$ states that a has to hold at every even position).

a) Formalize the semantics of these operators.

b) For each of the two new operators argue whether or not they can be expressed in terms of regular LTL.

Exercise 4 (Positive Normal Form): (2 points)

Provide a sequence (φ_n) of LTL formulae such that the LTL formulae ψ_n is in PNF (including weak-until), $\varphi_n \equiv \psi_n$, and ψ_n is exponentially longer than φ_n. Use the transformation rules from the lecture.