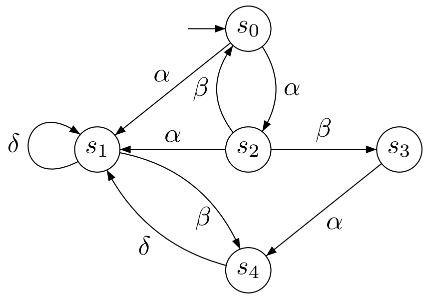
Exercise 1 (Realizability & Fairness): (2 points)

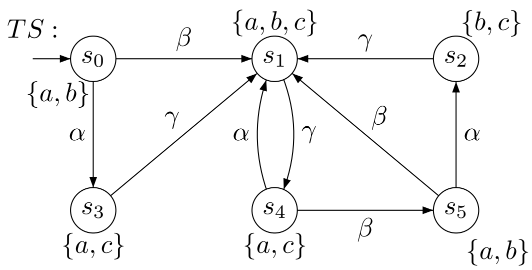
Consider the transition system TS on the right (where atomic propositions are omitted). Decide which of the following fairness assumptions $\mathcal{F}_i$ are realizable for TS. Justify your answers!



1. $\mathcal{F}_1 = (\{\{\alpha\}\}, \{\{\delta\}\}, \{\{\alpha, \beta\}\})$
2. $\mathcal{F}_2 = (\{\{\alpha, \delta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$
3. $\mathcal{F}_3 = (\{\{\alpha, \delta\}, \{\beta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$

Exercise 2 (Model Checking Regular Safety Properties): (3 points)

Consider the following transition system TS



and the regular safety property

$$P_{\text{safe}} = \begin{array}{c}\text{“always if } a \text{ is valid and } b \wedge \neg c \text{ was valid somewhere before,} \\ \text{then neither } a \text{ nor } b \text{ holds thereafter at least until } c \text{ holds”}\end{array}$$

As an example, it holds:

$$\{b\}\emptyset\{a, b\}\{a, b, c\} \in \mathit{pref}(P_{\text{safe}})$$
$$\{a, b\}\{a, b\}\emptyset\{b, c\} \in \mathit{pref}(P_{\text{safe}})$$
$$\{b\}\{a, c\}\{a\}\{a, b, c\} \in \mathit{BadPref}(P_{\text{safe}})$$
$$\{b\}\{a, c\}\{a, c\}\{a\} \in \mathit{BadPref}(P_{\text{safe}})$$

a) Define an NFA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = \mathit{MinBadPref}(P_{\text{safe}})$.

b) Decide whether $TS \models P_{\text{safe}}$ using the $TS \otimes \mathcal{A}$ construction.
Provide a counterexample if $TS \not\models P_{\text{safe}}$.

Exercise 3 (Quantitative Fairness): (3 points)

Let us introduce the notion of *quantitative fairness* $\exists_p X$, where X is a subset of some atomic propositions $AP \neq \emptyset$ and p is a real number. We are not only interested in something occurring (say event a, that is $X = \{a\}$) infinitely often, but also the ratio of the occurrence, say p.
For a finite word π, let $Freq_X$ be the number of times some Y with $X \subseteq Y$ occurs at some position $i < n$. For example, let $\pi = \{a\}\{b,c\}\{a,c\}\{c\}\{b,c\}\{c\}\{a\}\{c\}$, then $Freq_{\{a\}}(\pi) = 3$ and $Freq_{\{b,c\}}(\pi) = 2$.
For an infinite word π, let π_n be the finite prefix of length n, i.e., $\pi = \pi_n \cdot \pi'$, where $|\pi_n \models n$ and $\pi' \in \Sigma^\omega$. The semantics of quantitative fairness is as follows:

$$\pi \models \exists_p X \quad \text{iff} \quad \liminf_{n\to\infty} \left(\frac{1}{n} Freq_X(\pi_n) \right) = p$$

For example, the word $\pi = a^\omega$ satisfies $\exists_p\{a\}$ with $p = 1$.

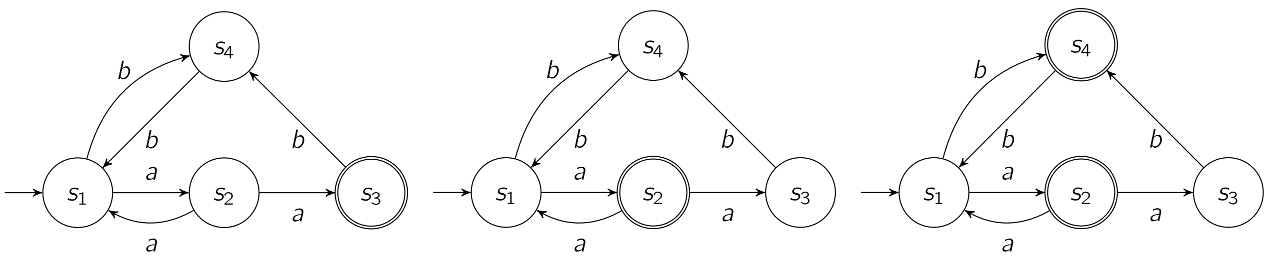
a) Give a formal definition for $Freq_X$.

b) Show that for any word π and letter a, $\liminf_{n\to\infty} \frac{1}{n} Freq_a(\pi_n) \leq 1$.

c) Show that $\exists_p\{a\}$ with $p = 0$ is not same as $\neg\exists^\infty a$. That is, find a word π such that $\pi \models \exists_p\{a\}$ and $\pi \models \exists^\infty a$.

Exercise 4 (NBA): (2 points)

a) Give the language for the following three NBA:



b) Give an NBA for:

- "initially a occurs, and at some point b occurs" with $\Sigma = \{a, b, c\}$.
- "if a occurs somewhere, then afterwards (b occurs infinitely often iff c occurs infinitely often).