



Compiler Construction

Lecture 9: Syntax Analysis V ($LR(0)$ and $SLR(1)$ Parsing)

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<https://moves.rwth-aachen.de/teaching/ss-16/cc/>

Recap: $LR(0)$ Grammars

Outline of Lecture 9

Recap: $LR(0)$ Grammars

Examples of $LR(0)$ Conflicts

$LR(0)$ Parsing

The $LR(0)$ Parsing Automaton

$SLR(1)$ Parsing

Recap: $LR(0)$ Grammars

$LR(0)$ Grammars

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, **without any lookahead**.

Corollary ($LR(0)$ grammar)

$G \in CFG_{\Sigma}$ has the **$LR(0)$ property** if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha \beta y \end{cases}$$

it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

Goal: derive a **finite information** from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by fo-sets)

Recap: $LR(0)$ Grammars

$LR(0)$ Items and Sets

Definition ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an **$LR(0)$ item** for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all **$LR(0)$ items** for γ , called the **$LR(0)$ set** (or: **$LR(0)$ information**) of γ .
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary

1. For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.
2. $LR(0)(G)$ is finite.
3. The item $[A \rightarrow \beta \cdot] \in LR(0)(\gamma)$ indicates the possible **reduction** $(w, \alpha \beta, z) \vdash (w, \alpha A, zi)$ where $\pi_i = A \rightarrow \beta$ and $\gamma = \alpha \beta$.
4. The item $[A \rightarrow \beta_1 \cdot Y \beta_2] \in LR(0)(\gamma)$ indicates an **incomplete handle** β_1 (to be completed by shift operations or ε -reductions).

Recap: $LR(0)$ Grammars

$LR(0)$ Conflicts

Definition ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $I \in LR(0)(G)$.

- I has a **shift/reduce conflict** if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

- I has a **reduce/reduce conflict** if there exist $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

Lemma

$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.

Proof.

omitted □

Recap: $LR(0)$ Grammars

Computing $LR(0)$ Sets

Theorem (Computing $LR(0)$ sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and reduced.

1. $LR(0)(\varepsilon)$ is the least set such that

- $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$ and
- if $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$.

2. $LR(0)(\alpha Y)$ ($\alpha \in X^*$, $Y \in X$) is the least set such that

- if $[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$,
then $[A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ and
- if $[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$.

Examples of $LR(0)$ Conflicts

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Examples of $LR(0)$ Conflicts

Shift/Reduce Conflicts

Example 9.1

$G : S' \rightarrow S$

$S \rightarrow aS \mid a$

Examples of $LR(0)$ Conflicts

Shift/Reduce Conflicts

Example 9.1

$G : S' \rightarrow S$

$S \rightarrow aS \mid a$

$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(a) : [S \rightarrow a \cdot S] \quad [S \rightarrow \cdot aS] \quad [S \rightarrow \cdot a] \quad [S \rightarrow a \cdot]$

$LR(0)(aS) : [S \rightarrow aS \cdot]$

Examples of $LR(0)$ Conflicts

Shift/Reduce Conflicts

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$LR(0)(aS) : [S \rightarrow aS \cdot]$

Note: G is unambiguous

Examples of $LR(0)$ Conflicts

Reduce/Reduce Conflicts

Example 9.2

$G : S' \rightarrow S$

$S \rightarrow Aa \mid Bb$

$A \rightarrow a$

$B \rightarrow a$

Examples of $LR(0)$ Conflicts

Reduce/Reduce Conflicts

Example 9.2

$G : S' \rightarrow S$

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$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(A) : [S \rightarrow A \cdot a]$

$LR(0)(B) : [S \rightarrow B \cdot b]$

$LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$

$LR(0)(Aa) : [S \rightarrow Aa \cdot]$

$LR(0)(Bb) : [S \rightarrow Bb \cdot]$

Examples of $LR(0)$ Conflicts

Reduce/Reduce Conflicts

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$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

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$LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$

$LR(0)(Aa) : [S \rightarrow Aa \cdot]$

$LR(0)(Bb) : [S \rightarrow Bb \cdot]$

Note: G is unambiguous

LR(0) Parsing

Outline of Lecture 9

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SLR(1) Parsing

The goto Function I

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields **deterministic shift/reduce decision** for $NBA(G)$ in a configuration with pushdown γ
 \implies **new pushdown alphabet:** $LR(0)(G)$ in place of X

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\implies **new pushdown alphabet:** $LR(0)(G)$ in place of X

Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but **independent from γ** in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

The goto Function I

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$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

Definition 9.3 ($LR(0)$ goto function)

The function $\text{goto} : LR(0)(G) \times X \rightarrow LR(0)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).$$

The goto Function II

Example 9.4 (cf. Example 8.16)

$$\begin{aligned}l_0 &:= LR(0)(\varepsilon) : \begin{array}{l} [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \\ [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array} \\l_1 &:= LR(0)(S) : [S' \rightarrow S \cdot] \\l_2 &:= LR(0)(B) : [S \rightarrow B \cdot] \\l_3 &:= LR(0)(C) : [S \rightarrow C \cdot] \\l_4 &:= LR(0)(a) : \begin{array}{l} [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array} \\l_5 &:= LR(0)(b) : [B \rightarrow b \cdot] \\l_6 &:= LR(0)(c) : [C \rightarrow c \cdot] \\l_7 &:= LR(0)(aB) : [B \rightarrow aB \cdot] \\l_8 &:= LR(0)(aC) : [C \rightarrow aC \cdot] \\l_9 &:= \emptyset\end{aligned}$$

The goto Function II

Example 9.4 (cf. Example 8.16)

$l_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C]$
 $\quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$
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 $l_8 := LR(0)(aC) : [C \rightarrow aC \cdot]$
 $l_9 := \emptyset$

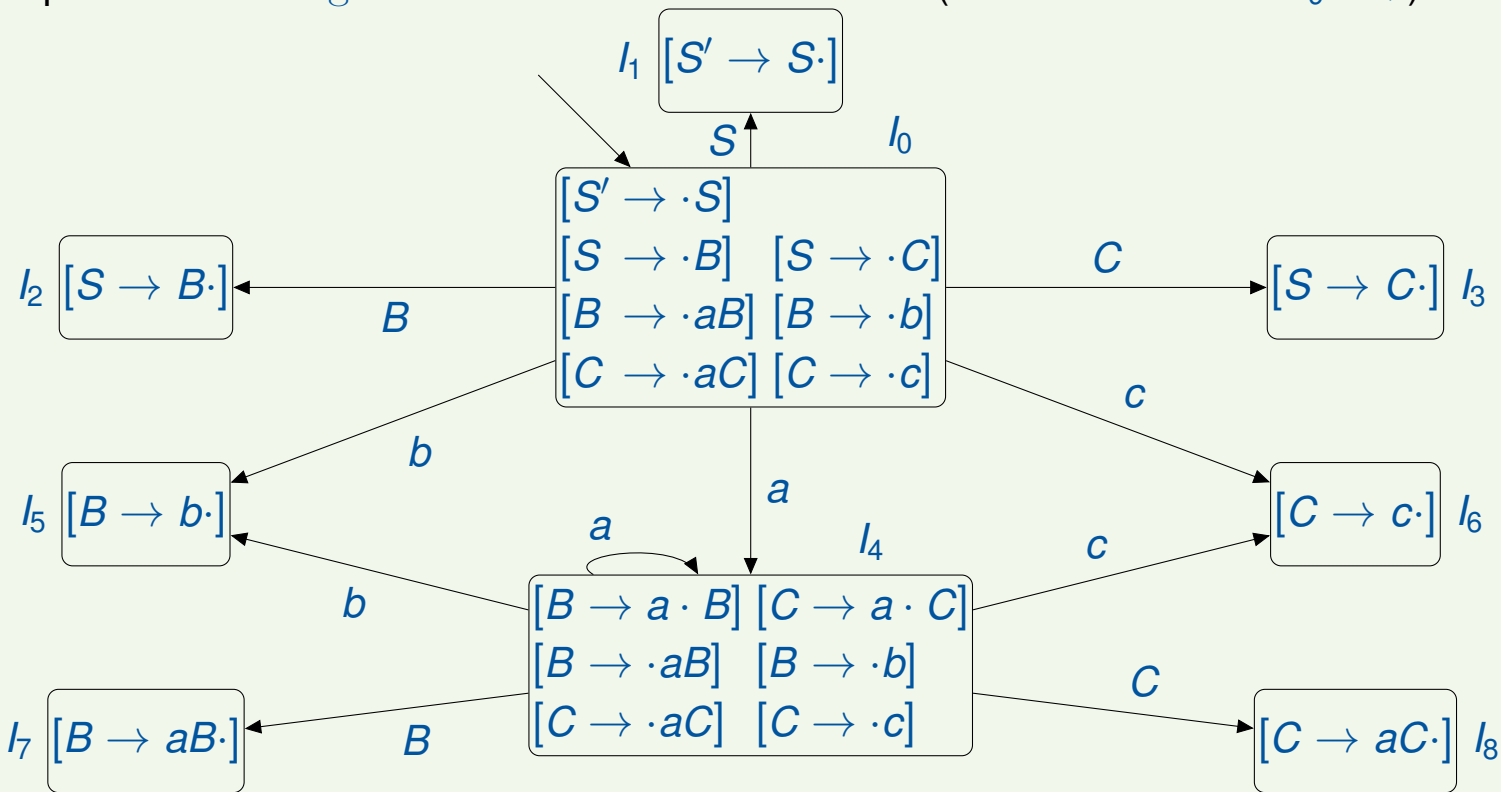
goto	S	B	C	a	b	c
l_0	l_1	l_2	l_3	l_4	l_5	l_6
l_1						
l_2						
l_3						
l_4		l_7	l_8	l_4	l_5	l_6
l_5						
l_6						
l_7						
l_8						
l_9						

(empty = l_9)

The goto Function III

Example 9.4 (continued)

Representation of goto function as finite automaton (omitted: sink state $l_9 = \emptyset$):



The LR(0) Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision (reminder: $\pi_0 = S' \rightarrow S$).

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Definition 9.5 (LR(0) action function)

The **LR(0) action function** $\text{act} : LR(0)(G) \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$ is defined by

$$\text{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot a\alpha_2] \in I \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

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Corollary 9.6

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act is well defined.

LR(0) Parsing

The LR(0) Action Function

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Corollary 9.6

For every $G \in CFG_{\Sigma}$, $G \in LR(0)$ iff act is well defined.

Together, act and goto form the **LR(0) parsing table** of G .

The $LR(0)$ Parsing Automaton

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The $LR(0)$ Parsing Table

Example 9.7 (cf. Example 9.4)

$G: S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

$l_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S]$
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The LR(0) Parsing Automaton

The LR(0) Parsing Table

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$LR(0)(G)$	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
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l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

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The $LR(0)$ Parsing Automaton

The $LR(0)$ Parsing Automaton I

Definition 9.8 ($LR(0)$ parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LR(0)$. The (deterministic) $LR(0)$ parsing automaton of G is defined by the following components:

- Input alphabet Σ
- Pushdown alphabet $\Gamma := LR(0)(G)$
- Output alphabet $\Delta := [\rho] \cup \{0, \text{error}\}$
- Configurations $\Sigma^* \times \Gamma^* \times \Delta^*$
- Initial configuration (w, l_0, ε) where $l_0 := LR(0)(\varepsilon)$
- Final configurations $\{\varepsilon\} \times \{\varepsilon\} \times \Delta^*$
- Transitions:

shift: $(aw, \alpha l, z) \vdash (w, \alpha l', z)$ if $\text{act}(l) = \text{shift}$ and $\text{goto}(l, a) = l'$

reduce: $(w, \alpha l_1 \dots l_n, z) \vdash (w, \alpha l', zi)$ if $\text{act}(l_n) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, $\text{goto}(l, A) = l'$

accept: $(\varepsilon, l_0 l, z) \vdash (\varepsilon, \varepsilon, z 0)$ if $\text{act}(l) = \text{accept}$

error: $(w, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(l) = \text{error}$

The $LR(0)$ Parsing Automaton

The $LR(0)$ Parsing Automaton II

Example 9.9 (cf. Example 9.7)

$G: S' \rightarrow S \quad (0)$
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$LR(0)(G)$	act	goto					
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(empty = l_9)

The LR(0) Parsing Automaton

The LR(0) Parsing Automaton II

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LR(0) parsing of *aac*:

(*aac*, l_0 , ϵ)

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The LR(0) Parsing Automaton

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LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0/l_4, \varepsilon)$

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 $\vdash (c, l_0l_4l_4, \varepsilon)$

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LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0l_4, \varepsilon)$
 $\vdash (c, l_0l_4l_4, \varepsilon)$
 $\vdash (\varepsilon, l_0l_4l_4l_6, \varepsilon)$

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LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0l_4, \varepsilon)$
 $\vdash (c, l_0l_4l_4, \varepsilon)$
 $\vdash (\varepsilon, l_0l_4l_4l_6, \varepsilon)$
 $\vdash (\varepsilon, l_0l_4l_4l_8, 6)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
l_2	red 1						
l_3	red 2						
l_4	shift		l_7	l_8	l_4	l_5	l_6
l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

The LR(0) Parsing Automaton

The LR(0) Parsing Automaton II

Example 9.9 (cf. Example 9.7)

$G: S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0) parsing of *aac*:

(aac, l_0, ε)
 $\vdash (ac, l_0l_4, \varepsilon)$
 $\vdash (c, l_0l_4l_4, \varepsilon)$
 $\vdash (\varepsilon, l_0l_4l_4l_6, \varepsilon)$
 $\vdash (\varepsilon, l_0l_4l_4l_8, 6)$
 $(*)$
 $\vdash (\varepsilon, l_0l_4l_8, 65)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
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The LR(0) Parsing Automaton

The LR(0) Parsing Automaton II

Example 9.9 (cf. Example 9.7)

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 $\vdash (\varepsilon, l_0l_4l_4l_8, 6)$
 $(*)$
 $\vdash (\varepsilon, l_0l_4l_8, 65)$
 $\vdash (\varepsilon, l_0l_3, 655)$

LR(0)(G)	act	goto					
		S	B	C	a	b	c
l_0	shift	l_1	l_2	l_3	l_4	l_5	l_6
l_1	accept						
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The LR(0) Parsing Automaton

The LR(0) Parsing Automaton II

Example 9.9 (cf. Example 9.7)

$G: S' \rightarrow S \quad (0)$
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l_8	red 5						
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(aac, l_0, ε)
 $\vdash (ac, l_0l_4, \varepsilon)$
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 $\vdash (\varepsilon, l_0l_3, 655)$
 $\vdash (\varepsilon, l_0l_1, 6552)$

The LR(0) Parsing Automaton

The LR(0) Parsing Automaton II

Example 9.9 (cf. Example 9.7)

$G: S' \rightarrow S \quad (0)$
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l_5	red 4						
l_6	red 6						
l_7	red 3						
l_8	red 5						
l_9	error						

(empty = l_9)

(aac, l_0, ε)
 $\vdash (ac, l_0l_4, \varepsilon)$
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 $\vdash (\varepsilon, l_0l_4l_4l_8, 6)$
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 $\vdash (\varepsilon, l_0l_4l_8, 65)$
 $\vdash (\varepsilon, l_0l_3, 655)$
 $\vdash (\varepsilon, l_0l_1, 6552)$
 $\vdash (\varepsilon, \varepsilon, 65520)$

The LR(0) Parsing Automaton

The LR(0) Parsing Automaton II

Example 9.9 (cf. Example 9.7)

$G: S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
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 $\vdash (\varepsilon, \varepsilon, 65520)$

Check by rightmost derivation (on the board)

The LR(0) Parsing Automaton

The LR(0) Parsing Automaton II

Example 9.9 (cf. Example 9.7)

$G: S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
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(empty = l_9)

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 $\vdash (\varepsilon, l_0l_4l_8, 65)$
 $\vdash (\varepsilon, l_0l_3, 655)$
 $\vdash (\varepsilon, l_0l_1, 6552)$
 $\vdash (\varepsilon, \varepsilon, 65520)$

Check by rightmost derivation (on the board)

Remark: in the corresponding computation of $NBA(G)$, (*) is nondeterministic

The $LR(0)$ Parsing Automaton

The $LR(0)$ Parsing Automaton III

Theorem 9.10 (Correctness of $LR(0)$ Parsing Automaton)

If $G \in LR(0)$, then the $LR(0)$ parsing automaton of G is deterministic, and for every $w \in \Sigma^*$ and $z \in \{0, \dots, p\}^*$:

$$(w, l_0, \varepsilon) \vdash^* (\varepsilon, \varepsilon, z) \quad \text{iff} \quad \overleftarrow{z} \text{ is a rightmost analysis of } w$$

Proof.

omitted □

SLR(1) Parsing

Outline of Lecture 9

Recap: *LR*(0) Grammars

Examples of *LR*(0) Conflicts

LR(0) Parsing

The *LR*(0) Parsing Automaton

SLR(1) Parsing

SLR(1) Parsing

Removing Conflicts in LR(0) Parsing

In practice: often $G \notin LR(0)$

Example 9.11

$$G_{AE} : \begin{array}{ll} E' \rightarrow E & E \rightarrow E+T \mid T \\ T \rightarrow T*F \mid F & F \rightarrow (E) \mid a \mid b \end{array}$$

SLR(1) Parsing

Removing Conflicts in LR(0) Parsing

In practice: often $G \notin LR(0)$

Example 9.11

$$G_{AE} : E' \rightarrow E \quad E \rightarrow E+T \mid T \\ T \rightarrow T*F \mid F \quad F \rightarrow (E) \mid a \mid b$$

LR(0)(G_{AE}) with conflicts:

$$I_0 : [E' \rightarrow \cdot E] \quad [E \rightarrow \cdot E+T] \quad [E \rightarrow \cdot T] \\ [T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F] \quad [F \rightarrow \cdot (E)] \\ [F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$

$$I_4 : [F \rightarrow (\cdot E)] \quad [E \rightarrow \cdot E+T] \quad [E \rightarrow \cdot T] \\ [T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F] \quad [F \rightarrow \cdot (E)] \\ [F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$

$$I_8 : [T \rightarrow T* \cdot F] \quad [F \rightarrow \cdot (E)] \\ [F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$

$$I_{11} : [T \rightarrow T*F \cdot]$$

$$I_1 : [E' \rightarrow E \cdot] \quad [E \rightarrow E \cdot +T]$$

$$I_2 : [E \rightarrow T \cdot] \quad [T \rightarrow T \cdot *F]$$

$$I_3 : [T \rightarrow F \cdot]$$

$$I_5 : [F \rightarrow a \cdot]$$

$$I_6 : [F \rightarrow b \cdot]$$

$$I_7 : [E \rightarrow E+ \cdot T] \quad [T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F] \\ [F \rightarrow \cdot (E)] \quad [F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$

$$I_9 : [F \rightarrow (E \cdot)] \quad [E \rightarrow E \cdot +T]$$

$$I_{10} : [E \rightarrow E+T \cdot] \quad [T \rightarrow T \cdot *F]$$

$$I_{12} : [F \rightarrow (E) \cdot]$$

Adding Lookahead I

Goal: resolving conflicts by considering next input symbol

SLR(1) Parsing

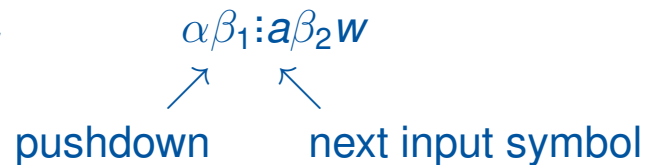
Adding Lookahead I

Goal: resolving conflicts by considering next input symbol

Observations:

- $[A \rightarrow \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$
 $\implies \exists \alpha \in X^*, w \in \Sigma^* :$

$$S' \Rightarrow_r^* \alpha A w \Rightarrow_r$$



Thus: shift only on lookahead *a*

SLR(1) Parsing

Adding Lookahead I

Goal: resolving conflicts by considering next input symbol

Observations:

- $[A \rightarrow \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$
 $\implies \exists \alpha \in X^*, w \in \Sigma^* :$

$$S' \Rightarrow_r^* \alpha A w \Rightarrow_r$$

$\alpha\beta_1 : a\beta_2 w$

↑ ↓
pushdown next input symbol

Thus: shift only on lookahead a

- $[A \rightarrow \beta \cdot] \in LR(0)(\alpha\beta)$
 $\implies \exists \alpha \in X^*, x \in \Sigma_\epsilon, w \in \Sigma^* (x = \epsilon \text{ only if } w = \epsilon):$

$$S' \Rightarrow_r^* \alpha A x w \Rightarrow_r$$

$\alpha\beta : xw$

↑ ↓
pushdown input

$$\implies x \in \text{fo}(A) \subseteq \Sigma_\epsilon$$

Thus: reduce with $A \rightarrow \beta$ only if lookahead $x \in \text{fo}(A)$

Adding Lookahead II

Example 9.12 (cf. Example 9.11)

$$\begin{array}{ll} G_{AE} : E' \rightarrow E & (0) \\ E \rightarrow E+T \mid T & (1, 2) \\ T \rightarrow T*F \mid F & (3, 4) \\ F \rightarrow (E) \mid a \mid b & (5, 6, 7) \end{array}$$

$A \in N$	$\text{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{+,), \varepsilon\}$

Adding Lookahead II

Example 9.12 (cf. Example 9.11)

$$\begin{aligned} G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7) \end{aligned}$$

$A \in N$	$\text{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{+,), \varepsilon\}$

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot + T]\}$:
 - accept on lookahead ε
 - shift on lookahead $+$

Adding Lookahead II

Example 9.12 (cf. Example 9.11)

$$\begin{aligned}G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7)\end{aligned}$$

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E'	$\{\varepsilon\}$
E	$\{+,), \varepsilon\}$

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot +T]\}$:
 - accept on lookahead ε
 - shift on lookahead $+$
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 2 on lookahead $+/)/\varepsilon$
 - shift on lookahead $*$

Adding Lookahead II

Example 9.12 (cf. Example 9.11)

$$\begin{aligned}G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7)\end{aligned}$$

$A \in N$	$\text{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{+,), \varepsilon\}$

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot +T]\}$:
 - accept on lookahead ε
 - shift on lookahead $+$
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 2 on lookahead $+/)/\varepsilon$
 - shift on lookahead $*$
- $I_{10} = \{[E \rightarrow E+T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 1 on lookahead $+/)/\varepsilon$
 - shift on lookahead $*$

SLR(1) Parsing

Adding Lookahead II

Example 9.12 (cf. Example 9.11)

$$\begin{aligned} G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7) \end{aligned}$$

$A \in N$	$\text{fo}(A)$
E'	$\{\varepsilon\}$
E	$\{+,), \varepsilon\}$

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot +T]\}$:
 - accept on lookahead ε
 - shift on lookahead $+$
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 2 on lookahead $+/)/\varepsilon$
 - shift on lookahead $*$
- $I_{10} = \{[E \rightarrow E+T\cdot], [T \rightarrow T\cdot *F]\}$:
 - red 1 on lookahead $+/)/\varepsilon$
 - shift on lookahead $*$

\implies **SLR(1) parsing** (Simple LR(1))

The SLR(1) Action Function

Definition 9.13 (SLR(1) action function)

The SLR(1) action function

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, \mathbf{x}) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I, \\ & \text{and } \mathbf{x} \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot \mathbf{x} \alpha_2] \in I \text{ and } \mathbf{x} \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } \mathbf{x} = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

The $SLR(1)$ Action Function

Definition 9.13 ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

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Definition 9.14 ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

SLR(1) Parsing

The SLR(1) Action Function

Definition 9.13 (SLR(1) action function)

The **SLR(1) action function**

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, \mathbf{x}) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I, \\ & \text{and } \mathbf{x} \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot \mathbf{x} \alpha_2] \in I \text{ and } \mathbf{x} \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } \mathbf{x} = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Definition 9.14 (SLR(1) grammar)

A grammar $G \in CFG_\Sigma$ has the **SLR(1) property** (notation: $G \in SLR(1)$) if its **SLR(1) action function** is well defined.

act and the **LR(0) goto** function (Definition 9.3) form the **SLR(1) parsing table** of G .

SLR(1) Parsing

The SLR(1) Parsing Table

Example 9.15 (cf. Example 9.11)

l_0 : $[E' \rightarrow \cdot E]$ $[E \rightarrow \cdot E+T]$ $[E \rightarrow \cdot T]$ l_1 : $[E' \rightarrow E \cdot]$ $[E \rightarrow E \cdot +T]$
 $[T \rightarrow \cdot T*F]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot (E)]$ l_2 : $[E \rightarrow T \cdot]$ $[T \rightarrow T \cdot *F]$
 $[F \rightarrow \cdot a]$ $[F \rightarrow \cdot b]$ l_3 : $[T \rightarrow F \cdot]$
 l_4 : $[F \rightarrow (\cdot E)]$ $[E \rightarrow \cdot E+T]$ $[E \rightarrow \cdot T]$ l_5 : $[F \rightarrow a \cdot]$
 $[T \rightarrow \cdot T*F]$ $[T \rightarrow \cdot F]$ $[F \rightarrow \cdot (E)]$ l_6 : $[F \rightarrow b \cdot]$
 $[F \rightarrow \cdot a]$ $[F \rightarrow \cdot b]$ l_7 : $[E \rightarrow E+ \cdot T]$ $[T \rightarrow \cdot T*F]$ $[T \rightarrow \cdot F]$
 l_8 : $[T \rightarrow T* \cdot F]$ $[F \rightarrow \cdot (E)]$ $[F \rightarrow \cdot a]$ $[F \rightarrow \cdot b]$
 $[F \rightarrow \cdot a]$ $[F \rightarrow \cdot b]$ l_9 : $[F \rightarrow (E \cdot)]$ $[E \rightarrow E \cdot +T]$
 l_{11} : $[T \rightarrow T*F \cdot]$ l_{10} : $[E \rightarrow E+T \cdot]$ $[T \rightarrow T \cdot *F]$
 l_{12} : $[F \rightarrow (E) \cdot]$

$A \in N$	$fo(A)$
E'	$\{\epsilon\}$
E	$\{+, \cdot, \epsilon\}$
T	$\{+, *, \cdot, \epsilon\}$
F	$\{+, *, \cdot, \epsilon\}$

SLR(1) Parsing

The SLR(1) Parsing Table

Example 9.15 (cf. Example 9.11)

$l_0 : [E' \rightarrow \cdot E] \quad [E \rightarrow \cdot E+T] \quad [E \rightarrow \cdot T] \quad [F \rightarrow \cdot (E)]$
 $l_1 : [E' \rightarrow E \cdot] \quad [E \rightarrow E \cdot +T]$
 $l_2 : [E \rightarrow T \cdot] \quad [T \rightarrow T \cdot *F]$
 $l_3 : [T \rightarrow F \cdot]$
 $l_4 : [F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$
 $l_5 : [F \rightarrow a \cdot]$
 $l_6 : [F \rightarrow b \cdot]$
 $l_7 : [E \rightarrow E+ \cdot T] \quad [T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F]$
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 $l_{12} : [F \rightarrow (E) \cdot]$

$A \in N$	$fo(A)$
E'	$\{\epsilon\}$
E	$\{+, \cdot, \epsilon\}$
T	$\{+, *, \cdot, \epsilon\}$
F	$\{+, *, \cdot, \epsilon\}$

$LR(0)(G_{AE})$	act							goto								
	+	*	()	a	b	ϵ	E	T	F	+	*	()	a	b
l_0			shift		shift	shift		l_1	l_2	l_3			l_4		l_5	l_6
l_1	shift						accept					l_7				
l_2	red 2	shift		red 2			red 2						l_8			
l_3	red 4	red 4		red 4			red 4									
l_4			shift		shift	shift		l_9	l_2	l_3			l_4		l_5	l_6
l_5	red 6	red 6		red 6			red 6									
l_6	red 7	red 7		red 7			red 7									
l_7			shift		shift	shift			l_{10}	l_3			l_4		l_5	l_6
l_8			shift		shift	shift				l_{11}			l_4		l_5	l_6
l_9	shift			shift							l_7			l_{12}		
l_{10}	red 1	shift		red 1			red 1					l_8				
l_{11}	red 3	red 3		red 3			red 3									
l_{12}	red 5	red 5		red 5			red 5									

The SLR(1) Parsing Automaton

Definition 9.16 (SLR(1) parsing automaton)

The **SLR(1) parsing automaton** is defined as in the **LR(0)** case (see Definition 9.8), except for the **transition relation**:

shift: $(aw, \alpha l, z) \vdash (w, \alpha l', z)$ if $\text{act}(l, a) = \text{shift}$ and $\text{goto}(l, a) = l'$

reduce_a: $(aw, \alpha ll_1 \dots l_n, z) \vdash (aw, \alpha l', zi)$ if $\text{act}(l_n, a) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(l, A) = l'$

reduce_ε: $(\varepsilon, \alpha ll_1 \dots l_n, z) \vdash (\varepsilon, \alpha l', zi)$ if $\text{act}(l_n, \varepsilon) = \text{red } i$, $\pi_i = A \rightarrow Y_1 \dots Y_n$, and $\text{goto}(l, A) = l'$

accept: $(\varepsilon, l_0 l, z) \vdash (\varepsilon, \varepsilon, z 0)$ if $\text{act}(l, \varepsilon) = \text{accept}$

error_a: $(aw, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(l, a) = \text{error}$

error_ε: $(\varepsilon, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$ if $\text{act}(l, \varepsilon) = \text{error}$