



Compiler Construction

Lecture 8: Syntax Analysis IV ($LR(k)$ Grammars)

Summer Semester 2016

Thomas Noll

Software Modeling and Verification Group

RWTH Aachen University

<https://moves.rwth-aachen.de/teaching/ss-16/cc/>



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***Rückenwind** ['rykn vɪnt]: Hilfe oder Beistand in einer schwierigen Situation oder das Engagement der Förderer für ein erfolgreiches Studium nutzen.

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RWTHAACHEN
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Einladung zum itestra Studententag

- **Von den Bytes bis zum Topmanagement** – Wie itestra Unternehmen verändert
- **Shaping Business** – Gestaltung einer webbasierten Anwendung zur Optimierung der Vertriebswege mit AngularJS
- Ausklang des Tages bei einem gemeinsamen Essen



Freitag, **10.06.2016**, 15.00-19.00 Uhr

itestra GmbH, Hansaring 20, 50670 Köln

Die Fahrtkosten in Höhe einer Bahnfahrt 2. Klasse trägt die itestra GmbH.

Melden Sie sich bis zum **03.06.2016** an.
Seien Sie mit dabei!

Anne-Kristin Hauk – hauk@itestra.de

Softwarequalität– Praxis-Workshop

Bringen Sie Informatik zur Wirkung!

Zuverlässig funktionierende Software, niedrige Kosten für Entwicklung und Betrieb oder schnelle Umsetzung neuer Anforderungen – Softwarequalität liefert einen wichtigen Beitrag zum Erfolg von Unternehmen. Dennoch existiert bis heute kein einheitliches Qualitätsverständnis für Software. Unterschiedlichste Verfahren und Kriterien werden eingesetzt.

Inhalte des Workshops:

- Wir stellen Ihnen im Workshop reale Beispiele aus der Praxis vor und zeigen, was neben Software-Tests und Code-Metriken wichtige Ansätze zur Qualitätssteigerung sind.
- Im Team erarbeiten Sie anschließend ein eigenes Software-Qualitätsmodell für Ihre Qualitätsziele und bewerten auf dieser Basis reale Softwaresysteme aus der Industrie. Unsere erfahrenen Kollegen unterstützen Sie dabei.



Dienstag, **28.06.2016**, 09.00–16.00 Uhr

Raum 9222, Geb. E3, 2. OG, Informatik-Zentrum, RWTH Aachen (Ahornstr. 55)

Melden Sie sich unter Angabe Ihres Semesters bis zum **19.06.2016** an.
Wir freuen uns auf Sie!

Kontakt: Anne-Kristin Hauk (hauk@itestra.de) – www.itestra.com

Bottom-Up Parsing

Outline of Lecture 8

Bottom-Up Parsing

Nondeterministic Bottom-Up Parsing

Resolving Termination Nondeterminism

$LR(k)$ Grammars

$LR(0)$ Grammars

Recap: Top-Down Parsing

Example 8.1

Grammar for
arithmetic expressions:

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T * F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Bottom-Up Parsing

Recap: Top-Down Parsing

Example 8.1

E

Grammar for
arithmetic expressions:

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Leftmost analysis of $(a)*b$:

(a) * b

Bottom-Up Parsing

Recap: Top-Down Parsing

Example 8.1

E
|
 T

Grammar for
arithmetic expressions:

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Leftmost analysis of $(a)*b$:

2

(a) * b

Bottom-Up Parsing

Recap: Top-Down Parsing

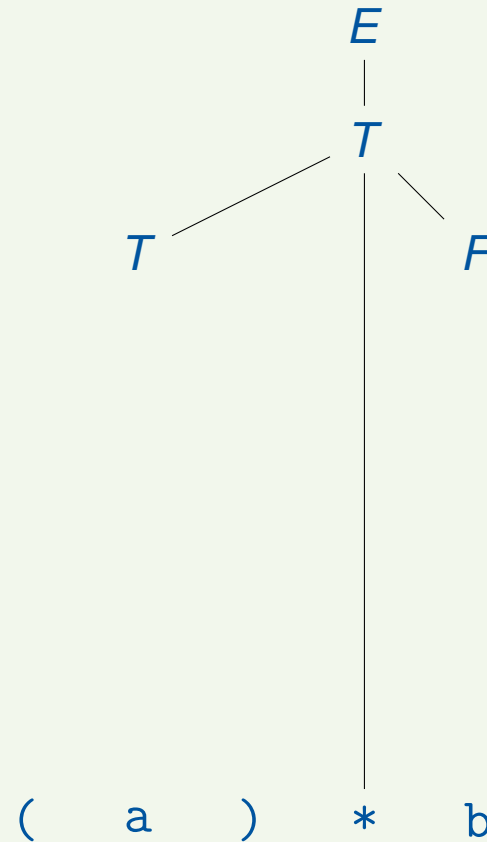
Example 8.1

Grammar for
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Leftmost analysis of $(a)*b$:

2 3



Bottom-Up Parsing

Recap: Top-Down Parsing

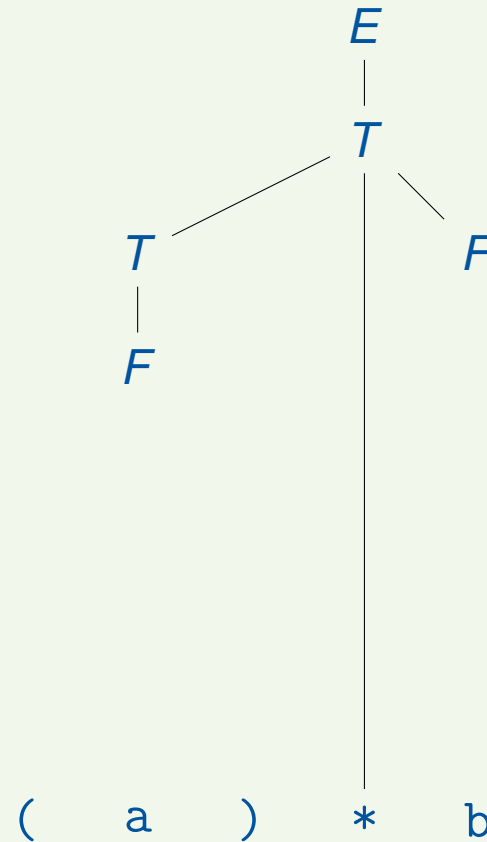
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Leftmost analysis of $(a)*b$:

2 3 4



Bottom-Up Parsing

Recap: Top-Down Parsing

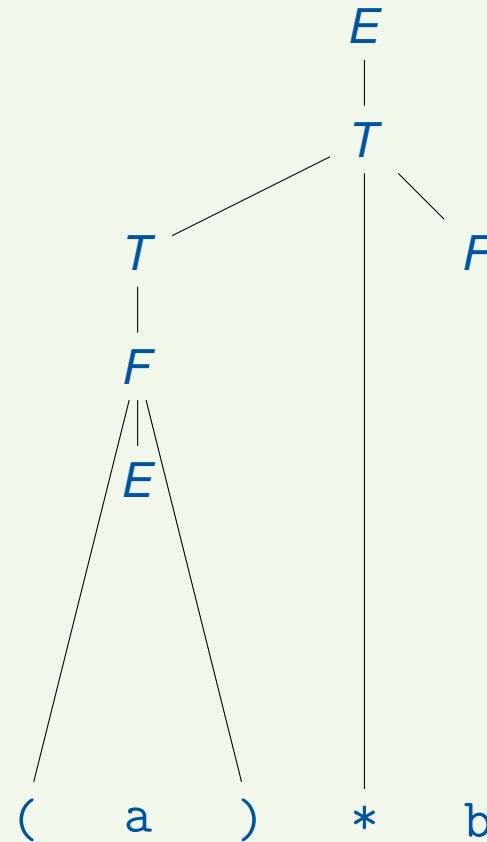
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Leftmost analysis of $(a)*b$:

2 3 4 5



Bottom-Up Parsing

Recap: Top-Down Parsing

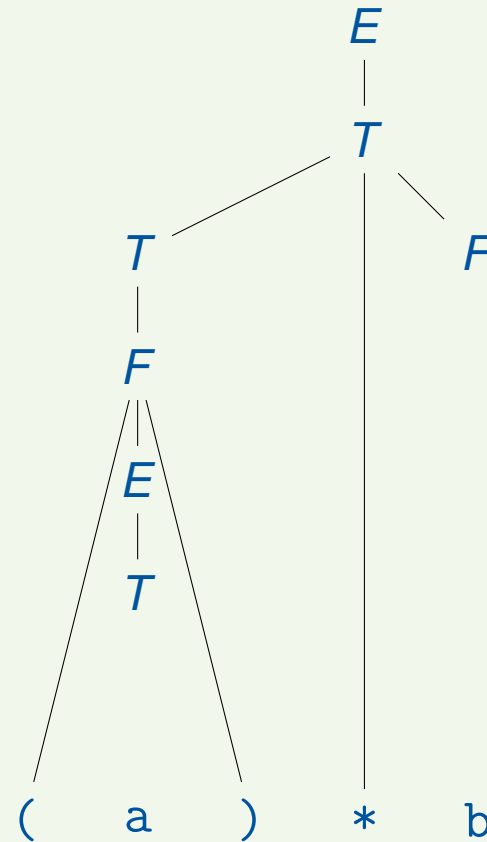
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Leftmost analysis of $(a)*b$:

2 3 4 5 2



Bottom-Up Parsing

Recap: Top-Down Parsing

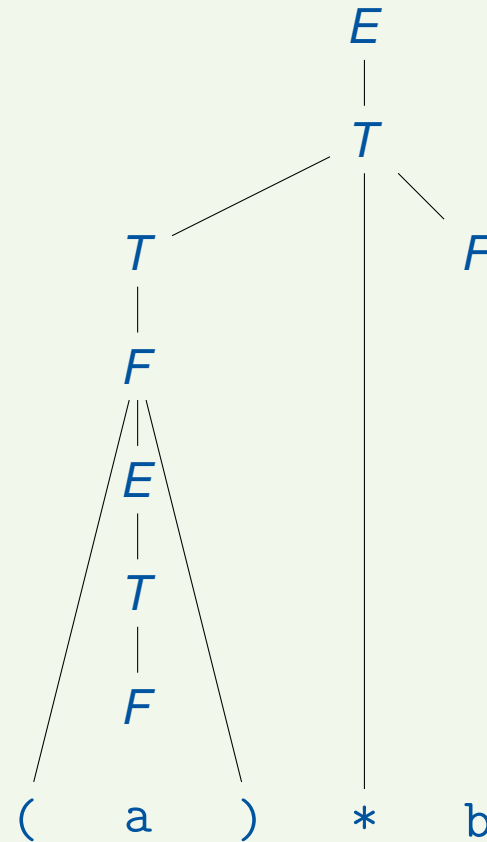
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Bottom-Up Parsing

Recap: Top-Down Parsing

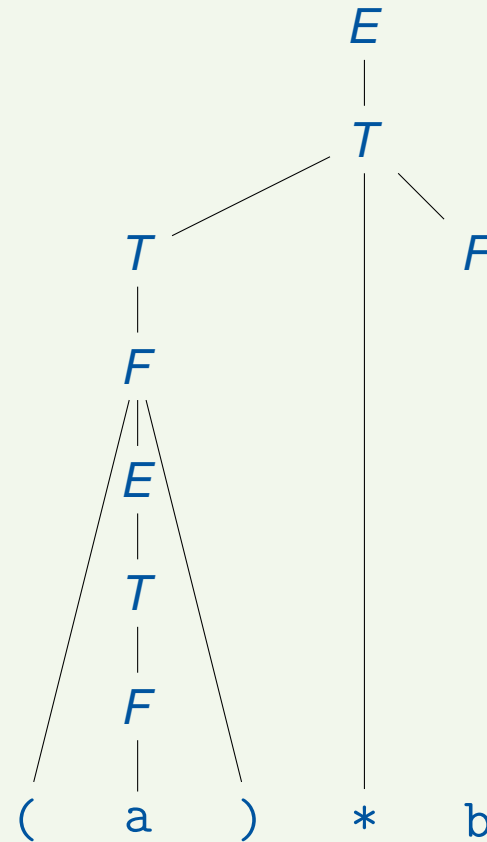
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Leftmost analysis of $(a)*b$:

2 3 4 5 2 4 6



Bottom-Up Parsing

Recap: Top-Down Parsing

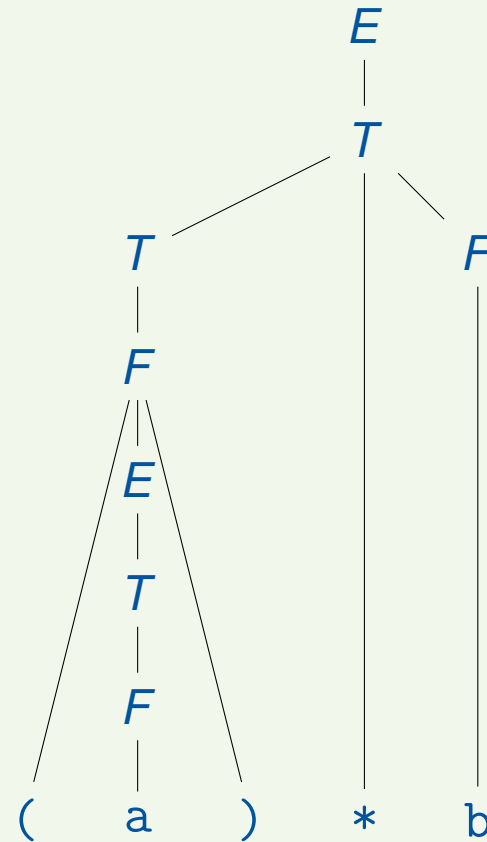
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Leftmost analysis of $(a)*b$:

2 3 4 5 2 4 6 7



Bottom-Up Parsing I

Example 8.2

Grammar for
arithmetic expressions:

$$\begin{aligned} G_{AE} : \quad & E \rightarrow E+T \mid T && (1, 2) \\ & T \rightarrow T*F \mid F && (3, 4) \\ & F \rightarrow (E) \mid a \mid b && (5, 6, 7) \end{aligned}$$

Bottom-Up Parsing

Bottom-Up Parsing I

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$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Reversed rightmost analysis
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(a) * b

Bottom-Up Parsing

Bottom-Up Parsing I

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$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Reversed rightmost analysis

of $(a)*b$:

6

$$\begin{array}{ccccccc} & & F & & & & \\ & & | & & & & \\ (& a &) & * & b \end{array}$$

Bottom-Up Parsing

Bottom-Up Parsing I

Example 8.2

Grammar for
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Reversed rightmost analysis

of $(a)*b$:

6 4

$$\begin{array}{c} T \\ | \\ F \\ | \\ (\quad a \quad) \quad * \quad b \end{array}$$

Bottom-Up Parsing

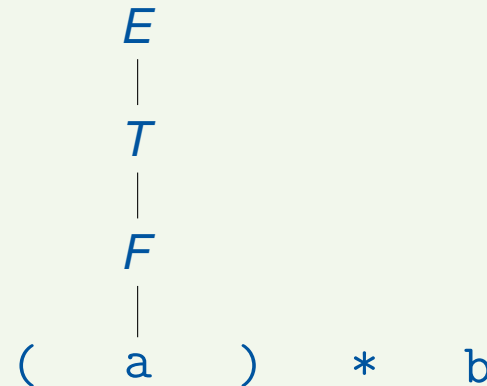
Bottom-Up Parsing I

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Reversed rightmost analysis
of $(a)*b$:
6 4 2



Bottom-Up Parsing

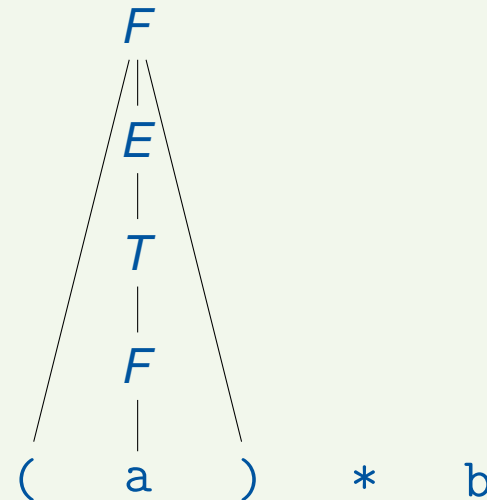
Bottom-Up Parsing I

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Reversed rightmost analysis
of $(a)*b$:
6 4 2 5



Bottom-Up Parsing

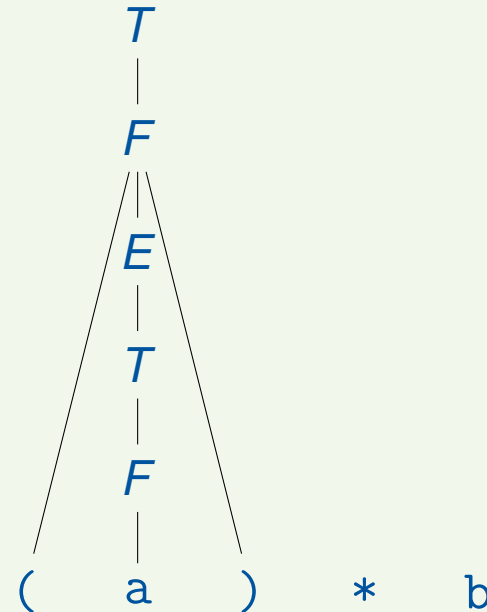
Bottom-Up Parsing I

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Reversed rightmost analysis
of $(a)*b$:
6 4 2 5 4



Bottom-Up Parsing

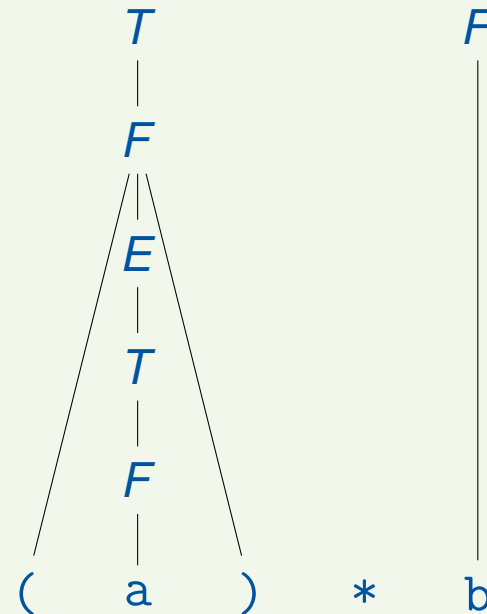
Bottom-Up Parsing I

Example 8.2

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Reversed rightmost analysis
of $(a)*b$:
6 4 2 5 4 7



Bottom-Up Parsing

Bottom-Up Parsing I

Example 8.2

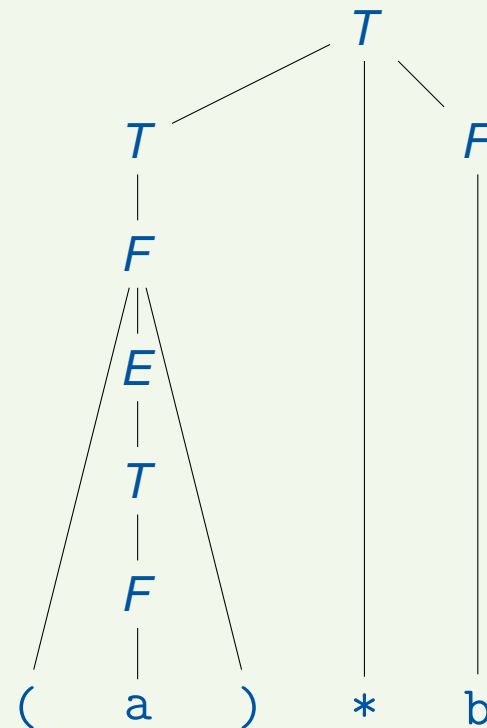
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Reversed rightmost analysis

of $(a)*b$:

6 4 2 5 4 7 3



Bottom-Up Parsing

Bottom-Up Parsing I

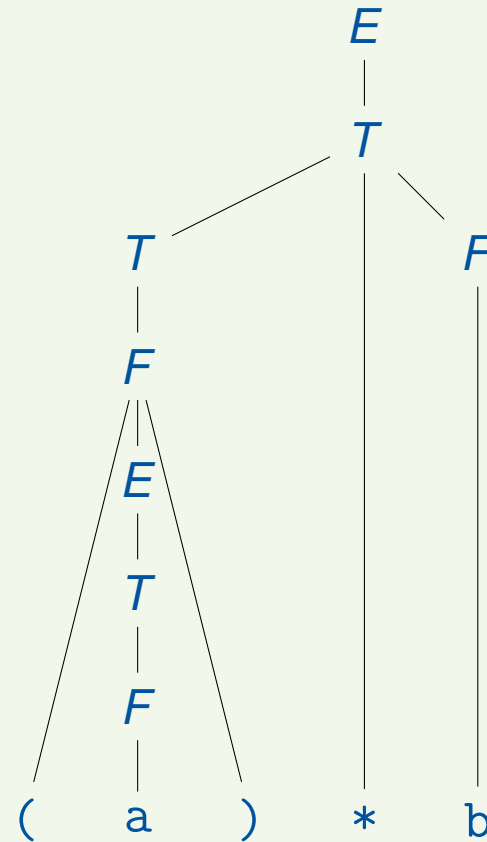
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Reversed rightmost analysis
of $(a)*b$:

6 4 2 5 4 7 3 2



Bottom-Up Parsing II

Approach:

1. Given $G \in CFG_{\Sigma}$, construct a **nondeterministic bottom-up parsing automaton** (NBA) which accepts $L(G)$ and which additionally computes corresponding (reversed) rightmost analyses
 - input alphabet: Σ
 - pushdown alphabet: X
 - output alphabet: $[p]$ (where $p := |P|$)
 - state set: omitted
 - transitions:
 - shift**: shifting input symbols onto the pushdown
 - reduce**: replacing the right-hand side of a production by its left-hand side (= inverse expansion step)

Bottom-Up Parsing II

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 - state set: omitted
 - transitions:
 - shift**: shifting input symbols onto the pushdown
 - reduce**: replacing the right-hand side of a production by its left-hand side (= inverse expansion step)
2. Remove nondeterminism by allowing **lookahead** on the input:
 $G \in LR(k)$ iff $L(G)$ recognisable by deterministic bottom-up parsing automaton with lookahead of k symbols

Nondeterministic Bottom-Up Parsing

Outline of Lecture 8

Bottom-Up Parsing

Nondeterministic Bottom-Up Parsing

Resolving Termination Nondeterminism

$LR(k)$ Grammars

$LR(0)$ Grammars

Nondeterministic Bottom-Up Automaton I

Definition 8.3 (Nondeterministic bottom-up parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$. The **nondeterministic bottom-up parsing automaton** of G , $NBA(G)$, is defined by the following components.

- **Input alphabet:** Σ
- **Pushdown alphabet:** X
- **Output alphabet:** $[p]$
- **Configurations:** $\Sigma^* \times X^* \times [p]^*$ (top of pushdown to the right)
- **Transitions** for $w \in \Sigma^*$, $\alpha \in X^*$, and $z \in [p]^*$:
 - shifting steps: $(aw, \alpha, z) \vdash (w, \alpha a, z)$ if $a \in \Sigma$
 - reduction steps: $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ if $\pi_i = A \rightarrow \beta$
- **Initial configuration** for $w \in \Sigma^*$: $(w, \varepsilon, \varepsilon)$
- **Final configurations:** $\{\varepsilon\} \times \{S\} \times [p]^*$

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
arithmetic expressions
(cf. Example 8.2):

$$\begin{array}{ll} G_{AE} : E \rightarrow E+T \mid T & (1, 2) \\ T \rightarrow T*F \mid F & (3, 4) \\ F \rightarrow (E) \mid a \mid b & (5, 6, 7) \end{array}$$

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
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Bottom-up parsing of $(a)*b$:

$$((a)*b, \varepsilon, \varepsilon)$$

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
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(cf. Example 8.2):

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Bottom-up parsing of $(a)*b$:

$$\begin{aligned} &((a)*b, \varepsilon, \varepsilon) \\ \vdash &(\textcolor{red}{a}) * b, (, \varepsilon) \end{aligned}$$

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
arithmetic expressions
(cf. Example 8.2):

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Bottom-up parsing of $(a)*b$:

$$\begin{aligned} &((a)*b, \varepsilon, \varepsilon) \\ \vdash & (a)*b, (, \varepsilon) \\ \vdash & ()*b, (a, \varepsilon) \end{aligned}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
arithmetic expressions
(cf. Example 8.2):

$\vdash ()*b, (F, 6)$

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Bottom-up parsing of $(a)*b$:

$$\begin{aligned} &((a)*b, \varepsilon, \varepsilon) \\ \vdash & (a)*b, (, \varepsilon) \\ \vdash & ()*b, (a, \varepsilon) \end{aligned}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
arithmetic expressions
(cf. Example 8.2):

$$\vdash () * b, (F, 6) \\ \vdash () * b, (T, 64)$$
$$G_{AE} : E \rightarrow E + T \mid T \quad (1, 2) \\ T \rightarrow T * F \mid F \quad (3, 4) \\ F \rightarrow (E) \mid a \mid b \quad (5, 6, 7)$$

Bottom-up parsing of $(a) * b$:

$$((a) * b, \varepsilon, \varepsilon) \\ \vdash (a) * b, (, \varepsilon) \\ \vdash () * b, (a, \varepsilon)$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
arithmetic expressions
(cf. Example 8.2):

$$\begin{array}{l} \vdash () * b, (F, 6) \\ \vdash () * b, (T, 64) \\ \vdash () * b, (E, 642) \end{array}$$
$$\begin{array}{ll} G_{AE} : E \rightarrow E + T \mid T & (1, 2) \\ T \rightarrow T * F \mid F & (3, 4) \\ F \rightarrow (E) \mid a \mid b & (5, 6, 7) \end{array}$$

Bottom-up parsing of $(a) * b$:

$$\begin{array}{l} ((a) * b, \varepsilon, \varepsilon) \\ \vdash (a) * b, (, \varepsilon) \\ \vdash () * b, (a, \varepsilon) \end{array}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
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$$\begin{aligned} \vdash ()*b, (F, 6) & \\ \vdash ()*b, (T, 64) & \\ \vdash ()*b, (E, 642) & \\ \vdash (*b, (E), 642) & \end{aligned}$$

Bottom-up parsing of $(a)*b$:

$$\begin{aligned} &((a)*b, \varepsilon, \varepsilon) \\ \vdash & (a)*b, (, \varepsilon) \\ \vdash & ()*b, (a, \varepsilon) \end{aligned}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

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Bottom-up parsing of $(a)*b$:

$$\begin{aligned} &((a)*b, \varepsilon, \varepsilon) \\ \vdash &(a)*b, (, \varepsilon) \\ \vdash &() *b, (a, \varepsilon) \end{aligned}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

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Bottom-up parsing of $(a)*b$:

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Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

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Bottom-up parsing of $(a)*b$:

$$\begin{aligned} &((a)*b, \varepsilon, \varepsilon) \\ \vdash & (a)*b, (, \varepsilon) \\ \vdash & ()*b, (a, \varepsilon) \end{aligned}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

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$$\begin{aligned} \vdash & ()*b, (F, 6) \\ \vdash & ()*b, (T, 64) \\ \vdash & ()*b, (E, 642) \\ \vdash & (*b, (E), 642) \\ \vdash & (*b, F, 6425) \\ \vdash & (*b, T, 64254) \\ \vdash & (b, T*, 64254) \\ \vdash & (\varepsilon, T*b, 64254) \end{aligned}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

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$$\begin{aligned} \vdash & ()*b, (F, 6) \\ \vdash & ()*b, (T, 64) \\ \vdash & ()*b, (E, 642) \\ \vdash & (*b, (E), 642) \\ \vdash & (*b, F, 6425) \\ \vdash & (*b, T, 64254) \\ \vdash & (b, T*, 64254) \\ \vdash & (\varepsilon, T*b, 64254) \\ \vdash & (\varepsilon, T*F, 642547) \end{aligned}$$

Nondeterministic Bottom-Up Parsing

Nondeterministic Bottom-Up Automaton II

Example 8.4

Grammar for
arithmetic expressions
(cf. Example 8.2):

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T & (1, 2) \\ T &\rightarrow T*F \mid F & (3, 4) \\ F &\rightarrow (E) \mid a \mid b & (5, 6, 7) \end{aligned}$$

Bottom-up parsing of $(a)*b$:

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Nondeterministic Bottom-Up Parsing

Correctness of $NBA(G)$

Theorem 8.5 (Correctness of $NBA(G)$)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $NBA(G)$ as before. Then, for every $w \in \Sigma^*$ and $z \in [p]^*$,

$$(w, \varepsilon, \varepsilon) \vdash^* (\varepsilon, S, z) \quad \text{iff} \quad \overleftarrow{z} \text{ is a rightmost analysis of } w$$

Nondeterministic Bottom-Up Parsing

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Proof.

similar to the top-down case (Theorem 6.1)



Nondeterministic Bottom-Up Parsing

Nondeterminism in $NBA(G)$

Observation: $NBA(G)$ is generally **nondeterministic**

- **Shift or reduce?** Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \text{ if } \pi_i = A \rightarrow a$$

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- **When to terminate parsing?** Example:

$$\underbrace{(\varepsilon, S, z)}_{\text{final}} \vdash (\varepsilon, A, zi) \text{ if } \pi_i = A \rightarrow S$$

Resolving Termination Nondeterminism

Outline of Lecture 8

Bottom-Up Parsing

Nondeterministic Bottom-Up Parsing

Resolving Termination Nondeterminism

$LR(k)$ Grammars

$LR(0)$ Grammars

Resolving Termination Nondeterminism

Resolving Termination Nondeterminism I

General assumption to avoid nondeterminism of last type:
every grammar is start separated

Definition 8.6 (Start separation)

A grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ is called **start separated** if S only occurs in productions of the form $S \rightarrow A$ where $A \neq S$.

Resolving Termination Nondeterminism

Resolving Termination Nondeterminism I

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A grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ is called **start separated** if S only occurs in productions of the form $S \rightarrow A$ where $A \neq S$.

Remarks:

- Start separation always possible by adding $S' \rightarrow S$ with **new start symbol** S'
- From now on consider only **reduced** grammars of this form (and let $\pi_0 := S' \rightarrow S$)

Resolving Termination Nondeterminism II

Start separation removes “When to terminate parsing?” nondeterminism:

Lemma 8.7

*If $G \in CFG_{\Sigma}$ is start separated, then no successor of a final configuration (ε, S', z) in $NBA(G)$ is again a final configuration.
(Thus parsing should be stopped in the first final configuration.)*

Resolving Termination Nondeterminism

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(Thus parsing should be stopped in the first final configuration.)*

Proof.

- To (ε, S', z) , only reductions by ε -productions can be applied:

$$(\varepsilon, S', z) \vdash (\varepsilon, S'A, zi) \quad \text{if } \pi_i = A \rightarrow \varepsilon$$

Resolving Termination Nondeterminism

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- Thereafter, only reductions by productions of the form $A_0 \rightarrow A_1 \dots A_n$ ($n \geq 0$) applicable
- Every resulting configuration is of the (non-final) form

$$(\varepsilon, S'B_1 \dots B_k, z) \quad \text{where } k \geq 1$$



Outline of Lecture 8

Bottom-Up Parsing

Nondeterministic Bottom-Up Parsing

Resolving Termination Nondeterminism

$LR(k)$ Grammars

$LR(0)$ Grammars

$LR(k)$ Grammars I

Goal: resolve remaining nondeterminism of $NBA(G)$ by supporting lookahead of $k \in \mathbb{N}$ symbols on the input

$\Rightarrow LR(k)$: reading of input from left to right with k -lookahead, computing a rightmost analysis

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$LR(k)$ Grammars I

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$\implies LR(k)$: reading of input from left to right with k -lookahead, computing a rightmost analysis

Definition 8.8 ($LR(k)$ grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated and $k \in \mathbb{N}$. Then G has the $LR(k)$ property (notation: $G \in LR(k)$) if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha \beta y \end{cases}$$

such that $\text{first}_k(w) = \text{first}_k(y)$, it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

$LR(k)$ Grammars II

Remarks:

- If $G \in LR(k)$, then the reduction of $\alpha\beta w$ to αAw is already determined by $\text{first}_k(w)$.
- Therefore $NBA(G)$ in configuration $(w, \alpha\beta, z)$ can decide to reduce and how to reduce.

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- **Computation of $NBA(G)$ for $S \Rightarrow_r^* \alpha Aw \Rightarrow_r \alpha\beta w$:**

$$(w'w, \varepsilon, \varepsilon) \vdash^* (w, \alpha\beta, z) \overset{\text{red } i}{\vdash} (w, \alpha A, zi) \vdash \dots$$

where $\pi_i = A \rightarrow \beta$

LR(k) Grammars II

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- **Computation of $NBA(G)$ for $S \Rightarrow_r^* \gamma Bx \Rightarrow_r \alpha\beta y$:**
 - with direct reduction $(y = x, \alpha\beta = \gamma\delta, \pi_j = B \rightarrow \delta)$:

$$(y'y, \varepsilon, \varepsilon) \vdash^* (y, \alpha\beta, z') = (x, \gamma\delta, z') \stackrel{\text{red } j}{\vdash} (x, \gamma B, z'j) \vdash \dots$$

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- with previous shifts ($y = x'x, \alpha\beta x' = \gamma\delta, \pi_j = B \rightarrow \delta$):

$$(y'y, \varepsilon, \varepsilon) \vdash^* (y, \alpha\beta, z') = (x'x, \alpha\beta, z') \stackrel{\text{shift}^*}{\vdash} (x, \alpha\beta x', z') = (x, \gamma\delta, z') \stackrel{\text{red } j}{\vdash} (x, \gamma B, z'j) \vdash \dots$$

Outline of Lecture 8

Bottom-Up Parsing

Nondeterministic Bottom-Up Parsing

Resolving Termination Nondeterminism

$LR(k)$ Grammars

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The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, **without any lookahead**.

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Corollary 8.9 (LR(0) grammar)

$G \in CFG_{\Sigma}$ has the **LR(0) property** if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha \beta y \end{cases}$$

it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

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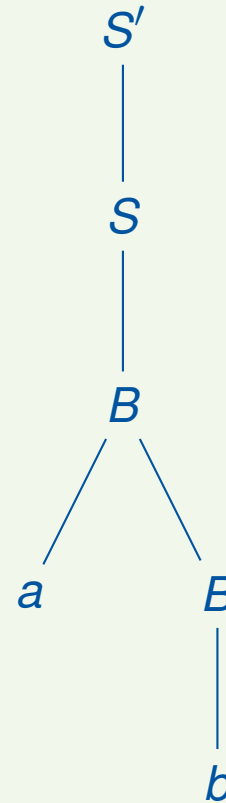
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it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

Goal: derive a **finite information** from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by fo-sets)

Example 8.10

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$



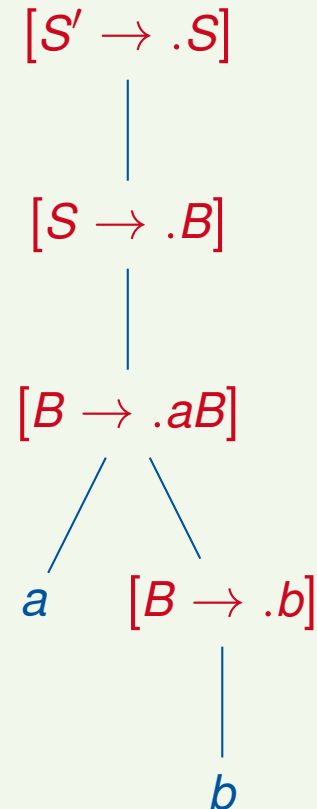
LR(0) Grammars

LR(0) Items and Sets I

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$NBA(G):$
 $(ab, \varepsilon, \varepsilon)$



LR(0) Grammars

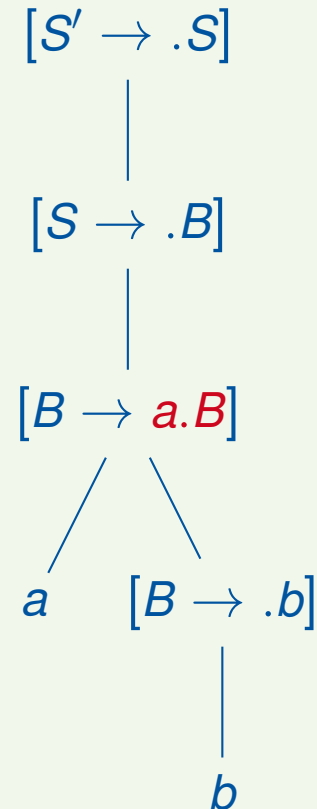
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 $\vdash (b, a, \varepsilon)$



LR(0) Grammars

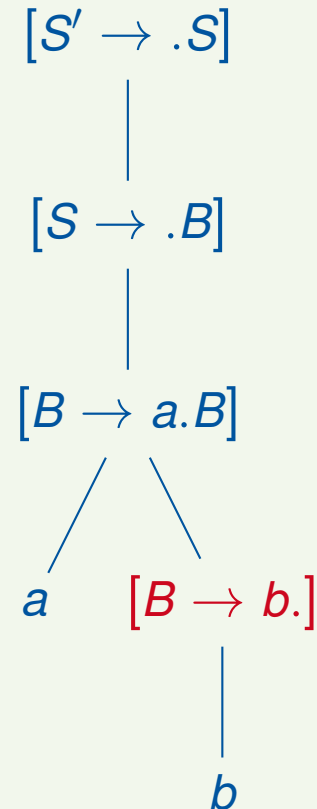
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 $\vdash (\varepsilon, ab, \varepsilon)$



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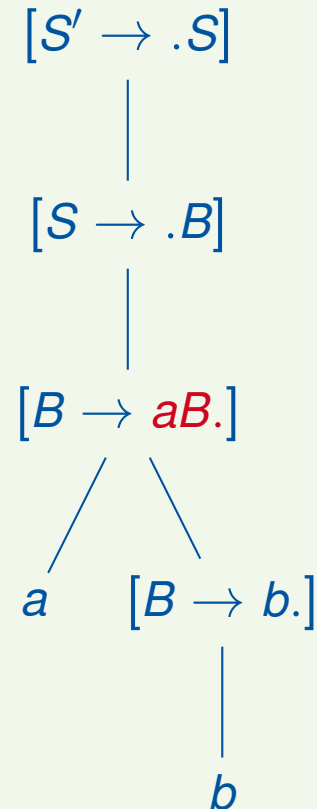
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 $\vdash (\varepsilon, aB, 4)$



LR(0) Grammars

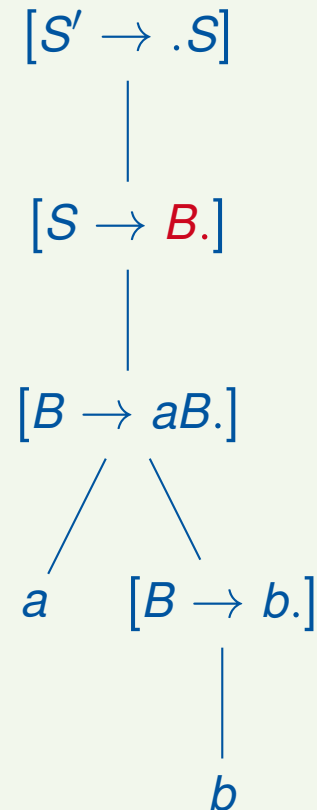
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LR(0) Grammars

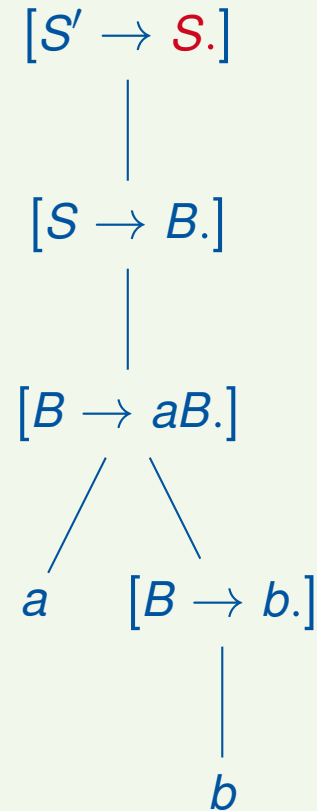
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 $\vdash (\varepsilon, S, 431)$



LR(0) Grammars

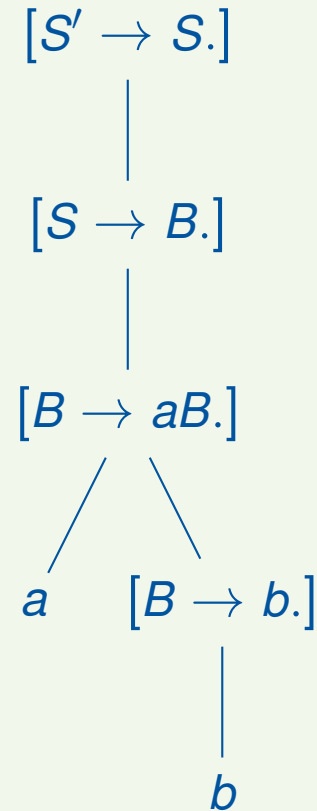
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LR(0) Items and Sets II

Definition 8.11 (LR(0) items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an **LR(0) item** for $\alpha \beta_1$.

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Corollary 8.12

1. For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.

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LR(0) Items and Sets II

Definition 8.11 (LR(0) items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an **LR(0) item** for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all **LR(0)** items for γ , called the **LR(0) set** (or: **LR(0) information**) of γ .
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 8.12

1. For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.
2. $LR(0)(G)$ is finite.
3. The item $[A \rightarrow \beta \cdot] \in LR(0)(\gamma)$ indicates the possible **reduction** $(w, \alpha \beta, z) \vdash (w, \alpha A, zi)$ where $\pi_i = A \rightarrow \beta$ and $\gamma = \alpha \beta$.

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4. The item $[A \rightarrow \beta_1 \cdot Y \beta_2] \in LR(0)(\gamma)$ indicates an **incomplete handle** β_1 (to be completed by shift operations or ε -reductions).

LR(0) Conflicts

Definition 8.13 (LR(0) conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $I \in LR(0)(G)$.

- I has a **shift/reduce conflict** if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

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- I has a **reduce/reduce conflict** if there exist $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

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LR(0) Grammars

LR(0) Conflicts

Definition 8.13 (LR(0) conflicts)

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$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

Lemma 8.14

$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.

Proof.

omitted □

Computing LR(0) Sets I

Theorem 8.15 (Computing LR(0) sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and reduced.

1. $LR(0)(\varepsilon)$ is the least set such that
 - $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$ and
 - if $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$.

Computing LR(0) Sets I

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1. $LR(0)(\varepsilon)$ is the least set such that

- $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$ and
- if $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$.

2. $LR(0)(\alpha Y)$ ($\alpha \in X^*$, $Y \in X$) is the least set such that

- if $[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$,
then $[A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ and
- if $[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$.

Computing $LR(0)$ Sets II

Example 8.16 (cf. Example 8.10)

$G: S' \rightarrow S \quad S \rightarrow B \mid C$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c$

Computing $LR(0)$ Sets II

Example 8.16 (cf. Example 8.10)

$G: S' \rightarrow S \quad S \rightarrow B \mid C$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c$ $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$
 $I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S]$

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$
 $I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B]$

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 $I_0 := LR(0)(\varepsilon): [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$

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Example 8.16 (cf. Example 8.10)

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 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$
 $I_0 := LR(0)(\varepsilon): [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

$$\begin{array}{lll} G: S' \rightarrow S & S \rightarrow B \mid C & [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ B \rightarrow aB \mid b & C \rightarrow aC \mid c & \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \\ \\ I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & \end{array}$$

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon): [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$
 $I_1 := LR(0)(S): [S' \rightarrow S \cdot]$
 $I_2 := LR(0)(B): [S \rightarrow B \cdot]$

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

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 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

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 $I_1 := LR(0)(S): [S' \rightarrow S \cdot]$
 $I_2 := LR(0)(B): [S \rightarrow B \cdot]$
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Computing LR(0) Sets II

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 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

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 $I_1 := LR(0)(S): [S' \rightarrow S \cdot]$
 $I_2 := LR(0)(B): [S \rightarrow B \cdot]$
 $I_3 := LR(0)(C): [S \rightarrow C \cdot]$
 $I_4 := LR(0)(a): [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C]$

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

$$\begin{array}{lll} G: S' \rightarrow S & S \rightarrow B \mid C & [A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P \\ & B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y) \end{array}$$
$$\begin{array}{l} I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_1 := LR(0)(S) : [S' \rightarrow S \cdot] \\ I_2 := LR(0)(B) : [S \rightarrow B \cdot] \\ I_3 := LR(0)(C) : [S \rightarrow C \cdot] \\ I_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \end{array}$$

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon): [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$
 $I_1 := LR(0)(S): [S' \rightarrow S \cdot]$
 $I_2 := LR(0)(B): [S \rightarrow B \cdot]$
 $I_3 := LR(0)(C): [S \rightarrow C \cdot]$
 $I_4 := LR(0)(a): [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

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Example 8.16 (cf. Example 8.10)

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 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon): [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$
 $I_1 := LR(0)(S): [S' \rightarrow S \cdot]$
 $I_2 := LR(0)(B): [S \rightarrow B \cdot]$
 $I_3 := LR(0)(C): [S \rightarrow C \cdot]$
 $I_4 := LR(0)(a): [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$
 $I_5 := LR(0)(b): [B \rightarrow b \cdot]$

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

$$\begin{array}{lll} G: S' \rightarrow S & S \rightarrow B \mid C & [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ & B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \end{array}$$
$$\begin{array}{l} I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_1 := LR(0)(S) : [S' \rightarrow S \cdot] \\ I_2 := LR(0)(B) : [S \rightarrow B \cdot] \\ I_3 := LR(0)(C) : [S \rightarrow C \cdot] \\ I_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_5 := LR(0)(b) : [B \rightarrow b \cdot] \\ I_6 := LR(0)(c) : [C \rightarrow c \cdot] \end{array}$$

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$I_6 := LR(0)(c) : [C \rightarrow c \cdot]$

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$I_8 := LR(0)(aC) : [C \rightarrow aC \cdot]$

$(LR(0)(aa) = LR(0)(a) = I_4, LR(0)(ab) = LR(0)(b) = I_5, LR(0)(ac) = LR(0)(c) = I_6, \dots,$

$I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases, e.g., for $\gamma = bB$)

Computing LR(0) Sets II

Example 8.16 (cf. Example 8.10)

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$(LR(0)(aa) = LR(0)(a) = I_4, LR(0)(ab) = LR(0)(b) = I_5, LR(0)(ac) = LR(0)(c) = I_6, \dots,$

$I_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases, e.g., for $\gamma = bB$)

No conflicts $\implies G \in LR(0)$ (but $G \notin LL(1)$)