Compiler Construction

Lecture 3: Lexical Analysis II (Extended Matching Problem)

Summer Semester 2016

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https://moves.rwth-aachen.de/teaching/ss-16/cc/

Outline of Lecture 3

Recap: Lexical Analysis

The Extended Matching Problem

First-Longest-Match Analysis

Implementation of FLM Analysis

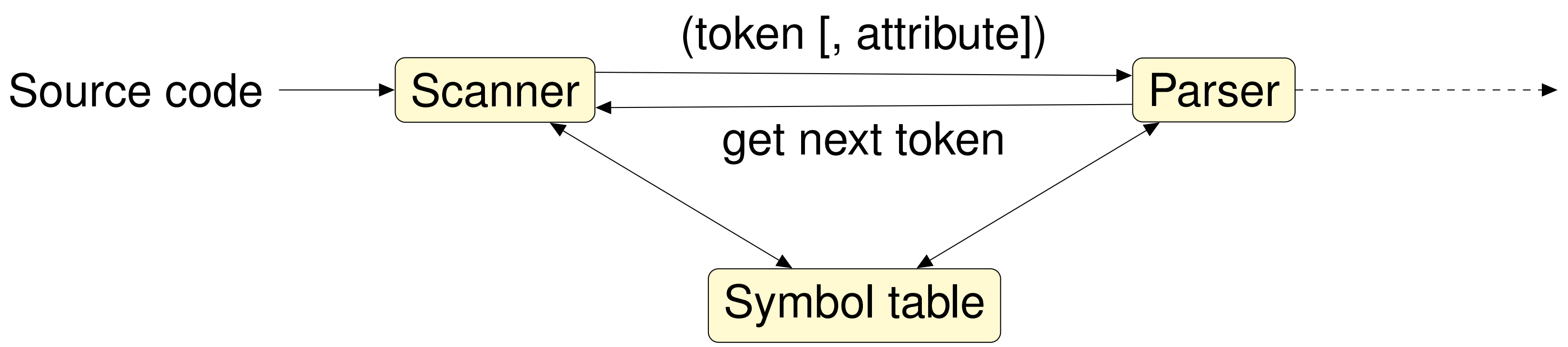
Recap: Lexical Analysis

Lexical Analysis

Definition

The goal of lexical analysis is the decomposition a source program into a sequence of lexemes and their transformation into a sequence of symbols.

The corresponding program is called a scanner (or lexer):



Example: . . . ␣x1␣:=y2+␣1␣;␣. . .

$\Downarrow$

$\ldots(\text{id}, p_1)(\text{gets},)(\text{id}, p_2)(\text{plus},)(\text{int}, 1)(\text{sem},)\ldots$

The DFA Method I

Known from *Formal Systems, Automata and Processes*:

Algorithm (DFA method)

Input: *regular expression* $\alpha \in RE_\Omega$, *input string* $w \in \Omega^*$

Procedure:
1. *using Kleene's Theorem, construct* $\mathfrak{A}_\alpha \in NFA_\Omega$ *such that* $L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$
2. *apply powerset construction (cf. Definition 2.11) to obtain* $\mathfrak{A}'_\alpha = \langle Q', \Omega, \delta', q'_0, F' \rangle \in DFA_\Omega$ *with* $L(\mathfrak{A}'_\alpha) = L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$
3. *solve the matching problem by deciding whether* $\delta'^*(q'_0, w) \in F'$

Output: *"yes" or "no"*

The DFA Method II

The powerset construction involves the following concept:

Definition (ε-closure)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in NFA_\Omega$. The ε-closure $\varepsilon(T) \subseteq Q$ of a subset $T \subseteq Q$ is the least set with (1) $T \subseteq \varepsilon(T)$ and (2) if $q \in \varepsilon(T)$, then $\delta(q, \varepsilon) \subseteq \varepsilon(T)$

Definition (Powerset construction)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in NFA_\Omega$. The powerset automaton $\mathfrak{A}' = \langle Q', \Omega, \delta', q_0', F' \rangle \in DFA_\Omega$ is defined by

- $Q' := 2^Q$
- $q_0' := \varepsilon(\{q_0\})$
- $\forall T \subseteq Q, a \in \Omega : \delta'(T, a) := \varepsilon\left(\bigcup_{q \in T} \delta(q, a)\right)$
- $F' := \{T \subseteq Q \mid T \cap F \neq \emptyset\}$

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The Extended Matching Problem I

Definition 3.1

Let $n \geq 1$ and $\alpha_1, \ldots, \alpha_n \in RE_\Omega$ with $\varepsilon \notin [\![\alpha_i]\!] \neq \emptyset$ for every $i \in [n]$ (where $[n] := \{1, \ldots, n\}$). Let $\Sigma := \{T_1, \ldots, T_n\}$ be an alphabet of corresponding tokens and $w \in \Omega^+$. If $w_1, \ldots, w_k \in \Omega^+$ such that

- $w = w_1 \ldots w_k$ and
- for every $j \in [k]$ there exists $i_j \in [n]$ such that $w_j \in [\![\alpha_{i_j}]\!]$,

then

- $(w_1, \ldots, w_k)$ is called a decomposition and
- $(T_{i_1}, \ldots, T_{i_k})$ is called an analysis

of w w.r.t. $\alpha_1, \ldots, \alpha_n$.

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Problem 3.2 (Extended matching problem)

Given $\alpha_1, \ldots, \alpha_n \in RE_\Omega$ and $w \in \Omega^+$, decide whether there exists a decomposition of w w.r.t. $\alpha_1, \ldots, \alpha_n$ and determine a corresponding analysis.

The Extended Matching Problem II

Observation: neither the decomposition nor the analysis are uniquely determined

Example 3.3

1. $\alpha = a^+$, $w = aa$
   $\Longrightarrow$ two decompositions (aa) and (a, a)
   with respective (unique) analyses (T_1) and (T_1, T_1)

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2. $\alpha_1 = a \mid b$, $\alpha_2 = a \mid c$, $w = a$
   $\Longrightarrow$ unique decomposition (a) but two analyses (T_1) and (T_2)

The Extended Matching Problem II

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   $\Longrightarrow$ unique decomposition (a) but two analyses (T_1) and (T_2)

Goal: make both unique $\Longrightarrow$ deterministic scanning

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First-Longest-Match Analysis

Ensuring Uniqueness

Two principles

1. Principle of the longest match ("maximal munch tokenisation")
   – for uniqueness of decomposition
   – make lexemes as long as possible
   – motivated by practical considerations: e.g., every proper prefix of an identifier is also an identifier

First-Longest-Match Analysis

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2. Principle of the first match
   - for uniqueness of analysis
   - choose first matching regular expression (in the given order)
   - therefore: arrange keywords before identifiers (if keywords protected)

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   - for uniqueness of analysis
   - choose first matching regular expression (in the given order)
   - therefore: arrange keywords before identifiers (if keywords protected)

Remark: uniqueness of analysis could also be achieved by requiring disjointness of symbol classes (i.e., $[\![\alpha_i]\!] \cap [\![\alpha_j]\!] = \emptyset$ for $i \neq j$)

- for example, $Id := (\mathtt{a} \mid \ldots)(\mathtt{a} \mid \ldots \mid \mathtt{0} \mid \ldots)^* \setminus \{\mathtt{if}, \mathtt{while}, \mathtt{begin}, \ldots\}$
- **but:** expensive implementation due to product construction ($A \setminus B = A \cap \overline{B}$)

Principle of the Longest Match

Definition 3.4 (Longest-match decomposition)

A decomposition $(w_1, \ldots, w_k)$ of $w \in \Omega^+$ w.r.t. $\alpha_1, \ldots, \alpha_n \in RE_\Omega$ is called a longest-match (LM) decomposition if, for every $i \in [k]$, $x \in \Omega^+$, and $y \in \Omega^*$,

$$w = w_1 \ldots w_i x y \implies \text{there is no } j \in [n] \text{ such that } w_i x \in [\![\alpha_j]\!]$$

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Corollary 3.5

Given w and $\alpha_1, \ldots, \alpha_n$,

- *at most one LM decomposition of w exists (clear by definition) and*

First-Longest-Match Analysis

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Corollary 3.5

Given w and $\alpha_1, \ldots, \alpha_n$,

- *at most one LM decomposition of w exists (clear by definition) and*
- *it is possible that w has a decomposition but no LM decomposition (see following example).*

Example 3.6

$w = aab$, $\alpha_1 = a^+$, $\alpha_2 = ab$

$\implies$ (a, ab) is a decomposition but no LM decomposition exists

First-Longest-Match Analysis

Principle of the First Match

Problem: a (unique) LM decomposition can have several associated analyses (since $[\![\alpha_i]\!] \cap [\![\alpha_j]\!] \neq \emptyset$ with $i \neq j$ is possible; cf. keyword/identifier problem)

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Definition 3.7 (First-longest-match analysis)

Let $(w_1, \ldots, w_k)$ be the LM decomposition of $w \in \Omega^+$ w.r.t. $\alpha_1, \ldots, \alpha_n \in RE_\Omega$. Its first-longest-match analysis (FLM analysis) $(T_{i_1}, \ldots, T_{i_k})$ is determined by

$$i_j := \min\{l \in [n] \mid w_j \in [\![\alpha_l]\!]\}$$

for every $j \in [k]$.

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for every $j \in [k]$.

Corollary 3.8

Given w and $\alpha_1, \ldots, \alpha_n$, there is at most one FLM analysis of w.
It exists iff the LM decomposition of w exists.

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Algorithm 3.9 (FLM analysis – overview)

Input: *expressions* $\alpha_1, \ldots, \alpha_n \in RE_\Omega$, *tokens* $\{T_1, \ldots, T_n\}$, *input word* $w \in \Omega^+$

Implementation of FLM Analysis

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Procedure: 1. *for every* $i \in [n]$, *construct* $\mathfrak{A}_i \in DFA_\Omega$ *such that* $L(\mathfrak{A}_i) = [\![\alpha_i]\!]$
(see DFA method; Algorithm 2.9)

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2. *construct the product automaton* $\mathfrak{A} \in DFA_\Omega$ *such that* $L(\mathfrak{A}) = \bigcup_{i=1}^{n} [\![\alpha_i]\!]$

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Output: *FLM analysis of* w *(if existing)*

(2) The Product Automaton

Definition 3.10 (Product automaton)

Let $\mathfrak{A}_i = \langle Q_i, \Omega, \delta_i, q_0^{(i)}, F_i \rangle \in DFA_\Omega$ for every $i \in [n]$. The product automaton $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in DFA_\Omega$ is defined by

- $Q := Q_1 \times \ldots \times Q_n$
- $q_0 := (q_0^{(1)}, \ldots, q_0^{(n)})$
- $\delta((q^{(1)}, \ldots, q^{(n)}), a) := (\delta_1(q^{(1)}, a), \ldots, \delta_n(q^{(n)}, a))$
- $(q^{(1)}, \ldots, q^{(n)}) \in F$ iff there ex. $i \in [n]$ such that $q^{(i)} \in F_i$

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The above construction yields $L(\mathfrak{A}) = \bigcup_{i=1}^n L(\mathfrak{A}_i)$ $(= \bigcup_{i=1}^n [\![\alpha_i]\!])$.

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Remark: similar construction for intersection ($F := F_1 \times \ldots \times F_n$)

(3) Partitioning the Final States

Definition 3.12 (Partitioning of final states)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in DFA_\Omega$ be the product automaton as constructed before. Its set of final states is partitioned into $F = \biguplus_{i=1}^{n} F^{(i)}$ by the requirement

$$(q^{(1)}, \ldots, q^{(n)}) \in F^{(i)} \iff q^{(i)} \in F_i \text{ and } \forall j \in [i-1] : q^{(j)} \notin F_j$$

(equivalently: $F^{(i)} := (Q_1 \setminus F_1) \times \ldots \times (Q_{i-1} \setminus F_{i-1}) \times F_i \times Q_{i+1} \times \ldots \times Q_n$)

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Corollary 3.13

The above construction yields ($w \in \Omega^+$, $i \in [n]$):

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Definition 3.14 (Productive states)

Given $\mathfrak{A}$ as above, $q \in Q$ is called productive if there exists $w \in \Omega^*$ such that $\delta^*(q, w) \in F$. Notation: productive states $P \subseteq Q$ (and thus $F \subseteq P$).

(4) The Backtracking DFA I

Goal: extend $\mathfrak{A}$ to the backtracking DFA $\mathfrak{B}$ with output by equipping the input tape with two pointers: a backtracking head for marking the last encountered match, and a lookahead for determining the longest match.

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A configuration of $\mathfrak{B}$ has three components
(remember: $\Sigma := \{T_1, \ldots, T_n\}$ denotes the set of tokens):

1. a mode $m \in \{N\} \uplus \Sigma$:
   - $m = N$ ("normal"): look for initial match (no final state reached yet)
   - $m = T \in \Sigma$: token T has been recognised, looking for possible longer match

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2. an input tape $vqw \in \Omega^* \cdot Q \cdot \Omega^*$:
   - v: lookahead part of input ($v \neq \varepsilon \implies m \in \Sigma$)
   - q: current state of $\mathfrak{A}$
   - w: remaining input

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   - q: current state of $\mathfrak{A}$
   - w: remaining input
3. an output tape $W \in \Sigma^* \cdot \{\varepsilon, \text{lexerr}\}$:
   - Σ^*: sequence of tokens recognised so far
   - lexerr: a lexical error has occurred (i.e., a non-productive state was entered or the suffix of the input is not a valid lexeme)

(4) The Backtracking DFA II

Definition 3.15 (Backtracking DFA)

- The set of configurations of $\mathfrak{B}$ is given by
$$(\{N\} \uplus \Sigma) \times \Omega^* \cdot Q \cdot \Omega^* \times \Sigma^* \cdot \{\varepsilon, \text{lexerr}\}$$
- The initial configuration for an input word $w \in \Omega^+$ is $(N, q_0 w, \varepsilon)$.

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- The transitions of $\mathfrak{B}$ are defined as follows (where $q' := \delta(q, a)$):
  - normal mode: look for initial match
$$(N, qaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} & (1) \\ (N, q'w, W) & \text{if } q' \in P \setminus F & (2) \\ \textbf{output: } W \cdot \text{lexerr} & \text{if } q' \notin P & (3) \end{cases}$$

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- The initial configuration for an input word $w \in \Omega^+$ is $(N, q_0 w, \varepsilon)$.
- The transitions of $\mathfrak{B}$ are defined as follows (where $q' := \delta(q, a)$):
  - normal mode: look for initial match
$$(N, qaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} & (1) \\ (N, q'w, W) & \text{if } q' \in P \setminus F & (2) \\ \textbf{output: } W \cdot \text{lexerr} & \text{if } q' \notin P & (3) \end{cases}$$
  - backtrack mode: look for longest match
$$(T, vqaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} & (4) \\ (T, vaq'w, W) & \text{if } q' \in P \setminus F & (5) \\ (N, q_0 vaw, WT) & \text{if } q' \notin P & (6) \end{cases}$$

(4) The Backtracking DFA II

Definition 3.15 (Backtracking DFA)

- The set of configurations of $\mathfrak{B}$ is given by
$$(\{N\} \uplus \Sigma) \times \Omega^* \cdot Q \cdot \Omega^* \times \Sigma^* \cdot \{\varepsilon, \text{lexerr}\}$$
- The initial configuration for an input word $w \in \Omega^+$ is $(N, q_0 w, \varepsilon)$.
- The transitions of $\mathfrak{B}$ are defined as follows (where $q' := \delta(q, a)$):
  - normal mode: look for initial match
$$(N, qaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} & (1) \\ (N, q'w, W) & \text{if } q' \in P \setminus F & (2) \\ \textbf{output: } W \cdot \text{lexerr} & \text{if } q' \notin P & (3) \end{cases}$$
  - backtrack mode: look for longest match
$$(T, vqaw, W) \vdash \begin{cases} (T_i, q'w, W) & \text{if } q' \in F^{(i)} & (4) \\ (T, vaq'w, W) & \text{if } q' \in P \setminus F & (5) \\ (N, q_0 vaw, WT) & \text{if } q' \notin P & (6) \end{cases}$$
  - end of input
$$\begin{aligned} (T, q, W) &\vdash \textbf{output: } WT && \text{if } q \in F && (7) \\ (N, q, W) &\vdash \textbf{output: } W \cdot \text{lexerr} && \text{if } q \in P \setminus F && (8) \\ (T, vaq, W) &\vdash (N, q_0 va, WT) && \text{if } q \in P \setminus F && (9) \end{aligned}$$

(4) The Backtracking DFA III

Lemma 3.16

Given the backtracking DFA $\mathfrak{B}$ as before and $w \in \Omega^+$,

$$(N, q_0 w, \varepsilon) \vdash^* \begin{cases} W \in \Sigma^* & \text{iff } W \text{ is the FLM analysis of } w \\ W \cdot \mathrm{lexerr} & \text{iff no FLM analysis of } w \text{ exists} \end{cases}$$

Proof.

(omitted) □

(4) The Backtracking DFA III

Lemma 3.16

Given the backtracking DFA $\mathfrak{B}$ *as before and* $w \in \Omega^+$,

$$(N, q_0 w, \varepsilon) \vdash^* \begin{cases} W \in \Sigma^* & \text{iff } W \text{ is the FLM analysis of } w \\ W \cdot \text{lexerr} & \text{iff no FLM analysis of } w \text{ exists} \end{cases}$$

Proof.

(omitted) □

Example 3.17

- $\Omega = \{a, b\}$, $w = baa$
- $n = 3$, $\Sigma = \{T_1, T_2, T_3\}$
- $\alpha_1 = a$ (“keyword”), $\alpha_2 = a^+b$ (“identifier”), $\alpha_3 = b$ (“operator”)

(on the board)