Compiler Construction

Lecture 16: Code Generation II (The Translation)

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https://moves.rwth-aachen.de/teaching/ss-16/cc/

Recap: Intermediate Code

Syntax of EPL

Definition (Syntax of EPL)

The syntax of EPL is defined as follows:

$$
\begin{array}{lll}
\mathbb{Z}: & z & \text{(* } z \text{ is an integer *)} \\
Ide: & I & \text{(* } I \text{ is an identifier *)} \\
AExp: & A ::= z \mid I \mid A_1 + A_2 \mid \ldots & \\
BExp: & B ::= A_1 < A_2 \mid \texttt{not}\ B \mid B_1\ \texttt{and}\ B_2 \mid B_1\ \texttt{or}\ B_2 & \\
Cmd: & C ::= I\ \texttt{:=}\ A \mid C_1\texttt{;}C_2 \mid \texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2 \mid \texttt{while}\ B\ \texttt{do}\ C \mid I\texttt{()} & \\
Dcl: & D ::= D_C\ D_V\ D_P & \\
 & D_C ::= \varepsilon \mid \texttt{const}\ I_1\ \texttt{:=}\ z_1\texttt{,}\ldots\texttt{,}I_n\ \texttt{:=}\ z_n\texttt{;} & \\
 & D_V ::= \varepsilon \mid \texttt{var}\ I_1\texttt{,}\ldots\texttt{,}I_n\texttt{;} & \\
 & D_P ::= \varepsilon \mid \texttt{proc}\ I_1\texttt{;}K_1\texttt{;}\ldots\texttt{;}\texttt{proc}\ I_n\texttt{;}K_n\texttt{;} & \\
Blk: & K ::= D\ C & \\
Pgm: & P ::= \texttt{in/out}\ I_1\texttt{,}\ldots\texttt{,}I_n\texttt{;}K\texttt{.} &
\end{array}
$$

Recap: Intermediate Code

The Abstract Machine AM

Definition (Abstract machine for EPL)

The abstract machine for EPL (AM) is defined by the state space

$$S := PC \times DS \times PS$$

with

- the program counter $PC := \mathbb{N}$,
- the data stack $DS := \mathbb{Z}^*$ (top of stack to the right), and
- the procedure stack (or: runtime stack) $PS := \mathbb{Z}^*$ (top of stack to the left).

Thus a state $s = (pc, d, p) \in S$ is given by

- a program counter $pc \in PC$,
- a data stack $d = d.r : \ldots : d.1 \in DS$, and
- a procedure stack $p = p.1 : \ldots : p.t \in PS$.

Recap: Intermediate Code

AM Instructions

Definition (AM instructions)

The set of AM instructions is divided into

arithmetic instructions: $\texttt{ADD}$, $\texttt{MULT}$, ...

Boolean instructions: $\texttt{NOT}$, $\texttt{AND}$, $\texttt{OR}$, $\texttt{LT}$, ...

jumping instructions: $\texttt{JMP}(ca)$, $\texttt{JFALSE}(ca)$ ($ca \in PC$)

procedure instructions: $\texttt{CALL}(ca, dif, loc)$ ($ca \in PC$, $dif, loc \in \mathbb{N}$), $\texttt{RET}$

transfer instructions: $\texttt{LOAD}(dif, off)$, $\texttt{STORE}(dif, off)$ ($dif, off \in \mathbb{N}$), $\texttt{LIT}(z)$ ($z \in \mathbb{Z}$)

Recap: Intermediate Code

Structure of Procedure Stack I

The semantics of procedure and transfer instructions requires a particular structure of the procedure stack $p \in PS$: it must be composed of frames (or: activation records) of the form

$$sl : dl : ra : v_1 : \ldots : v_k$$

where

static link sl: points to frame of surrounding declaration environment

$\Longrightarrow$ used to access non-local variables

dynamic link dl: points to previous frame (i.e., of calling procedure)

$\Longrightarrow$ used to remove topmost frame after termination of procedure call

return address ra: program counter after termination of procedure call

$\Longrightarrow$ used to continue program execution after termination of procedure call

local variables v_i: values of locally declared variables

Recap: Intermediate Code

Structure of Procedure Stack II

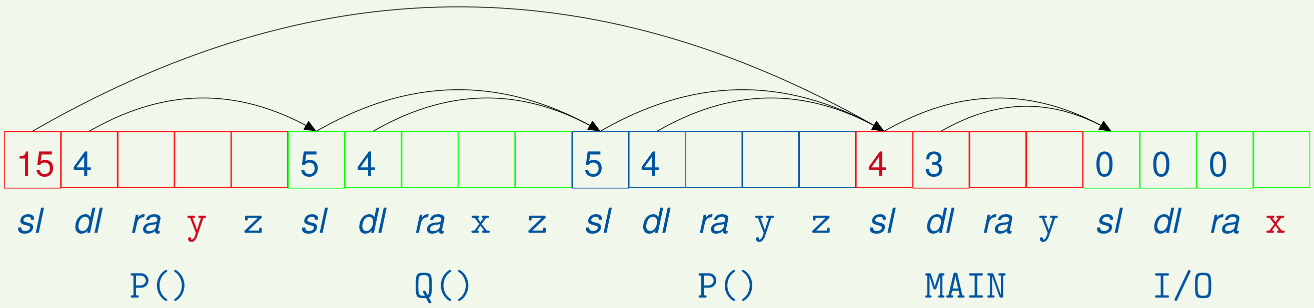
Observation:

- The usage of a variable in a procedure body refers to its innermost declaration.
- If the level difference between the usage and the declaration is *dif*, then a chain of *dif* static links has to be followed to access the corresponding frame.

Example (cf. Example 15.9)

```
in/out x;
  const c = 10;
  var y;
  proc P;
    var y, z;
    proc Q;
      var x, z;
      [... P() ...]
    [... x ... y ... Q() ...]
  proc R;
    [... P() ...]
  [... P() ...].
```

Procedure stack after second call of `P`:



Frame	Field	Value
`P()`	*sl*	15
	dl	4
	ra	
	`y`	
	`z`	
`Q()`	*sl*	5
	dl	4
	ra	
	`x`	
	`z`	
`P()`	*sl*	5
	dl	4
	ra	
	`y`	
	`z`	
`MAIN`	*sl*	4
	dl	3
	ra	
	`y`	
`I/O`	*sl*	0
	dl	0
	ra	0
	`x`	

P uses `x` $\Longrightarrow$ $\mathit{dif} = 2$

P uses `y` $\Longrightarrow$ $\mathit{dif} = 0$

Semantics of Procedure and Transfer Instructions

The base Function

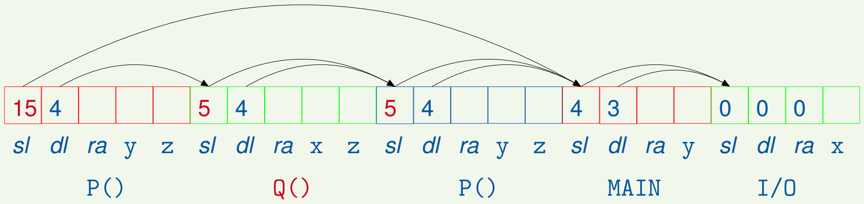
Computes static link information upon procedure call, given procedure stack and level difference

Definition (base function)

The function $\mathrm{base} : PS \times \mathbb{N} \dashrightarrow \mathbb{N}$ is given by

$$\begin{aligned} \mathrm{base}(p, 0) &:= 1 \\ \mathrm{base}(p, dif + 1) &:= \mathrm{base}(p, dif) + p.\mathrm{base}(p, dif) \end{aligned}$$

Example (cf. Example 16.4)



In the second call of P (from Q): $dif = 2$

$$\begin{aligned} & \mathrm{base}(p, 0) = 1 \\ \Longrightarrow\ & \mathrm{base}(p, 1) = 1 + p.1 = 6 \\ \Longrightarrow\ & \mathrm{base}(p, 2) = 6 + p.6 = 11 \\ \Longrightarrow\ & sl = \mathrm{base}(p, 2) + \underbrace{2}_{dl,ra} + \underbrace{2}_{\mathtt{y},\mathtt{z}} = 15 \end{aligned}$$

Semantics of Procedure and Transfer Instructions

Semantics of Procedure Instructions

- `CALL(`ca, dif, loc`)` with
  - code address $ca \in PC$
  - level difference $dif \in \mathbb{N}$
  - number of local variables $loc \in \mathbb{N}$

  creates new frame and jumps to given address (= starting address of procedure)
- `RET` removes the topmost frame and returns to the calling site

Definition (Semantics of procedure instructions)

The semantics of a procedure instruction O, $[\![O]\!] : S \dashrightarrow S$, is defined as follows:

$$[\![\texttt{CALL}(ca, dif, loc)]\!](pc, d, p)$$

$$:= \; (ca, d, \underbrace{(\mathrm{base}(p, dif) + loc + 2)}_{sl} : \underbrace{(loc + 2)}_{dl} : \underbrace{(pc + 1)}_{ra} : \underbrace{0 : \ldots : 0}_{\times loc} : p)$$

$$[\![\texttt{RET}]\!](pc, d, p.1 : \ldots : p.t)$$

$$:= \; (\underbrace{p.3}_{ra}, d, p.(\underbrace{p.2}_{dl} + 2) : \ldots : p.t) \qquad \text{if } t \geq p.2 + 2$$

Semantics of Procedure and Transfer Instructions

Semantics of Transfer Instructions

- $\texttt{LOAD}(dif, off)$ and $\texttt{STORE}(dif, off)$ with
  - level difference $dif \in \mathbb{N}$
  - variable offset $off \in \mathbb{N}$

  respectively load and store variable values between data and procedure stack, following a chain of *dif* static links
- $\texttt{LIT}(z)$ loads the literal constant $z \in \mathbb{Z}$

Definition (Semantics of transfer instructions)

The semantics of a transfer instruction O, $[\![O]\!] : S \dashrightarrow S$, is defined as follows:

$$\begin{aligned}
[\![\texttt{LOAD}(dif, off)]\!](pc, d, p) &:= (pc + 1, d : p.(\mathrm{base}(p, dif) + off + 2), p) \\
[\![\texttt{STORE}(dif, off)]\!](pc, d : z, p) &:= (pc + 1, d, p[\mathrm{base}(p, dif) + off + 2 \mapsto z]) \\
[\![\texttt{LIT}(z)]\!](pc, d, p) &:= (pc + 1, d : z, p)
\end{aligned}$$

Semantics of Procedure and Transfer Instructions

AM Programs and Their Semantics

Definition (Semantics of AM programs)

An AM program is a sequence of $k \geq 1$ labelled AM instructions:

$$P = 1 : O_1; \ldots ; k : O_k$$

The set of all AM programs is denoted by AM.

The semantics of AM programs is determined by

$$[\![.]\!] : AM \times S \dashrightarrow S$$

with

$$[\![P]\!](pc, d, p) := \begin{cases} [\![P]\!]([\![O_{pc}]\!](pc, d, p)) & \text{if } pc \in [k] \\ (pc, d, p) & \text{otherwise} \end{cases}$$

The Symbol Table

Structure of Symbol Table

Goal: define translation mapping $\mathrm{trans} : \mathit{Pgm} \dashrightarrow \mathit{AM}$

The translation employs a symbol table:

$$\begin{aligned} \mathit{Tab} := \{\mathrm{st} \mid \mathrm{st} : \mathit{Ide} \dashrightarrow & (\{\texttt{const}\} \times \mathbb{Z}) \\ & \cup (\{\texttt{var}\} \times \mathit{Lev} \times \mathit{Off}) \\ & \cup (\{\texttt{proc}\} \times \mathit{PC} \times \mathit{Lev} \times \mathit{Size})\} \end{aligned}$$

whose entries are created by declarations:

- constant declarations: $(\texttt{const}, z)$
  - value $z \in \mathbb{Z}$
- variable declarations: $(\texttt{var}, \mathit{lev}, \mathit{off})$
  - declaration level $\mathit{lev} \in \mathit{Lev} := \mathbb{N}$ (0 $\cong$ I/O, 1 $\cong$ MAIN, ...)
  - offset $\mathit{off} \in \mathit{Off} := \mathbb{N}$
  - offset and difference between usage and declaration level determine procedure stack entry
- procedure declarations: $(\texttt{proc}, \mathit{ca}, \mathit{lev}, \mathit{loc})$
  - code address $\mathit{ca} \in \mathit{PC}$
  - declaration level $\mathit{lev} \in \mathit{Lev}$
  - number of local variables $\mathit{loc} \in \mathit{Size} := \mathbb{N}$

The Symbol Table

Maintaining the Symbol Table

Function $\mathrm{update}(D, \mathrm{st}, \mathit{lev})$ specifies update of symbol table st according to declaration D (with respect to current level lev):

Definition (update function)

$\mathrm{update} : \mathit{Dcl} \times \mathit{Tab} \times \mathit{Lev} \dashrightarrow \mathit{Tab}$ is defined by

$$
\begin{aligned}
&\mathrm{update}(D_C\ D_V\ D_P, \mathrm{st}, \mathit{lev})\\
&\quad := \mathrm{update}(D_P, \mathrm{update}(D_V, \mathrm{update}(D_C, \mathrm{st}, \mathit{lev}), \mathit{lev}), \mathit{lev})\\
&\qquad\quad \text{if all identifiers in } D_C\ D_V\ D_P \text{ different}\\
&\mathrm{update}(\varepsilon, \mathrm{st}, \mathit{lev})\\
&\quad := \mathrm{st}\\
&\mathrm{update}(\mathtt{const}\ I_1\ \mathtt{:=}\ z_1\mathtt{,}\ \ldots\mathtt{,}\ I_n\ \mathtt{:=}\ z_n\mathtt{;}, \mathrm{st}, \mathit{lev})\\
&\quad := \mathrm{st}[I_1 \mapsto (\mathtt{const}, z_1), \ldots, I_n \mapsto (\mathtt{const}, z_n)]\\
&\mathrm{update}(\mathtt{var}\ I_1\mathtt{,}\ \ldots\mathtt{,}\ I_n\mathtt{;}, \mathrm{st}, \mathit{lev})\\
&\quad := \mathrm{st}[I_1 \mapsto (\mathtt{var}, \mathit{lev}, 1), \ldots, I_n \mapsto (\mathtt{var}, \mathit{lev}, n)]\\
&\mathrm{update}(\mathtt{proc}\ I_1\mathtt{;}K_1\mathtt{;}\ \ldots\ \mathtt{;proc}\ I_n\mathtt{;}K_n\mathtt{;}, \mathrm{st}, \mathit{lev})\\
&\quad := \mathrm{st}[I_1 \mapsto (\mathtt{proc}, a_1, \mathit{lev}, \mathrm{size}(K_1)), \ldots, I_n \mapsto (\mathtt{proc}, a_n, \mathit{lev}, \mathrm{size}(K_n))]\\
&\qquad\quad \text{with “fresh” addresses } a_1, \ldots, a_n \text{ where } \mathrm{size}(D_C\ \mathtt{var}\ I_1\mathtt{,}\ \ldots\mathtt{,}\ I_n\mathtt{;}\ D_P C) := n
\end{aligned}
$$

The Symbol Table

The Initial Symbol Table

Reminder: an EPL program $P = \mathtt{in/out}\ I_1\,,\ \ldots\ ,I_n\,;K\,.\ \in Pgm$ has a semantics of type $\mathbb{Z}^n \dashrightarrow \mathbb{Z}^n$.

Given input values $(z_1, \ldots, z_n) \in \mathbb{Z}^n$, we choose the initial state

$$s := (1, \varepsilon, \underbrace{0 : 0 : 0 : z_1 : \ldots : z_n}_{\text{I/O frame}}) \in S = PC \times DS \times PS$$

Thus the corresponding initial symbol table has n entries:

$$\mathrm{st}_{I/O}(I_j) := (\mathtt{var}, \underbrace{0}_{lev}, \underbrace{j}_{off}) \qquad \text{for every } j \in [n]$$

Translation of Programs

Translation of Programs

Translation of $\texttt{in/out}\ I_1\texttt{,}\ldots\texttt{,}I_n\texttt{;}\,D\ C\,\texttt{.}$:

1. Create MAIN frame for executing C
2. Stop program execution after return

Definition (Translation of programs)

The mapping

$$\mathrm{trans} : \mathit{Pgm} \dashrightarrow \mathit{AM}$$

is defined by

$$\begin{aligned}\mathrm{trans}(\texttt{in/out}\ I_1\texttt{,}\ldots\texttt{,}I_n\texttt{;}K\,\texttt{.}) := {} & 1 : \texttt{CALL(}a\texttt{,0,}\mathrm{size}(K)\texttt{);}\\ & 2 : \texttt{JMP(0);}\\ & \mathrm{kt}(K, \mathrm{st}_{I/O}, a, 1)\end{aligned}$$

with “fresh” address a

Translation of Blocks

Translation of Blocks

Translation of $D\ C$:

1. Update symbol table according to D
2. Create code for procedures declared in D (using the updated symbol table – recursion!)
3. Create code for C (using the updated symbol table)

Definition (Translation of blocks)

The mapping

$$\mathrm{kt} : \mathit{Blk} \times \mathit{Tab} \times \mathit{PC} \times \mathit{Lev} \dashrightarrow \mathit{AM}$$

("block translation") is defined by

$$\begin{aligned}\mathrm{kt}(D\ C, \mathrm{st}, a, \mathit{lev}) := {} & \mathrm{dt}(D, \mathrm{update}(D, \mathrm{st}, \mathit{lev}), \mathit{lev}) \\ & \mathrm{ct}(C, \mathrm{update}(D, \mathrm{st}, \mathit{lev}), a, \mathit{lev}) \\ & a' : \mathtt{RET};\end{aligned}$$

Translation of Declarations

Translation of Declarations

Translation of D: generate code for the procedures declared in D

Definition (Translation of declarations)

The mapping

$$\mathrm{dt} : \mathit{Dcl} \times \mathit{Tab} \times \mathit{Lev} \dashrightarrow \mathit{AM}$$

("declaration translation") is defined by

$$\begin{aligned}
\mathrm{dt}(D_C\ D_V\ D_P, \mathrm{st}, \mathit{lev}) &:= \mathrm{dt}(D_P, \mathrm{st}, \mathit{lev})\\
\mathrm{dt}(\varepsilon, \mathrm{st}, \mathit{lev}) &:= \varepsilon\\
\mathrm{dt}(\texttt{proc}\ I_1\,;K_1\,;\ \ldots\ ;\texttt{proc}\ I_n\,;K_n\,;, \mathrm{st}, \mathit{lev}) &:= \mathrm{kt}(K_1, \mathrm{st}, a_1, \mathit{lev}+1)\\
&\quad\ \vdots\\
&\quad\ \mathrm{kt}(K_n, \mathrm{st}, a_n, \mathit{lev}+1)\\
&\quad\ \text{where } \mathrm{st}(I_j) = (\texttt{proc}, a_j, \ldots, \ldots)\\
&\quad\ \text{for every } j \in [n]
\end{aligned}$$

Translation of Commands

Definition (Translation of commands)

The mapping

$$\mathrm{ct} : \mathit{Cmd} \times \mathit{Tab} \times \mathit{PC} \times \mathit{Lev} \dashrightarrow \mathit{AM}$$

("command translation") is defined by

$$\begin{aligned}
\mathrm{ct}(I := A, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{at}(A, \mathrm{st}, a, \mathit{lev}); a' : \texttt{STORE}(\mathit{lev} - \mathit{lev}', \mathit{off});\\
&\ \text{if } \mathrm{st}(I) = (\texttt{var}, \mathit{lev}', \mathit{off})\\
\mathrm{ct}(I(), \mathrm{st}, a, \mathit{lev}) &:= a : \texttt{CALL}(\mathit{ca}, \mathit{lev} - \mathit{lev}', \mathit{loc});\\
&\ \text{if } \mathrm{st}(I) = (\texttt{proc}, \mathit{ca}, \mathit{lev}', \mathit{loc})\\
\mathrm{ct}(C_1 ; C_2, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{ct}(C_1, \mathrm{st}, a, \mathit{lev}); \mathrm{ct}(C_2, \mathrm{st}, a', \mathit{lev})\\
\mathrm{ct}(\texttt{if } B \texttt{ then } C_1 \texttt{ else } C_2, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{bt}(B, \mathrm{st}, a, \mathit{lev}); a' : \texttt{JFALSE}(a'');\\
&\ \mathrm{ct}(C_1, \mathrm{st}, a' + 1, \mathit{lev}); a'' - 1 : \texttt{JMP}(a''');\\
&\ \mathrm{ct}(C_2, \mathrm{st}, a'', \mathit{lev}); a''' :\\
\mathrm{ct}(\texttt{while } B \texttt{ do } C, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{bt}(B, \mathrm{st}, a, \mathit{lev}); a' : \texttt{JFALSE}(a'' + 1);\\
&\ \mathrm{ct}(C, \mathrm{st}, a' + 1, \mathit{lev}); a'' : \texttt{JMP}(a);
\end{aligned}$$

Translation of Expressions

Translation of Boolean Expressions

Definition (Translation of Boolean expressions)

The mapping

$$\mathrm{bt} : \mathit{BExp} \times \mathit{Tab} \times \mathit{PC} \times \mathit{Lev} \dashrightarrow \mathit{AM}$$

(“Boolean expression translation”) is defined by

$$\begin{aligned}
\mathrm{bt}(A_1 < A_2, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{at}(A_1, \mathrm{st}, a, \mathit{lev}); \mathrm{at}(A_2, \mathrm{st}, a', \mathit{lev}); a'' : \mathtt{LT};\\
\mathrm{bt}(\mathtt{not}\ B, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{bt}(B, \mathrm{st}, a, \mathit{lev}); a' : \mathtt{NOT};\\
\mathrm{bt}(B_1\ \mathtt{and}\ B_2, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{bt}(B_1, \mathrm{st}, a, \mathit{lev}); \mathrm{bt}(B_2, \mathrm{st}, a', \mathit{lev}); a'' : \mathtt{AND};\\
\mathrm{bt}(B_1\ \mathtt{or}\ B_2, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{bt}(B_1, \mathrm{st}, a, \mathit{lev}); \mathrm{bt}(B_2, \mathrm{st}, a', \mathit{lev}); a'' : \mathtt{OR};
\end{aligned}$$

Translation of Expressions

Translation of Arithmetic Expressions

Definition (Translation of arithmetic expressions)

The mapping

$$\mathrm{at} : \mathit{AExp} \times \mathit{Tab} \times \mathit{PC} \times \mathit{Lev} \dashrightarrow \mathit{AM}$$

("arithmetic expression translation") is defined by

$$\begin{aligned}
\mathrm{at}(z, \mathrm{st}, a, \mathit{lev}) &:= a : \mathtt{LIT}(z)\,; \\
\mathrm{at}(I, \mathrm{st}, a, \mathit{lev}) &:= \begin{cases} a : \mathtt{LIT}(z)\,; & \text{if } \mathrm{st}(I) = (\mathtt{const}, z) \\ a : \mathtt{LOAD}(\mathit{lev} - \mathit{lev}'\,, \mathit{off})\,; & \text{if } \mathrm{st}(I) = (\mathtt{var}, \mathit{lev}', \mathit{off}) \end{cases} \\
\mathrm{at}(A_1 + A_2, \mathrm{st}, a, \mathit{lev}) &:= \mathrm{at}(A_1, \mathrm{st}, a, \mathit{lev}) \\
&\ \mathrm{at}(A_2, \mathrm{st}, a', \mathit{lev}) \\
&\ a'' : \mathtt{ADD}\,;
\end{aligned}$$

A Translation Example

Example: Factorial Function I

Example (Factorial function; cf. Example 15.3)

Source code:

```
in/out x;
  var y;
  proc F;
    if x > 1 then
      y := y * x;
      x := x - 1;
      F()
  y := 1;
  F();
  x := y.
```

$\mathrm{trans}(\texttt{in/out } I_1\texttt{,}\ldots\texttt{,}I_n\texttt{;}K\texttt{.}) :=$

$1 : \texttt{CALL(}a\texttt{,0,}\mathrm{size}(K)\texttt{)}\texttt{;}$ $\mathrm{kt}(D\ C, \mathrm{st}, a, lev) :=$

$2 : \texttt{JMP(0);}$

$\mathrm{kt}(K, \mathrm{st}_{I/O}, a, 1)$

$\mathrm{dt}(D, \mathrm{update}(D, \mathrm{st}, lev), lev)$

$\mathrm{ct}(C, \mathrm{update}(D, \mathrm{st}, lev), a, lev)$

$a' : \texttt{RET;}$

$\mathrm{update}(\texttt{var } I_1\texttt{,}\ldots\texttt{,}I_n\texttt{;}, \mathrm{st}, lev) :=$

$\mathrm{st}[I_1 \mapsto (\mathtt{var}, l, 1), \ldots, I_n \mapsto (\mathtt{var}, l, n)]$

$\mathrm{update}(\texttt{proc } I_1\texttt{;}K_1\texttt{;}\ldots\texttt{;proc } I_n\texttt{;}K_n\texttt{;}, \mathrm{st}, lev) :=$

$\mathrm{st}[I_1 \mapsto (\mathtt{proc}, a_1, l, \mathrm{size}(K_1)), \ldots, I_n \mapsto (\mathtt{proc}, a_n, l, \mathrm{size}(K_n))]$

$\mathrm{kt}(K_1, \mathrm{st}, a_1, lev + 1)$

$\vdots$

$\mathrm{kt}(K_n, \mathrm{st}, a_n, lev + 1)$

where $\mathrm{st}(I_j) = (\mathtt{proc}, a_j, \ldots, \ldots)$ for every $j \in [n]$

$\mathrm{ct}(\texttt{if } B \texttt{ then } C_1 \texttt{ else } C_2, \mathrm{st}, a, lev) :=$

$\mathrm{bt}(B, \mathrm{st}, a, lev)$

$a' : \texttt{JFALSE(}a''\texttt{);}$

$\mathrm{bt}(A_1 \texttt{>} A_2, \mathrm{st}, a, lev) :=$

Intermediate code:

$\mathrm{trans}(\texttt{in/out x;}K\texttt{.})$

$1 : \texttt{CALL(}a_0\texttt{,0,1);}$

$2 : \texttt{JMP(0);}$

$\mathrm{kt}(K, \mathrm{st}_{I/O}, a_0, 1)$

$1 : \texttt{CALL(}a_0\texttt{,0,1);}$

$2 : \texttt{JMP(0);}$

$\mathrm{dt}(D, \mathrm{update}(D, \mathrm{st}_{I/O}, 1), 1)$

$\mathrm{ct}(C, \mathrm{update}(D, \mathrm{st}_{I/O}, 1), a_0, 1)$

$a_2 : \texttt{RET;}$

$1 : \texttt{CALL(}a_0\texttt{,0,1);}$

$2 : \texttt{JMP(0);}$

$\mathrm{dt}(D, \mathrm{st}', 1)$

$\mathrm{ct}(C, \mathrm{st}', a_0, 1)$

$a_2 : \texttt{RET;}$

$1 : \texttt{CALL(}a_0\texttt{,0,1);}$

$2 : \texttt{JMP(0);}$

$\mathrm{dt}(\texttt{proc } I_1\texttt{;}K_1\texttt{;}\ldots\texttt{;pr}$

$\mathrm{kt}(K_{\mathrm{F}}, \mathrm{st}', a_1, 2)$

$\mathrm{ct}(C, \mathrm{st}', a_0, 1)$

$a_2 : \texttt{RET;}$

$1 : \texttt{CALL(}a_0\texttt{,0,1);}$

$2 : \texttt{JMP(0);}$

$\mathrm{ct}(C_{\mathrm{F}}, \mathrm{st}', a_1, 2)$

$a_3 : \texttt{RET;}$

$\mathrm{ct}(C, \mathrm{st}', a$

$a_2 : \texttt{RET;}$

$1 : \texttt{CALL(}a_0\texttt{,0,1);}$

$2 : \texttt{JMP(0);}$

A Translation Example

Example: Factorial Function II

Example (Factorial function; continued)

Code with symbolic addresses:	**Linearised** ($a_0 = 17, a_1 = 3, a_2 = 22, a_3 = 16, a_4 = 6$):
1 : CALL(a_0,0,1);	1 : CALL(17,0,1);
2 : JMP(0);	2 : JMP(0);
a_1 : LOAD(2,1);	3 : LOAD(2,1);
LIT(1);	4 : LIT(1);
GT;	5 : GT;
a_4 : JFALSE(a_3);	6 : JFALSE(16);
LOAD(1,1);	7 : LOAD(1,1);
LOAD(2,1);	8 : LOAD(2,1);
MULT;	9 : MULT;
STORE(1,1);	10 : STORE(1,1);
LOAD(2,1);	11 : LOAD(2,1);
LIT(1);	12 : LIT(1);
SUB;	13 : SUB;
STORE(2,1);	14 : STORE(2,1);
CALL(a_1,1,0);	15 : CALL(3,1,0);
a_3 : RET;	16 : RET;
a_0 : LIT(1);	17 : LIT(1);
STORE(0,1);	18 : STORE(0,1);
CALL(a_1,0,0);	19 : CALL(3,0,0);
LOAD(0,1);	20 : LOAD(0,1);
STORE(1,1);	21 : STORE(1,1);
a_2 : RET;	22 : RET;

A Translation Example

Example: Factorial Function III

Example (Factorial function; continued)

Computation for $\mathtt{x} = 2$:

```
 1 : CALL(17,0,1);
 2 : JMP(0);
 3 : LOAD(2,1);
 4 : LIT(1);
 5 : GT;
 6 : JFALSE(16);
 7 : LOAD(1,1);
 8 : LOAD(2,1);
 9 : MULT;
10 : STORE(1,1);
11 : LOAD(2,1);
12 : LIT(1);
13 : SUB;
14 : STORE(2,1);
15 : CALL(3,1,0);
16 : RET;
17 : LIT(1);
18 : STORE(0,1);
19 : CALL(3,0,0);
20 : LOAD(0,1);
21 : STORE(1,1);
22 : RET;
```

PC	*DS*	*PS*
1	ε	$\underline{0:0:0:2}$
17	ε	$\underline{4:3:2:0}:\underline{0:0:0:2}$
18	1	$\underline{4:3:2:0}:\underline{0:0:0:2}$
19	ε	$\underline{4:3:2:1}:\underline{0:0:0:2}$
3	ε	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
4	2	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
5	2 : 1	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
6	1	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
7	ε	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
8	1	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
9	1 : 2	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
10	2	$\underline{3:2:20}:\underline{4:3:2:1}:\underline{0:0:0:2}$
11	ε	$\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:2}$
12	2	$\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:2}$
13	2 : 1	$\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:2}$
14	1	$\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:2}$
15	ε	$\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:1}$
3	ε	$\underline{6:2:16}:\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:1}$
4	1	$\underline{6:2:16}:\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:1}$
5	1 : 1	$\underline{6:2:16}:\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:1}$
6	0	$\underline{6:2:16}:\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:1}$
16	ε	$\underline{6:2:16}:\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:1}$
16	ε	$\underline{3:2:20}:\underline{4:3:2:2}:\underline{0:0:0:1}$
20	ε	$\underline{4:3:2:2}:\underline{0:0:0:1}$
21	2	$\underline{4:3:2:2}:\underline{0:0:0:1}$
22	ε	$\underline{4:3:2:2}:\underline{0:0:0:2}$
2	ε	$\underline{0:0:0:2}$
0	ε	$\underline{0:0:0:2}$

Correctness of the Translation

Correctness of the Translation

Theorem (Correctness of translation)

For every $P \in Pgm$, $n \in \mathbb{N}$, *and* $(z_1, \ldots, z_n), (z'_1, \ldots, z'_n) \in \mathbb{Z}^n$*:*

$$\begin{aligned} & [\![P]\!](z_1, \ldots, z_n) = (z'_1, \ldots, z'_n) \\ \iff \quad & [\![\mathrm{trans}(P)]\!](1, \varepsilon, 0 : 0 : 0 : z_1 : \ldots : z_n) = (0, \varepsilon, 0 : 0 : 0 : z'_1 : \ldots : z'_n) \end{aligned}$$

Proof.

see M. Mohnen: *A Compiler Correctness Proof for the Static Link Technique by means of Evolving Algebras*, Fundamenta Informaticae 29(3), 1997, pp. 257–303 $\square$