Compiler Construction

Lecture 14: Semantic Analysis III (Attribute Evaluation)

Summer Semester 2016

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https://moves.rwth-aachen.de/teaching/ss-16/cc/

Outline of Lecture 14

Circularity of Attribute Grammars

Goal: unique solvability of equation system
$\Longrightarrow$ avoid cyclic dependencies

Definition (Circularity)

An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is called circular if there exists a syntax tree t such that the attribute equation system E_t is recursive (i.e., some attribute variable of t depends on itself). Otherwise it is called noncircular.

Remark: because of the division of Var_π into In_π and Out_π, cyclic dependencies cannot occur at production level.

Attribute Dependency Graphs and Circularity I

Observation: a cycle in the dependency graph D_t of a given syntax tree t is caused by the occurrence of a "cover" production $\pi = A_0 \to w_0 A_1 w_1 \ldots A_r w_r \in P$ in a node k_0 of t such that

- the dependencies in E_{k_0} yield the "upper end" of the cycle and
- for at least one $i \in [r]$, some attributes in $\mathrm{syn}(A_i)$ depend on attributes in $\mathrm{inh}(A_i)$.

Example

on the board

To identify such "critical" situations we need to determine for each $i \in [r]$ the possible ways in which attributes in $\mathrm{syn}(A_i)$ can depend on attributes in $\mathrm{inh}(A_i)$.

Attribute Dependency Graphs and Circularity II

Definition (Attribute dependence)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- If t is a syntax tree with root label $A \in N$ and root node k, $\alpha \in \mathrm{syn}(A)$, and $\beta \in \mathrm{inh}(A)$ such that $\beta.k \to_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \overset{A}{\hookrightarrow} \alpha$).
- For every syntax tree t with root label $A \in N$,

$$is(A, t) := \{(\beta, \alpha) \in \mathrm{inh}(A) \times \mathrm{syn}(A) \mid \beta \overset{A}{\hookrightarrow} \alpha \text{ in } t\}.$$

- For every $A \in N$, $\quad IS(A) := \{is(A, t) \mid t \text{ syntax tree with root label A}\} \subseteq 2^{Inh \times Syn}$.

Remark: it is important that $IS(A)$ is a system of attribute dependence sets, not a union (otherwise: strong noncircularity – see exercises).

Example

on the board

Outline of Lecture 14

The Circularity Check I

In the circularity check, the dependency systems $IS(A)$ are iteratively computed. The following notation is employed:

Definition 14.1

Given $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ and $is_i \subseteq \mathrm{inh}(A_i) \times \mathrm{syn}(A_i)$ for each $i \in [r]$,

$$is[\pi; is_1, \ldots, is_r] \subseteq \mathrm{inh}(A) \times \mathrm{syn}(A)$$

is defined by

$$is[\pi; is_1, \ldots, is_r] := \left\{ (\beta, \alpha) \mid (\beta.0, \alpha.0) \in \left(\to_\pi \cup \bigcup_{i=1}^{r} \{(\beta'.p_i, \alpha'.p_i) \mid (\beta', \alpha') \in is_i\}\right)^+ \right\}$$

where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$.

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where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$.

Example 14.2

on the board

The Circularity Check II

Algorithm 14.3 (Circularity check for attribute grammars)

Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ *with* $G = \langle N, \Sigma, P, S \rangle$

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Procedure: 1. *for every* $A \in N$, *iteratively construct* $IS(A)$ *as follows:*

i. *if* $\pi = A \to w \in P$, *then* $is[\pi] \in IS(A)$

ii. *if* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ *and* $is_i \in IS(A_i)$ *for every* $i \in [r]$, *then* $is[\pi; is_1, \ldots, is_r] \in IS(A)$

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2. *test whether* $\mathfrak{A}$ *is circular by checking if there exist* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ *and* $is_i \in IS(A_i)$ *for every* $i \in [r]$ *such that the following relation is cyclic:*

$$\to_\pi \cup \bigcup_{i=1}^{r}\{(\beta.p_i, \alpha.p_i) \mid (\beta, \alpha) \in is_i\}$$

(where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$*)*

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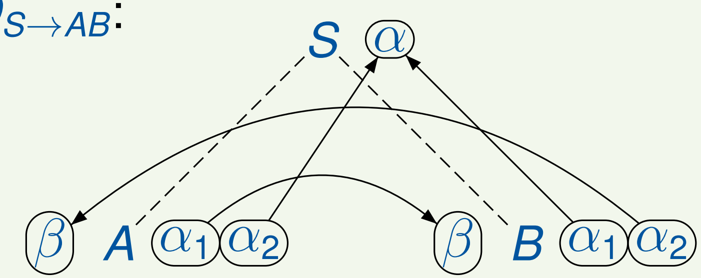
(where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$*)*

Output: *"yes" or "no"*

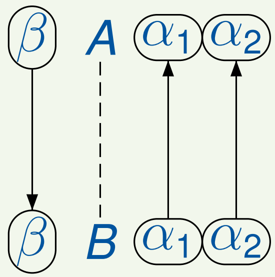
The Circularity Check III

Example 14.4

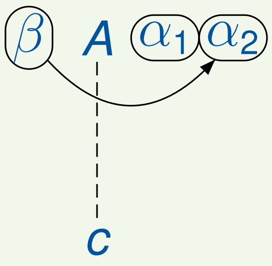
$D_{S \to AB}$:



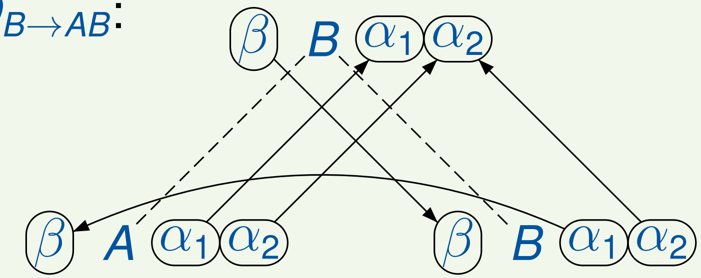
$D_{A \to B}$:



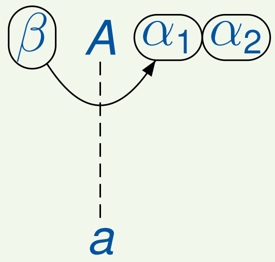
$D_{A \to c}$:



$D_{B \to AB}$:



$D_{A \to a}$:



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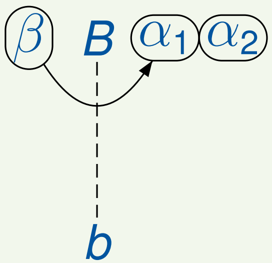

Application of Algorithm 14.3: on the board

Outline of Lecture 14

Correctness and Complexity of Circularity Check

Theorem 14.5 (Correctness of circularity check)

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Lemma 14.6

The time complexity of the circularity check is exponential in the size of the attribute grammar (= maximal length of right-hand sides of productions).

Proof.

by reduction of the word problem of alternating Turing machines (see M. Jazayeri: *A Simpler Construction for Showing the Intrinsically Exponential Complexity of the Circularity Problem for Attribute Grammars*, Comm. ACM 28(4), 1981, pp. 715–720) □

Outline of Lecture 14

Attribute Evaluation Methods

Given:
- noncircular attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$
- syntax tree t of G
- valuation $v : Syn_\Sigma \to V$ for $Syn_\Sigma := \{\alpha.k \mid k \text{ labelled by } a \in \Sigma, \alpha \in \mathrm{syn}(a)\} \subseteq Var_t$

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2. Strongly noncircular AGs: recursive functions (details omitted)
   i. for every $A \in N$ and $\alpha \in \mathrm{syn}(A)$, define evaluation function $g_{A,\alpha}$ with the following parameters:
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4. S-attributed grammars (i.e., $Inh = \emptyset$): `yacc`

Outline of Lecture 14

Attribute Evaluation by Topological Sorting

Algorithm 14.7 (Evaluation by topological sorting)

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2. *while* $Var \neq \emptyset$ *do*
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   ii. *let* $x = f(x_1, \ldots, x_n) \in E_t$
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Example 14.8

see Examples 12.1 and 12.2 (Knuth's binary numbers)

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L-Attributed Grammars I

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Remark: note that no restrictions are imposed for $\beta \in Syn$ (for $i = 0$) or $\alpha \in Inh$ (for $j = 0$). Thus, in an L-attributed grammar,

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Corollary 14.2

Every $\mathfrak{A} \in LAG$ is noncircular.

L-Attributed Grammars II

Example 14.3

L-attributed grammar:

$$
\begin{array}{lll}
S \rightarrow AB & i.1 & = 0 \\
 & i.2 & = s.1 + 1 \\
 & s.0 & = s.2 + 1 \\
A \rightarrow \mathrm{a}A & i.2 & = i.0 + 1 \\
 & s.0 & = s.2 + 1 \\
A \rightarrow \mathrm{c} & s.0 & = i.0 + 1 \\
B \rightarrow \mathrm{b} & s.0 & = i.0 + 1
\end{array}
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L-Attributed Grammars II

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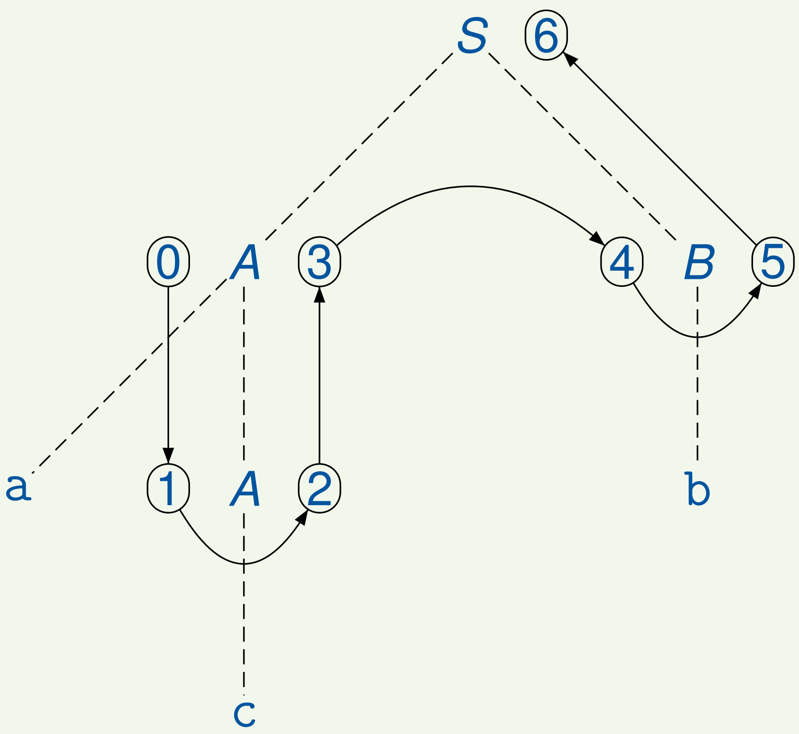

Evaluation of L-Attributed Grammars

Observation 1: the syntax tree of an L-attributed grammar can be attributed by a depth-first, left-to-right tree traversal with two visits to each node

1. top-down: evaluation of inherited attributes
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1. top-down: expansion steps
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2. bottom-up: reduction steps

Idea: extend LL parsing to support reduction steps, and integrate attribute evaluation
$\Longrightarrow$

- use recursive-descent parser and
- add variables and operations for attribute evaluation

L-Attributed Grammars

Recursive-Descent Parsing I

Ingredients:
- variable `token` for current token
- function `next()` for invoking the scanner
- procedure `print(i)` for displaying the leftmost analysis (or errors)

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Recursive-Descent Parsing I

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- variable `token` for current token
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Method: to every $A \in N$ we assign a procedure

`A()`

which

- tests `token` with regard to the lookahead sets of the A-productions,
- prints the corresponding rule number and
- evaluates the corresponding right-hand side as follows:
  - for $a \in \Sigma$: check `token`; call `next()`
  - for $A \in N$: call `A`

Recursive-Descent Parsing and Attribute Evaluation I

Ingredients:
- variable `token` for current token
- function `next()` for invoking the scanner
- procedure `print(i)` for displaying the leftmost analysis (or errors)

Method: to every $A \in N$ we assign a procedure

$$\texttt{A(in:}\ \mathrm{inh}(A), \texttt{out:}\ \mathrm{syn}(A))$$

which

- declares local variables for synthesized attributes on right-hand sides,
- tests `token` with regard to the lookahead sets of the A-productions,
- prints the corresponding rule number and
- evaluates the corresponding right-hand side as follows:
  - for $a \in \Sigma$: check `token`; call `next()`
  - for $A \in N$: call `A` with appropriate parameters

Recursive-Descent Parsing II

Example 14.4 (cf. Example 14.3)

```
proc main();
  token := next(); S()
proc S();
  if token in {'a','c'} then   (* S → AB *)
    print(1); A(); B()
  else print(error); stop fi
proc A();
  if token = 'a' then   (* A → aA *)
    print(2); token := next(); A()
  elsif token = 'c' then   (* A → c *)
    print(3); token := next()
  else print(error); stop fi
proc B();
  if token = 'b' then   (* B → b *)
    print(4); token := next()
  else print(error); stop fi
```

Recursive-Descent Parsing and Attribute Evaluation II

Example 14.5 (cf. Example 14.3)

```
proc main(); var s;
  token := next(); S(s); print(s)
proc S(out s0); var s1,s2;
  if token in {'a','c'} then   (* S → AB : i.1 = 0, i.2 = s.1 + 1, s.0 = s.2 + 1 *)
    print(1); A(0,s1); B(s1+1,s2); s0 := s2+1
  else print(error); stop fi
proc A(in i0,out s0); var s2;
  if token = 'a' then   (* A → aA : i.2 = i.0 + 1, s.0 = s.2 + 1 *)
    print(2); token := next(); A(i0+1,s2); s0 := s2+1
  elsif token = 'c' then   (* A → c : s.0 = i.0 + 1 *)
    print(3); token := next(); s0 := i0+1
  else print(error); stop fi
proc B(in i0,out s0);
  if token = 'b' then   (* B → b : s.0 = i.0 + 1 *)
    print(4); token := next(); s0 := i0+1
  else print(error); stop fi
```