



Compiler Construction

Lecture 13: Semantic Analysis II (Circularity Check)

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Thomas Noll

Software Modeling and Verification Group

RWTH Aachen University

<https://moves.rwth-aachen.de/teaching/ss-16/cc/>

Recap: Attribute Grammars

Outline of Lecture 13

Recap: Attribute Grammars

Attribute Dependency Graphs

Testing Attribute Grammars for Circularity

Recap: Attribute Grammars

Attribute Grammars

Goal: compute context-dependent but runtime-independent properties of a given program

Idea: enrich context-free grammar by **semantic rules** which annotate syntax tree with **attribute values**

⇒ **Semantic analysis = attribute evaluation**

Result: **attributed syntax tree**

In greater detail:

- With every nonterminal a set of attributes is associated.
- Two types of attributes are distinguished:
 - Synthesized:** bottom-up computation (from the leaves to the root)
 - Inherited:** top-down computation (from the root to the leaves)
- With every production a set of semantic rules is associated.

Recap: Attribute Grammars

Formal Definition of Attribute Grammars I

Definition (Attribute grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesized or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^{\alpha}$ be a union of value sets.
- Let $att : X \rightarrow 2^{Att}$ be an attribute assignment, and let $\text{syn}(Y) := att(Y) \cap Syn$ and $\text{inh}(Y) := att(Y) \cap Inh$ for every $Y \in X$.

- Every production $\pi = Y_0 \rightarrow Y_1 \dots Y_r \in P$ determines the set

$$Var_{\pi} := \{\alpha.i \mid \alpha \in att(Y_i), i \in \{0, \dots, r\}\}$$

of attribute variables of π with the subsets of inner and outer variables:

$$In_{\pi} := \{\alpha.i \mid (i = 0, \alpha \in \text{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \text{inh}(Y_i))\}, \quad Out_{\pi} := Var_{\pi} \setminus In_{\pi}$$

- A semantic rule of π is an equation of the form

$$\alpha.i = f(\alpha_1.i_1, \dots, \alpha_n.i_n)$$

where $n \in \mathbb{N}$, $\alpha.i \in In_{\pi}$, $\alpha_j.i_j \in Out_{\pi}$, and $f : V^{\alpha_1} \times \dots \times V^{\alpha_n} \rightarrow V^{\alpha}$.

- For each $\pi \in P$, let E_{π} be a set with exactly one semantic rule for every inner variable of π , and let $E := (E_{\pi} \mid \pi \in P)$.

Then $\mathfrak{A} := \langle G, E, V \rangle$ is called an attribute grammar: $\mathfrak{A} \in AG$.

Recap: Attribute Grammars

Attribution of Syntax Trees I

Definition (Attribution of syntax trees)

Let $\mathcal{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G with the set of nodes K .

- K determines the set of **attribute variables of t** :

$$Var_t := \{\alpha.k \mid k \in K \text{ labelled with } Y \in X, \alpha \in \text{att}(Y)\}.$$

- Let $k_0 \in K$ be an (inner) node where production $\pi = Y_0 \rightarrow Y_1 \dots Y_r \in P$ is applied, and let $k_1, \dots, k_r \in K$ be the corresponding successor nodes. The **attribute equation system E_{k_0}** of k_0 is obtained from E_π by substituting every attribute index $i \in \{0, \dots, r\}$ by k_i .
- The **attribute equation system** of t is given by

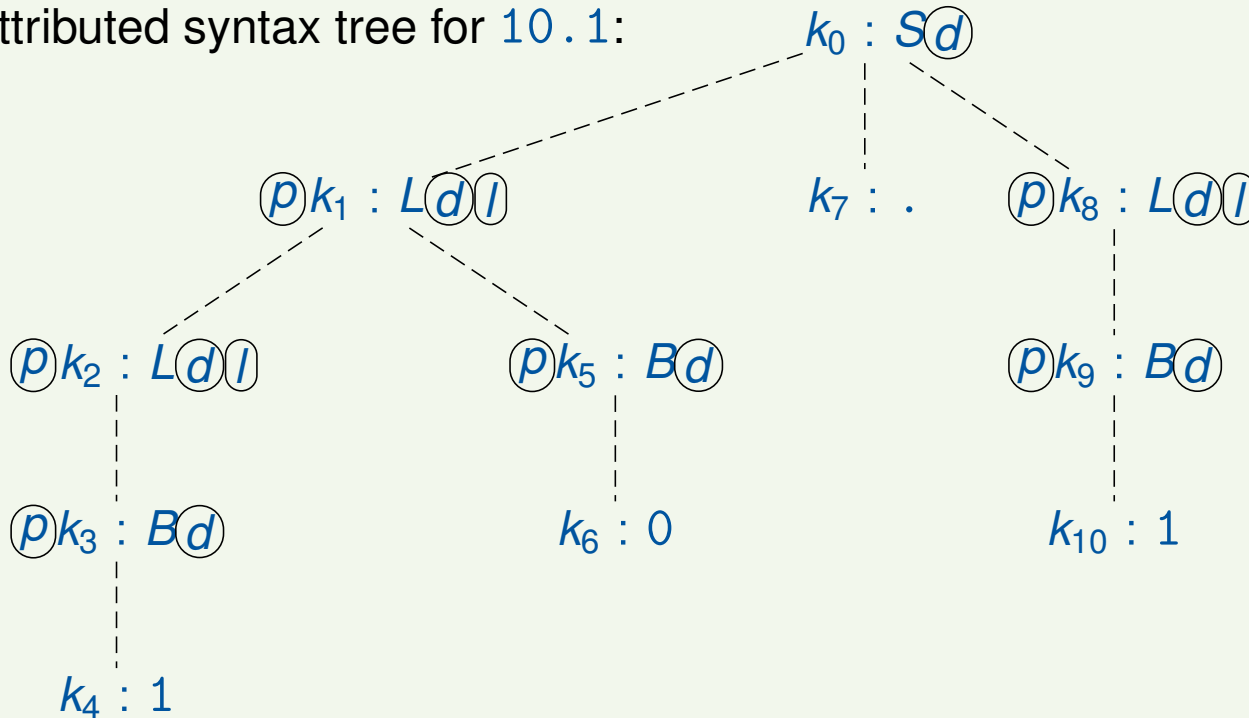
$$E_t := \bigcup \{E_k \mid k \text{ inner node of } t\}.$$

Recap: Attribute Grammars

Attribution of Syntax Trees II

Example (cf. Example 12.2)

Attributed syntax tree for 10.1:

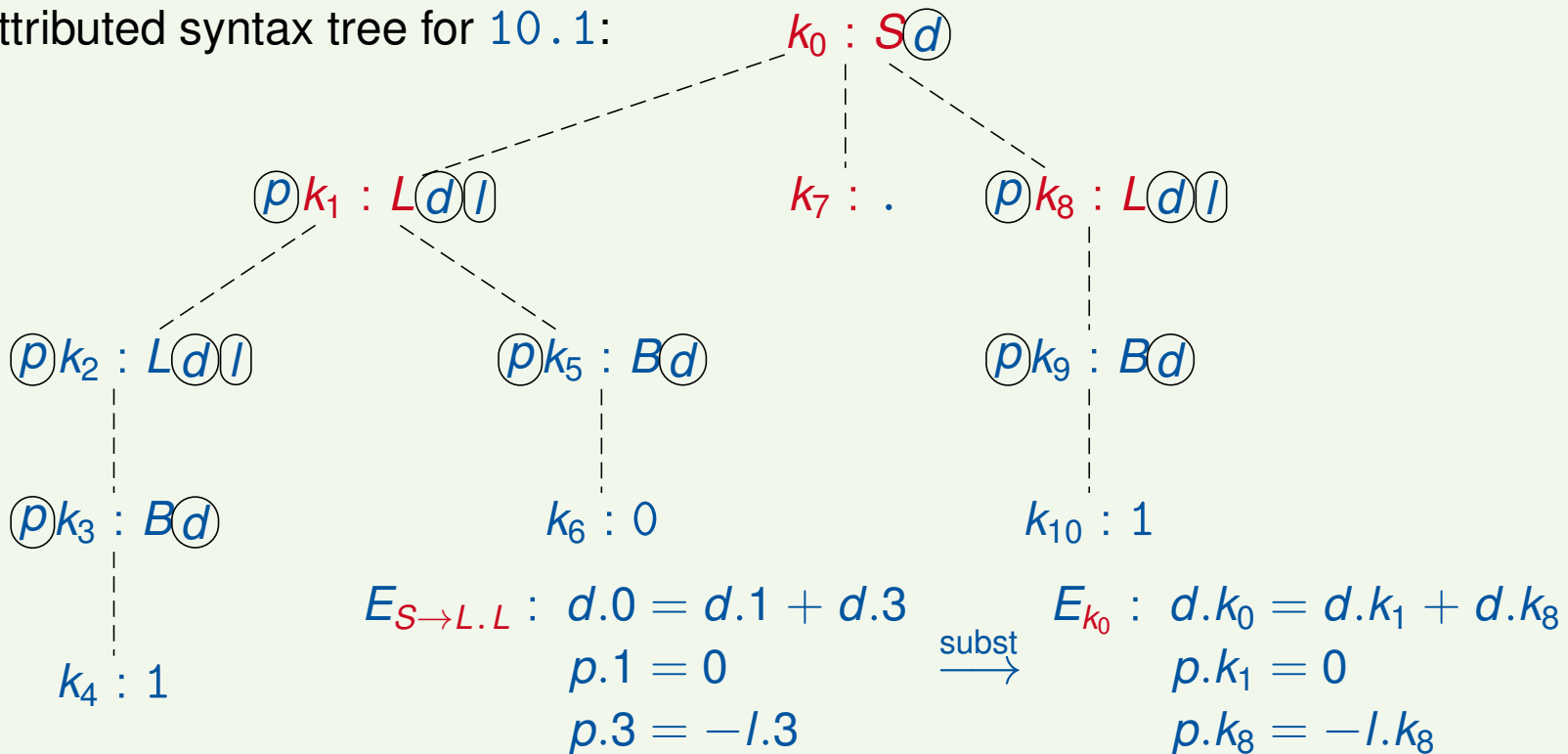


Recap: Attribute Grammars

Attribution of Syntax Trees II

Example (cf. Example 12.2)

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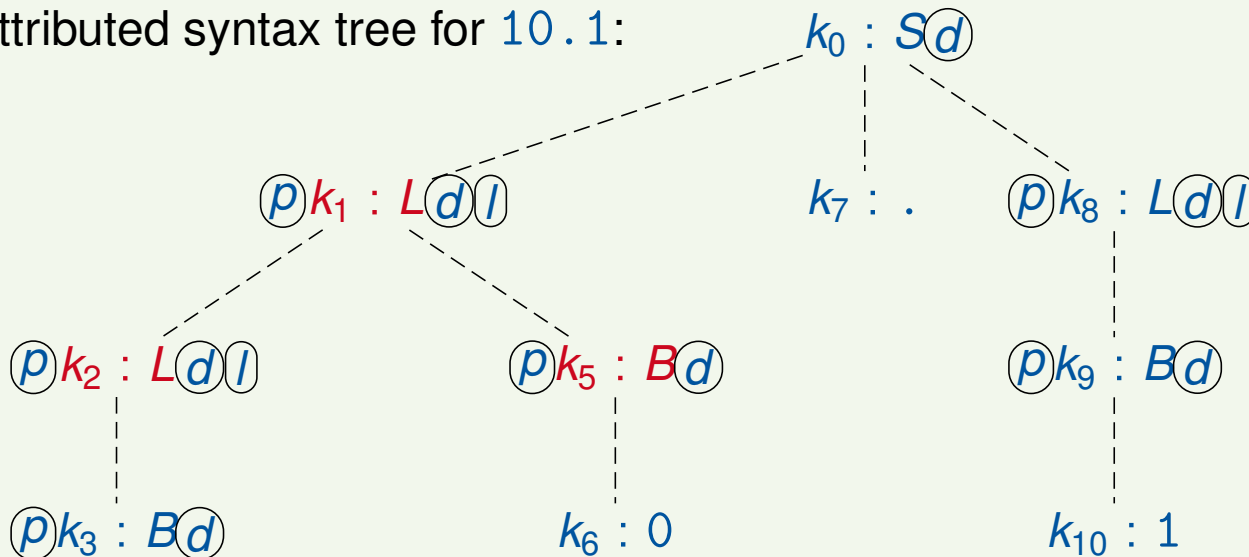


Recap: Attribute Grammars

Attribution of Syntax Trees II

Example (cf. Example 12.2)

Attributed syntax tree for 10.1:



$$E_{L \rightarrow LB} : \begin{aligned} d.0 &= d.1 + d.2 \\ l.0 &= l.1 + 1 \\ p.1 &= p.0 + 1 \\ p.2 &= p.0 \end{aligned}$$

subst
→

$$E_{k_1} : \begin{aligned} d.k_1 &= d.k_2 + d.k_5 \\ l.k_1 &= l.k_2 + 1 \\ p.k_2 &= p.k_1 + 1 \\ p.k_5 &= p.k_1 \end{aligned}$$

Attribution of Syntax Trees III

Corollary

For each $\alpha.k \in \text{Var}_t$ except for the inherited attribute variables at the root and the synthesized attribute variables at the leaves of t , E_t contains **exactly one equation** with left-hand side $\alpha.k$.

Assumptions:

- The **start symbol** does not have inherited attributes: $\text{inh}(S) = \emptyset$.
- **Synthesized attributes of terminal symbols** are provided by the scanner.

Recap: Attribute Grammars

Solvability of Attribute Equation System I

Definition (Solution of attribute equation system)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G . A **solution** of E_t is a mapping

$$v : Var_t \rightarrow V$$

such that, for every $\alpha.k \in Var_t$ and $\alpha.k = f(\alpha.k_1, \dots, \alpha.k_n) \in E_t$,

$$v(\alpha.k) = f(v(\alpha.k_1), \dots, v(\alpha.k_n)).$$

In general, the attribute equation system E_t of a given syntax tree t can have

- no solution,
- exactly one solution, or
- several solutions.

Solvability of Attribute Equation System II

Example

- $A \rightarrow aB, B \rightarrow b \in P$
- $\alpha \in \text{syn}(B), \beta \in \text{inh}(B)$
- $\beta.2 = f(\alpha.2) \in E_{A \rightarrow aB}$
- $\alpha.0 = \beta.0 \in E_{B \rightarrow b}$

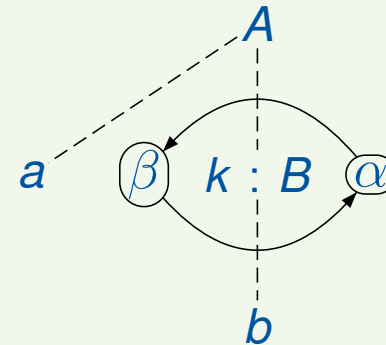
Recap: Attribute Grammars

Solvability of Attribute Equation System II

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\Rightarrow cyclic dependency:



$$E_t : \begin{aligned} \beta.k &= f(\alpha.k) \\ \alpha.k &= \beta.k \end{aligned}$$

Recap: Attribute Grammars

Solvability of Attribute Equation System II

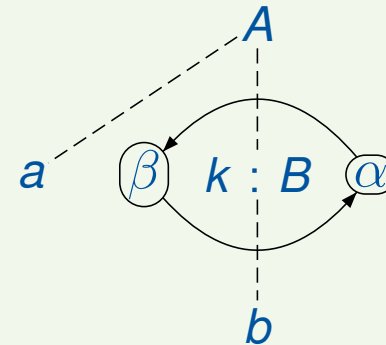
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\implies for $V^\alpha := V^\beta := \mathbb{N}$ and

- $f(x) := x + 1$: **no solution**
- $f(x) := 2x$: **exactly one solution**
($v(\alpha.k) = v(\beta.k) = 0$)
- $f(x) := x$: **infinitely many solutions**
($v(\alpha.k) = v(\beta.k) = y$ for any $y \in \mathbb{N}$)

\implies **cyclic dependency:**



$$E_t : \begin{aligned} \beta.k &= f(\alpha.k) \\ \alpha.k &= \beta.k \end{aligned}$$

Recap: Attribute Grammars

Circularity of Attribute Grammars

Goal: **unique solvability** of equation system

⇒ avoid cyclic dependencies

Definition (Circularity)

An attribute grammar $\mathcal{A} = \langle G, E, V \rangle \in AG$ is called **circular** if there exists a syntax tree t such that the attribute equation system E_t is recursive (i.e., some attribute variable of t depends on itself). Otherwise it is called **noncircular**.

Remark: because of the division of Var_π into In_π and Out_π , cyclic dependencies cannot occur at production level.

Attribute Dependency Graphs

Outline of Lecture 13

Recap: Attribute Grammars

Attribute Dependency Graphs

Testing Attribute Grammars for Circularity

Attribute Dependency Graphs

Attribute Dependency Graphs I

Goal: graphic representation of attribute dependencies

Definition 13.1 (Production dependency graph)

Let $\mathcal{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$. Every production $\pi \in P$ determines the **dependency graph** $D_\pi := \langle Var_\pi, \rightarrow_\pi \rangle$ where the set of edges $\rightarrow_\pi \subseteq Var_\pi \times Var_\pi$ is given by

$$x \rightarrow_\pi y \quad \text{iff} \quad y = f(\dots, x, \dots) \in E_\pi.$$

Attribute Dependency Graphs

Attribute Dependency Graphs I

Goal: graphic representation of attribute dependencies

Definition 13.1 (Production dependency graph)

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$$x \rightarrow_\pi y \quad \text{iff} \quad y = f(\dots, x, \dots) \in E_\pi.$$

Corollary 13.2

*The dependency graph of a production is acyclic
(since $\rightarrow_\pi \subseteq Out_\pi \times In_\pi$ and $Out_\pi \cap In_\pi = \emptyset$).*

Attribute Dependency Graphs

Attribute Dependency Graphs II

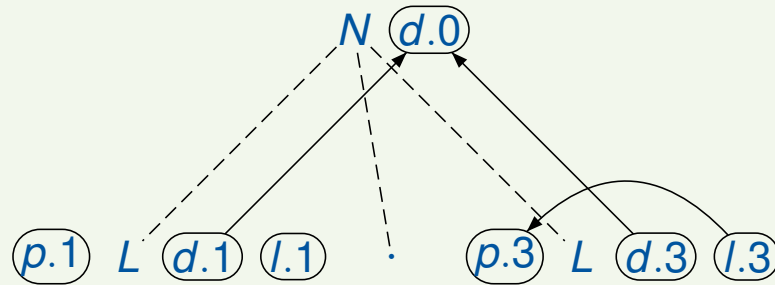
Example 13.3 (cf. Example 12.2)

1. $N \rightarrow L.L :$ $\implies D_{N \rightarrow L.L} :$

$$d.0 = d.1 + d.3$$

$$p.1 = 0$$

$$p.3 = -l.3$$



Attribute Dependency Graphs

Attribute Dependency Graphs II

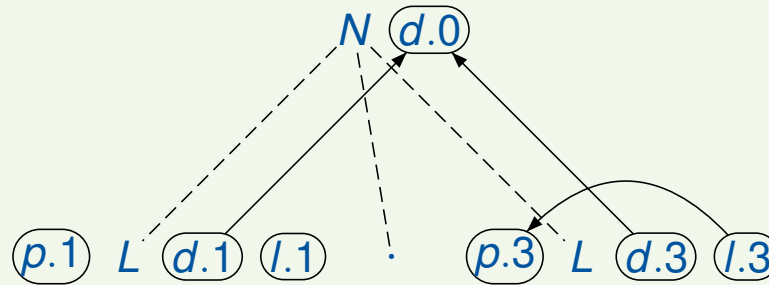
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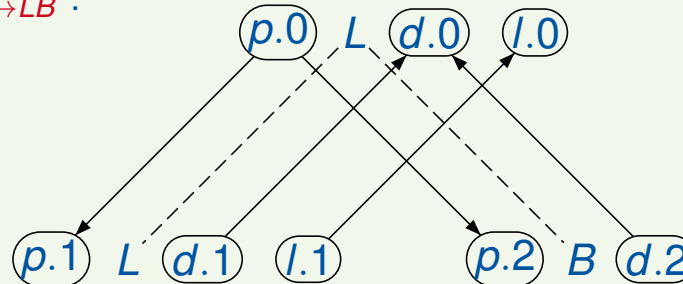
2. $L \rightarrow LB :$ $\implies D_{L \rightarrow LB} :$

$$d.0 = d.1 + d.2$$

$$l.0 = l.1 + 1$$

$$p.1 = p.0 + 1$$

$$p.2 = p.0$$



Attribute Dependency Graphs III

Just as the attribute equation system E_t of a syntax tree t is obtained from the semantic rules of the contributing productions, the dependency graph of t is obtained by “glueing together” the dependency graphs of the productions.

Attribute Dependency Graphs

Attribute Dependency Graphs III

Just as the attribute equation system E_t of a syntax tree t is obtained from the semantic rules of the contributing productions, the dependency graph of t is obtained by “glueing together” the dependency graphs of the productions.

Definition 13.4 (Tree dependency graph)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G .

- The **dependency graph** of t is defined by $D_t := \langle Var_t, \rightarrow_t \rangle$ where the set of edges, $\rightarrow_t \subseteq Var_t \times Var_t$, is given by

$$x \rightarrow_t y \quad \text{iff} \quad y = f(\dots, x, \dots) \in E_t.$$

- D_t is called **cyclic** if there exists $x \in Var_t$ such that $x \rightarrow_t^+ x$.

Attribute Dependency Graphs

Attribute Dependency Graphs III

Just as the attribute equation system E_t of a syntax tree t is obtained from the semantic rules of the contributing productions, the dependency graph of t is obtained by “glueing together” the dependency graphs of the productions.

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Corollary 13.5

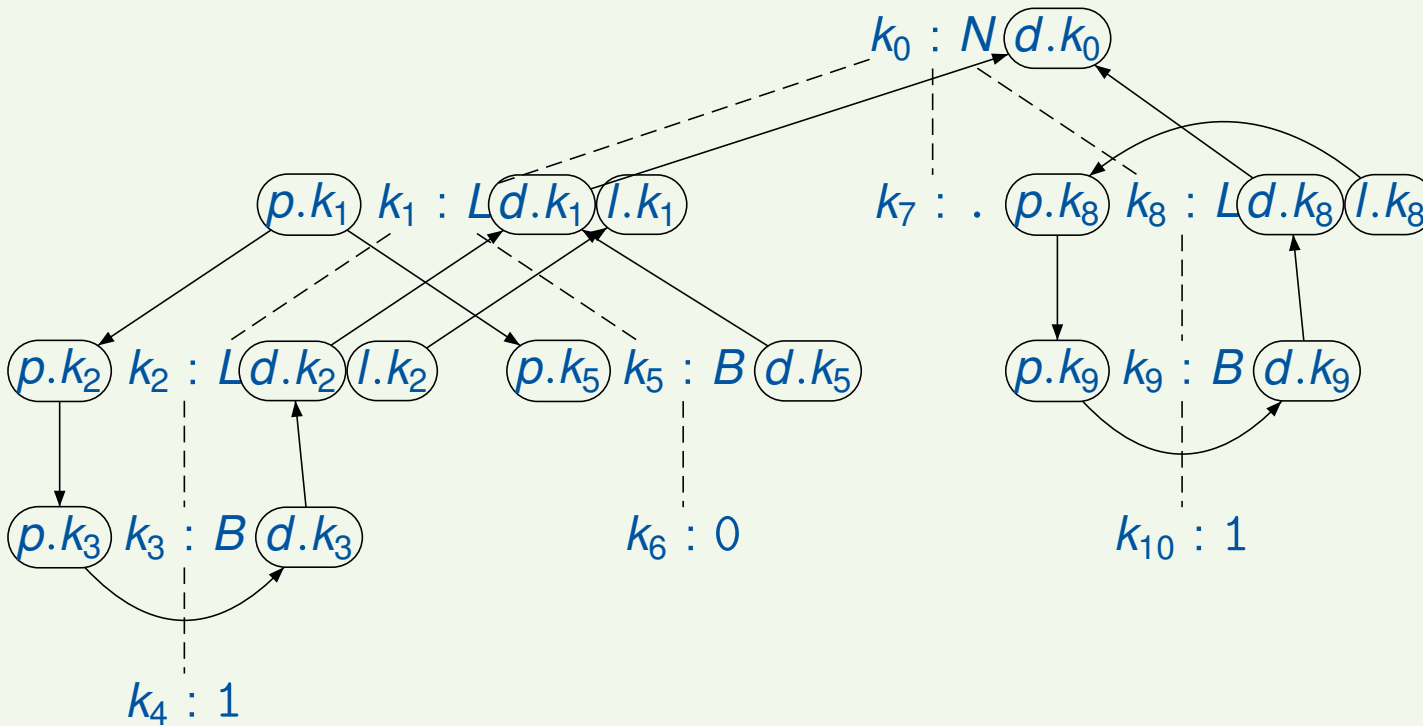
An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is **circular** iff there exists a syntax tree t of G such that D_t is **cyclic**.

Attribute Dependency Graphs

Attribute Dependency Graphs IV

Example 13.6 (cf. Example 12.2)

(Acyclic) dependency graph of the syntax tree for 10.1:



Testing Attribute Grammars for Circularity

Outline of Lecture 13

Recap: Attribute Grammars

Attribute Dependency Graphs

Testing Attribute Grammars for Circularity

Attribute Dependency Graphs and Circularity I

Observation: a cycle in the dependency graph D_t of a given syntax tree t is caused by the occurrence of a “cover” production $\pi = A_0 \rightarrow w_0 A_1 w_1 \dots A_r w_r \in P$ in a node k_0 of t such that

- the dependencies in E_{k_0} yield the “upper end” of the cycle and
- for at least one $i \in [r]$, some attributes in $\text{syn}(A_i)$ depend on attributes in $\text{inh}(A_i)$.

Testing Attribute Grammars for Circularity

Attribute Dependency Graphs and Circularity I

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Example 13.7

on the board

Testing Attribute Grammars for Circularity

Attribute Dependency Graphs and Circularity I

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Example 13.7

on the board

To identify such “critical” situations we need to determine for each $i \in [r]$ the possible ways in which attributes in $\text{syn}(A_i)$ can depend on attributes in $\text{inh}(A_i)$.

Attribute Dependency Graphs and Circularity II

Definition 13.8 (Attribute dependence)

Let $\mathcal{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- If t is a syntax tree with root label $A \in N$ and root node k , $\alpha \in \text{syn}(A)$, and $\beta \in \text{inh}(A)$ such that $\beta.k \rightarrow_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \xrightarrow{A} \alpha$).

Attribute Dependency Graphs and Circularity II

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- For every syntax tree t with root label $A \in N$,

$$is(A, t) := \{(\beta, \alpha) \in \text{inh}(A) \times \text{syn}(A) \mid \beta \xrightarrow{A} \alpha \text{ in } t\}.$$

Testing Attribute Grammars for Circularity

Attribute Dependency Graphs and Circularity II

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Testing Attribute Grammars for Circularity

Attribute Dependency Graphs and Circularity II

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Remark: it is important that $IS(A)$ is a **system** of attribute dependence sets, not a **union** (otherwise: **strong noncircularity** – see exercises).

Testing Attribute Grammars for Circularity

Attribute Dependency Graphs and Circularity II

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Example 13.9

on the board