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Compiler Construction 2016

— Series 3 —

Hand in until May 24th before the exercise class.

General Remarks

- Follow the naming convention for the zip file: `ex2_MATRNO1_MATRNO2_MATRNO3` and include the complete framework provided to you via our webpage.
- It is allowed to hand your solutions for the theoretical part via email **as a separately attached PDF file**.
- Please hand in your solutions in groups of 3 or 4.

Exercise 1 (2 Points)

Consider the context-free grammar G given by the following rules:

$$S \rightarrow S + S \mid S\,S \mid (\,S\,) \mid S^* \mid a$$

(a) Provide a leftmost analysis of the string $(a + a)^*a$.

(b) Provide a rightmost analysis of the string $(a + a)^*a$.

(c) Prove or disprove: G is unambiguous.

Exercise 2 (2 Points)

Complete the correctness proof of Theorem 6.1 by showing the direction omitted in the lecture. More precisely, let $G = \langle N, \Sigma, P, S\rangle \in \mathrm{CFG}_\Sigma$ and $NTA(G)$ as in the lecture (lecture 6, slide 5). Show that for each $w \in \Sigma^*$ and $z \in [p]^*$ it holds that

$$(w, S, \varepsilon) \;\vdash^*\; (\varepsilon, \varepsilon, z) \quad \text{implies} \quad z \text{ is a leftmost analysis of } w.$$

Exercise 3 (1 Points)

Show that for each context free grammar G it holds that $G \in LL(1)$ implies that G is unambiguous.

Exercise 4 (2 Points)

Two characterizations of $LL(1)$ have been given in the lecture.

First, by Lemma 6.5, a context free grammar $G = \langle N, \Sigma, P, S\rangle$ is in $LL(1)$ if and only if for all leftmost derivations of the form

$$S \Rightarrow_l^* wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \\ \Rightarrow_l w\gamma\alpha \end{cases}$$

such that $\beta \neq \gamma$, it follows that $first_1(\beta\alpha) \cap first_1(\gamma\alpha) = \emptyset$.

Second, by Theorem 6.10, G is in $LL(1)$ if and only if for all pairs of rules $A \to \beta|\gamma \in P$, where $\beta \neq \gamma$, we have

$$la(A \to \beta) \cap la(A \to \gamma) \ = \ \emptyset.$$

(a) Lift the statement of Theorem 6.10 to $LL(k)$ for arbitrary $k \in \mathbb{N}_{>0}$.

(b) Show that the two characterizations are not equivalent for $k > 1$ by showing that the following grammar is in $LL(2)$ according to the first characterization, but not according to your modified version of the second one.

$$\begin{aligned} S &\to aAab \mid bAbb \\ A &\to a \mid \varepsilon \end{aligned}$$

Exercise 5 (3 Points)

Consider the following grammar G:

$$\begin{aligned} S &\to (L) \mid a \\ L &\to L, S \mid L, SS \mid S \mid SS \end{aligned}$$

(a) Show that $G \notin LL(1)$.

(b) Transform G into an equivalent grammar $LL(1)$, i.e. provide a grammar $G' \in LL(1)$ such that $L(G') = L(G)$.

(c) Prove that G' has the $LL(1)$ property