Theoretical Foundations of the UML

Lecture 9: Bounded MSC and CFMs

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moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

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Outline

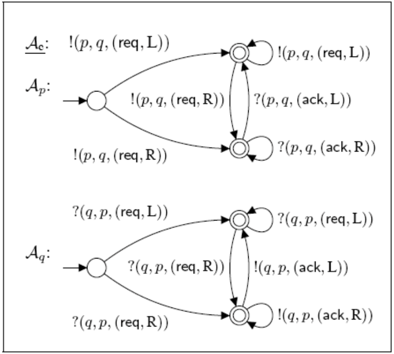
Overview

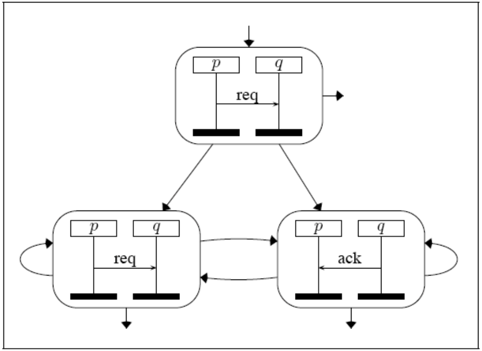
1. Communicating finite-state machines: a refresher
2. Well-formedness of CFMs
3. Bounded CFMs
   - Bounded words
   - Bounded MSCs
   - Bounded CFMs

Communicating finite-state machines

- A communicating finite-state machine (CFM) is a collection of finite-state machines, one for each process
- Communication between these machines takes place via (a priori) unbounded reliable FIFO channels
- The underlying system architecture is parametrised by the set $\mathcal{P}$ of processes and the set $\mathcal{C}$ of messages
- Action $!(p, q, m)$ puts message m at the end of the channel (p, q)
- Action $?(q, p, m)$ is enabled only if m is at head of buffer, and its execution by process q removes m from the channel (p, q)
- Synchronisation messages are used to avoid deadlocks

Example communicating finite-state machine





This CFM accepts if $\mathcal{A}_p$ and $\mathcal{A}_q$ are in some local state, and (as usual) all channels are empty

Formal definition

Definition (What is a CFM?)

A communicating finite-state machine (CFM) over $\mathcal{P}$ and $\mathcal{C}$ is a tuple

$$\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$$

where

- for each $p \in \mathcal{P}$:

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- for each $p \in \mathcal{P}$:
  - S_p is a non-empty finite set of local states (the S_p are disjoint)
  - $\Delta_p \subseteq S_p \times Act_p \times \mathbb{D} \times S_p$ is a set of local transitions

In sequel, let $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ be a CFM over $\mathcal{P}$ and $\mathcal{C}$.

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- $F \subseteq S_{\mathcal{A}}$ is the set of global final states

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Formal semantics of CFMs

Definition (Configuration)

Configurations of $\mathcal{A}$: $Conf_{\mathcal{A}} := S_{\mathcal{A}} \times \{\eta \mid \eta : Ch \to (\mathcal{C} \times \mathbb{D})^*\}$

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Definition (Transitions between configurations)

$\Longrightarrow_{\mathcal{A}} \subseteq Conf_{\mathcal{A}} \times Act \times \mathbb{D} \times Conf_{\mathcal{A}}$ is defined as follows:

- sending a message: $((\overline{s}, \eta), !(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], !(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta' = \eta[(p, q) := (a, m) \cdot \eta((p, q))]$
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- receipt of a message: $((\overline{s}, \eta), ?(p, q, a), m, (\overline{s}', \eta')) \in \Longrightarrow_{\mathcal{A}}$ if
  - $(\overline{s}[p], ?(p, q, a), m, \overline{s}'[p]) \in \Delta_p$
  - $\eta((q, p)) = w \cdot (a, m) \neq \epsilon$ and $\eta' = \eta[(q, p) := w]$
  - $\overline{s}[r] = \overline{s}'[r]$ for all $r \in \mathcal{P} \setminus \{p\}$

Linearizations of a CFM

Definition ((Accepting) Runs)

A run of $\mathcal{A}$ on $\sigma_1 \ldots \sigma_n \in Act^*$ is a sequence $\rho = \gamma_0\, m_1\, \gamma_1 \ldots \gamma_{n-1}\, m_n\, \gamma_n$ such that

- $\gamma_0 = (s_{init}, \eta_\varepsilon)$ with η_ε mapping any channel to ε
- $\gamma_{i-1} \xRightarrow{\sigma_i, m_i}_{\mathcal{A}} \gamma_i$ for any $i \in \{1, \ldots, n\}$

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Definition (Linearizations)

The set of linearizations of CFM $\mathcal{A}$:

$Lin(\mathcal{A}) := \{w \in Act^* \mid \text{there is an accepting run of } \mathcal{A} \text{ on } w\}$

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Well-formedness (reminder)

Let $Ch := \{(p,q) \mid p \neq q\,,\, p,q \in \mathcal{P}\}$ be a set of channels over $\mathcal{P}$.

We call $w = a_1 \ldots a_n \in Act^*$ proper if

1. every receive in w is preceded by a corresponding send, i.e.: $\forall (p,q) \in Ch$ and prefix u of w, we have:

$$\underbrace{\sum_{m \in \mathcal{C}} |u|_{!(p,q,m)}}_{\#\text{ sends from } p \text{ to } q} \quad \geqslant \quad \underbrace{\sum_{m \in \mathcal{C}} |u|_{?(q,p,m)}}_{\#\text{ receipts by } q \text{ from } p}$$

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2. the FIFO policy is respected, i.e.:
   $\forall 1 \leqslant i < j \leqslant n$, $(p,q) \in Ch$, and $a_i = !(p,q,m_1)$, $a_j = ?(q,p,m_2)$:

$$\sum_{m \in \mathcal{C}} |a_1 \ldots a_{i-1}|_{!(p,q,m)} = \sum_{m \in \mathcal{C}} |a_1 \ldots a_{j-1}|_{?(q,p,m)} \quad \text{implies} \quad m_1 = m_2$$

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A proper word w is well-formed if $\sum_{m \in \mathcal{C}} |w|_{!(p,q,m)} = \sum_{m \in \mathcal{C}} |w|_{?(q,p,m)}$

Well-formedness and CFMs

Lemma

For any CFM $\mathcal{A}$ and $w \in Lin(\mathcal{A})$, w is well-formed.

Recall that there is a strong correspondence between well-formed linearizations and MSCs.

From linearizations to partial orders (reminder)

Associate to $w = a_1 \dots a_n \in Act^*$ an Act-labelled poset

$$M(w) = (E, \preceq, \ell)$$

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  - $i \prec_{\text{msg}} j$ if for some $(p, q) \in Ch$ and $m \in \mathcal{C}$ we have:

$$\ell(i) = !(p, q, m) \text{ and } \ell(j) = ?(q, p, m) \text{ and}$$

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Relating well-formed words to CFMs

For any well-formed words u and v with $M(u)$ is isomorphic to $M(v)$:

$$\text{for any CFM } \mathcal{A}: \ u \in \mathcal{L}(\mathcal{A}) \quad \text{iff} \quad v \in \mathcal{L}(\mathcal{A}).$$

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Emptiness problem is undecidable for CFMs

Theorem: [Brand & Zafiropulo 1983]

The following (emptiness) problem:

INPUT:	CFM $\mathcal{A}$ over processes $\mathcal{P}$ and message contents $\mathcal{C}$
QUESTION:	Is $\mathcal{L}(\mathcal{A})$ empty?

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is undecidable. (Even if $\mathcal{C}$ is a singleton set).

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We define bounded CFMs, by first considering bounded words and bounded MSCs. Bounded CFMs will then generate bounded MSCs.

Bounded words

Definition (B-bounded words)

Let $B \in \mathbb{N}$ and $B > 0$. A word $w \in Act^*$ is called B-bounded if for any prefix u of w and any channel $(p, q) \in Ch$:

$$0 \leqslant \sum_{a \in \mathcal{C}} |u|_{!(p,q,a)} - \sum_{a \in \mathcal{C}} |u|_{?(q,p,a)} \leqslant B$$

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Word w is B-bounded if for any pair of processes (p, q), the number of sends from p to q cannot be more than B ahead of the number of receipts by q from p (for every message a).

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Example

$!(1, 2, a)\ !(1, 2, b)\ ?(2, 1, a)\ ?(2, 1, b)$ is 2-bounded but not 1-bounded.

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So: if M is B-bounded, then a buffer capacity B is sufficient for all possible runs of MSC M.

Bounded MSCs

Definition (Existentially bounded MSCs)

Let $B \in \mathbb{N}$ and $B > 0$. An MSC $M \in \mathbb{M}$ is called existentially B-bounded ($\exists B$-bounded, for short) if $Lin(M) \cap Lin^B(M) \neq \varnothing$.

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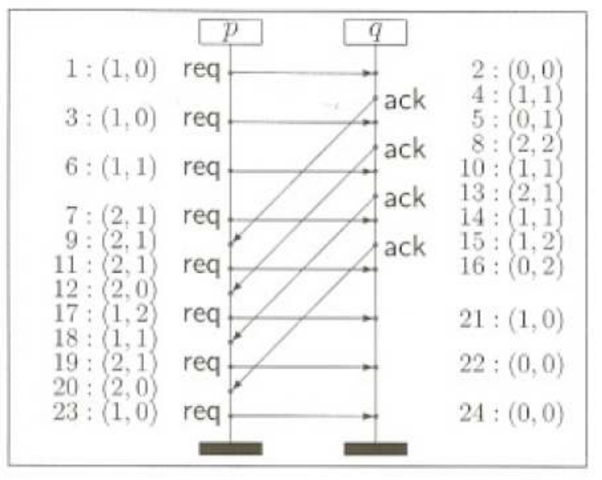
MSC M is $\exists B$-bounded if at least one linearization of M is B-bounded.

Consequence

The MSC M can be "scheduled" in such a way that no channel ever contains more than B messages.

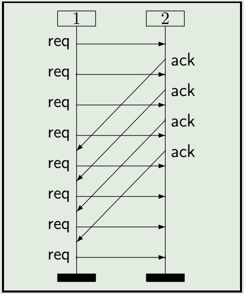
Bounded MSCs

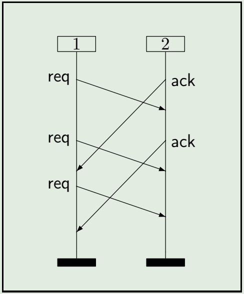


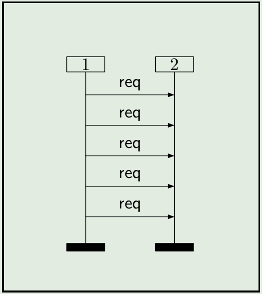
An ∃2-bounded MSC with a corresponding justification

Bounded MSCs

Example







Bounded MSCs

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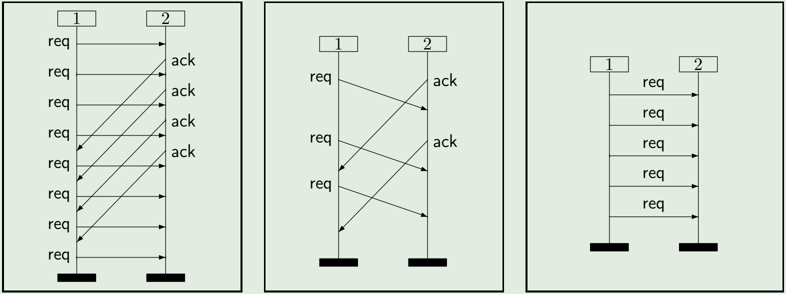


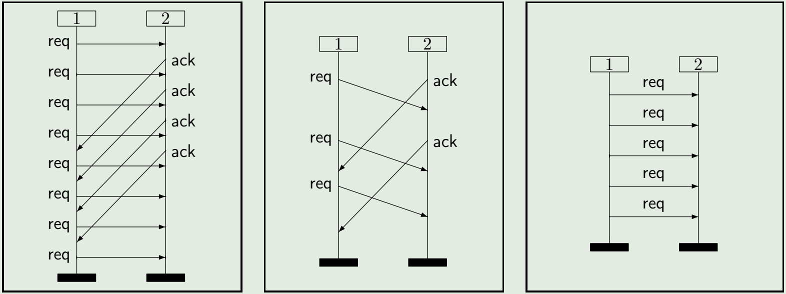
$\forall$4-bounded

$\exists$2-bounded

not $\exists$1-bounded

Bounded MSCs

Example



$\forall$4-bounded	$\forall$3-bounded
$\exists$2-bounded	$\exists$1-bounded
not $\exists$1-bounded	

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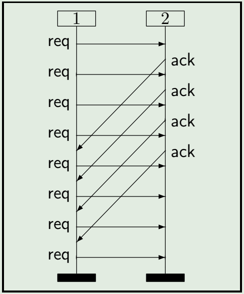
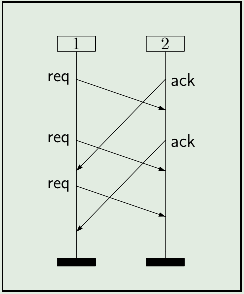
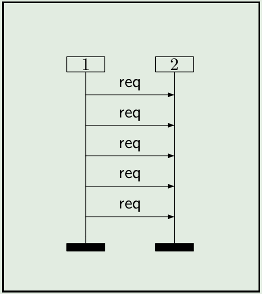


$\forall$4-bounded
$\exists$2-bounded
not $\exists$1-bounded



$\forall$3-bounded
$\exists$1-bounded



$\forall$5-bounded
$\exists$1-bounded

Bounded CFMs

Definition (Universally bounded CFM)

1. Let $B \in \mathbb{N}$ and $B > 0$. CFM $\mathcal{A}$ is *universally B-bounded* if each MSC in $\mathcal{L}(\mathcal{A})$ is $\forall B$-bounded.
2. CFM $\mathcal{A}$ is *universally bounded* if it is $\forall B$-bounded for some $B \in \mathbb{N}$ and $B > 0$.

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Definition (Existentially bounded CFM)

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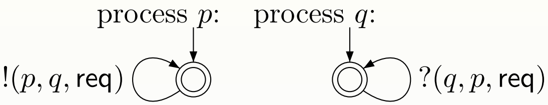
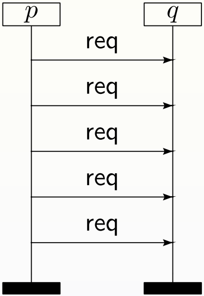
Example (1)



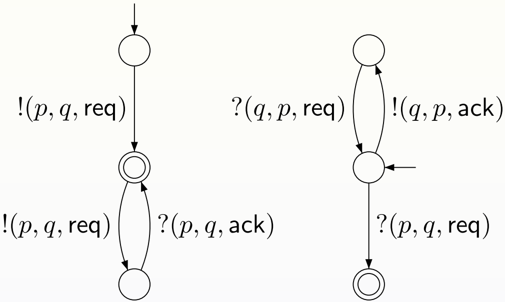


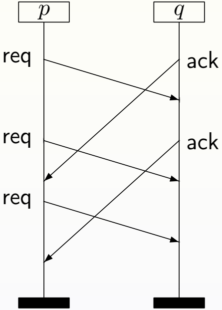
$\exists$1-bounded, but not $\forall B$-bounded for any B

so, not $\forall$-bounded.

Example (2)

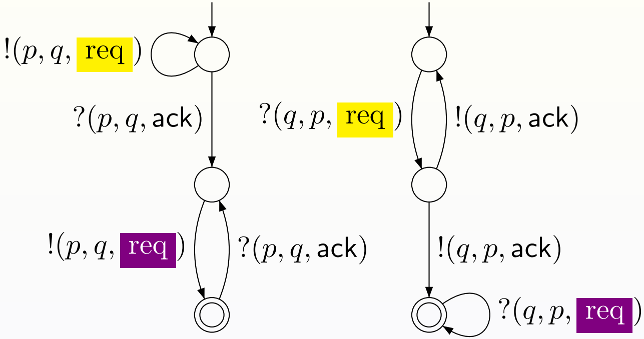
process p: process q:





∃1-bounded, and ∀3-bounded

Example (3)



$\exists\lceil\frac{n}{2}\rceil$-bounded, but not $\forall B$-bounded for any B

Justification

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  - messages of phase 1 are tagged with data req

- … and waits for the first acknowledgement of q

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- So, p sends $2n$ reqs to q and q sends n acks
  - existentially $\lceil \frac{n}{2} \rceil$-bounded
  - q starts to send acks after $\lceil \frac{n}{2} \rceil$ requests have been sent by p
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Emptiness is decidable for ∃-bounded CFMs

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Note:

This decision problem is undecidable for arbitrary CFM, and is obviously decidable for ∀-bounded CFMs, as ∀-bounded CFMs have finitely many configurations, and thus one can check whether a configuration (s, η_ε) with $s \in F$ is reachable by a simple graph analysis.

Some (un)decidability results

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the proofs of all these facts are left as an exercise

Deadlocks

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Deadlock-free CFMs

$(\overline{s}, \eta) \in Conf_{\mathcal{A}}$ is a deadlock configuration of CFM $\mathcal{A}$ if there is no 'accepting" configuration $(\overline{s}', \eta') \in F \times \{\eta_\varepsilon\}$ with $(\overline{s}, \eta) \Longrightarrow^*_{\mathcal{A}} (\overline{s}', \eta')$.

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The decision problem: Is CFM $\mathcal{A}$ deadlock free? is undecidable.

Checking B-boundedness for deadlock-free CFMs is decidable

The decision problem: for deadlock-free CFM $\mathcal{A}$ and $B \in \mathbb{N}$ with $B > 0$, is $\mathcal{A}$ $\forall B$-bounded? is decidable.