Theoretical Foundations of the UML

Lecture 6: Compositional Message Sequence Graphs

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moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

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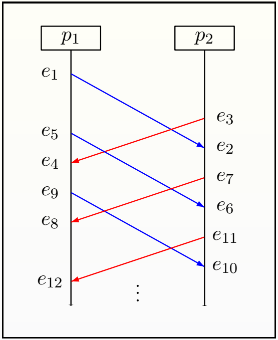
Outline

Overview

An MSC that cannot be decomposed [Yannakakis 1999]



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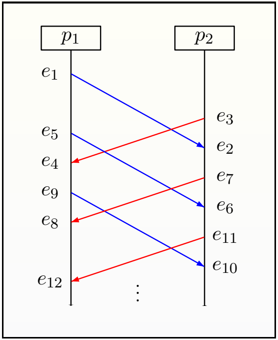

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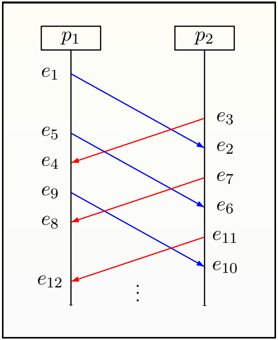

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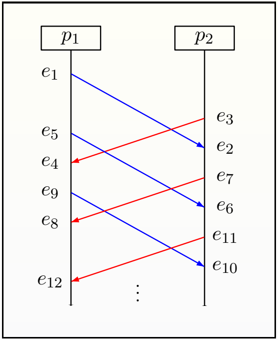

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- $e_3 \preceq e_2$ and $e_1 \preceq e_4$ thus
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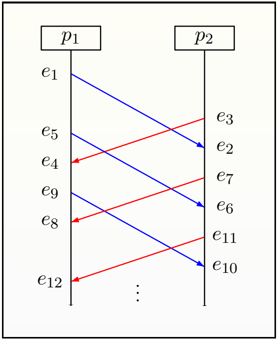

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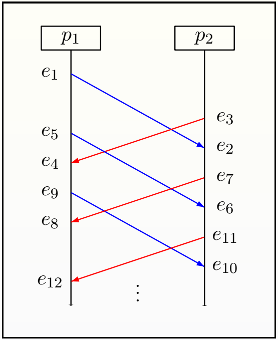

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Problem:

Compulsory matching between send and receive events in the same MSG vertex (i.e., send e and receive $m(e)$ must belong to the same MSC).

Overview

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$$m(e) = e' \wedge l(e) = !(p, q, a) \quad \text{implies} \quad l(e') = ?(q, p, a)$$

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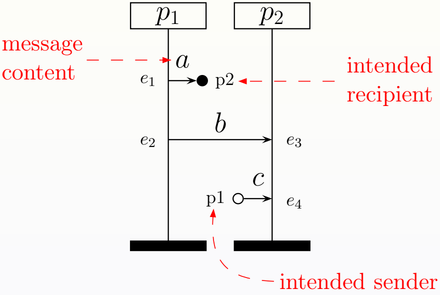
Note:

An MSC is a CMSC where m is total and bijective.

CMSC example



$m(e_2) = e_3$
$e_1 \notin dom(m)$
$e_4 \notin rng(m)$

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  3. $M_1 \bullet M_2$ is FIFO (when restricted to matched events)

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- l and m are defined as on the previous slide
- $\preceq$ is the reflexive and transitive closure of:

$$\left(\bigcup_{p \in \mathcal{P}} <_{p,1} \cup <_{p,2} \right) \quad \cup \quad \{(e, e') \mid e \in E_1 \cap E_p \, , \; e' \in E_2 \cap E_p\} \quad \cup \quad \{(e, m(e) \mid e \in dom(m)\}$$

Examples

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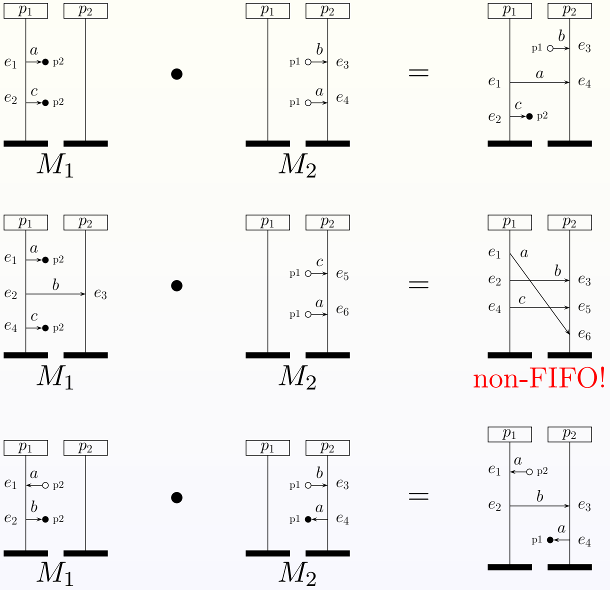

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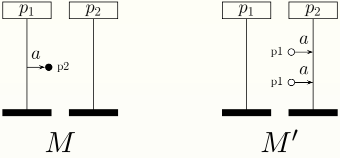

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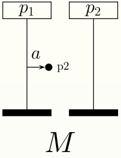

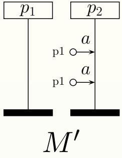

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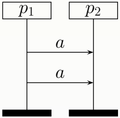

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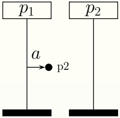

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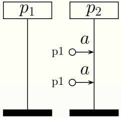

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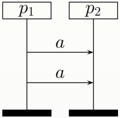

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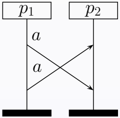

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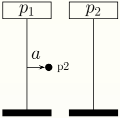

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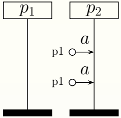

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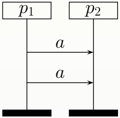

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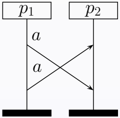

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Note:

Concatenation of CMSCs is <u>not</u> associative.

Overview

Compositional MSG

Let $\mathbb{CM}$ be the set of all CMSCs.

Definition (Compositional MSG)

A compositional MSG (CMSG) $G = (V, \rightarrow, v_0, F, \lambda)$ with $\lambda : V \rightarrow \mathbb{CM}$, where $V, \rightarrow, v_0,$ and F as for MSGs.

The difference with an MSG is that the vertices in a CMSG are labeled with compositional MSCs (rather than "real" MSCs).

Paths

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Definition (Path in a CMSG)

A path π of G is a finite sequence

$$\pi = u_0\, u_1 \; \ldots \; u_n \text{ with } u_i \in V \;\; (0 \leq i \leq n) \text{ and } u_i \rightarrow u_{i+1} \;\; (0 \leq i < n)$$

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Definition (CMSC of a path)

The CMSC of a path $\pi = u_0 \, \ldots \, u_n$ is:

$$M(\pi) \;=\; (\ldots(\lambda(u_0) \bullet \lambda(u_1)) \bullet \lambda(u_2) \ldots) \bullet \lambda(u_n)$$

where CMSC concatenation is left associative.

The MSC language of a CMSG

Definition (Language of a CMSG)

The (MSC) language of CMSG G is defined by:

$$L(G) = \{ \underbrace{M(\pi) \in \mathbb{M}}_{\text{only "real" MSCs}} \mid \pi \text{ is an accepting path of } G \}.$$

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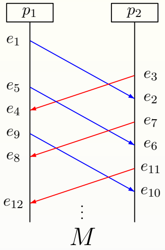
Note: Accepting paths that give rise to an CMSC (which is not an MSC) are not part of $L(G)$.

Yannakakis' example as compositional MSG



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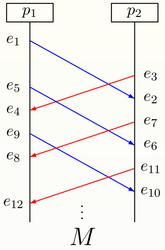

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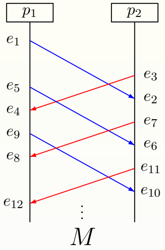

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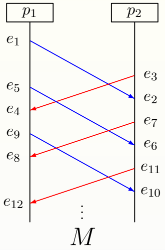

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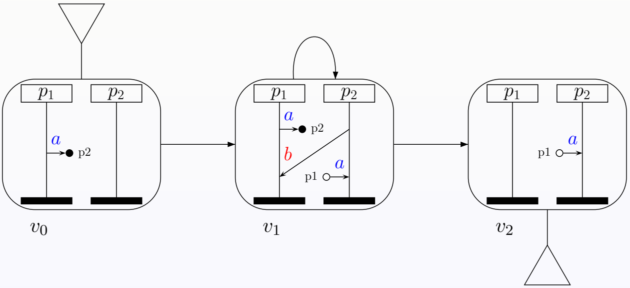

Overview

1. A non-decomposable MSC
2. Compositional Message Sequence Charts
3. Compositional Message Sequence Graphs
4. Safe Compositional Message Sequence Graphs
5. Existence of Safe Paths
6. Universality of Safe Paths

Safe paths and CMSGs

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Definition (Safe path)

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So:

CMSG G is safe if on any of its accepting paths there are no unmatched sends and receipts, i.e., if any of its accepting paths is indeed an MSC.

Overview

Existence of a safe accepting path

Theorem: undecidability of existence of a safe path

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. . . black board . . . □

The complement decision problem "does CMSG G have no safe, accepting path?" is undecidable too.

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Universality of safe accepting paths

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Proof.

Polynomial reduction to reachability problem in (non-deterministic) pushdown automata.

... see details on the next slides ... □

Pushdown automata

Definition (Pushdown automaton)

A pushdown automaton (PDA, for short) $K = (Q, q_0, \Gamma, \Sigma, \Delta)$ with

- Q, a finite set of control states
- $q_0 \in Q$, the initial state
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Transition relation

$(q, a, \gamma, q', \text{pop}) \in \Delta$ means: in state q, on reading input symbol a and top of stack is symbol γ, change to q' and pop γ from the stack.

Reachability in pushdown automata

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Theorem: [Esparza et al. 2000]

The reachability problem for PDA is decidable in PTIME.

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  CMSG G is not safe wrt. (p_i, p_j) iff PDA $K_{i,j}$ accepts

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- For accepting path $u_0 \ldots u_k$ in G, feed $K_{i,j}$ with the word

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  such that unmatched sends (of some type) precede all unmatched receipts (of the same type)
- Possible violations that $K_{i,j}$ may encounter:
  1. nr. of unmatched $!(p_i, p_j, \cdot)$ $>$ nr. of unmatched $?(p_j, p_i, \cdot)$
  2. type of k-th unmatched send $\neq$ type of k-th unmatched receive
  3. non-FIFO communication

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- Transition function Δ is described on next slide

Safeness of CMSGs (2)

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- Remaining input w empty? Accept, if stack non-empty; else reject

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The behaviour in state q_{a_m} for $0 < m \leqslant k$:

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    - if 1 is on top of stack, pop it
- If stack is empty:
    - if last receive differs from a_m, accept
    - otherwise reject, while ignoring the rest (if any) of the input

Safeness of CMSGs (4)

It follows: PDA $K_{i,j}$ accepts iff CMSG G is not safe wrt. (p_i, p_j)

$\Longrightarrow$ CMSG G is not safe wrt. (p_i, p_j) iff configuration $(q_F, \cdot, \cdot)$ is reachable.

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Time complexity

The worst-case time complexity of checking whether CMSG G is safe is in $\mathcal{O}(k^2 \cdot N^2 \cdot L \cdot |E|^2)$ where $k = |\mathcal{P}|$, $N = |V|$, and $L = |\mathcal{C}|$.

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Proof.

Checking reachability in PDA $K_{i,j}$ is in $\mathcal{O}(L \cdot |E|^2)$. The number of PDAs is k^2, as we consider ordered pairs in $\mathcal{P}$. The number of paths in the CMSG G for each pair that need to be checked is in $\mathcal{O}(N^2)$, as a single traversal for each loop in G suffices. □