Overview

RWTH AACHEN UNIVERSITY

Theoretical Foundations of the UML

Lecture 4: Message Sequence Graphs

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moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

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Summary of Lecture #3

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1. A Message Sequence Chart is a visual partial order
   - between send and receive events
   - totally ordered per process — vertical ordering
   - receive events happen after their send events — horizontal ordering
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4. A MSC has a race if causal order $\neq$ visual order
   - checking whether an MSC has a race can be done in quadratic time (in number of events)
   - using an optimized version of Warshall's algorithm

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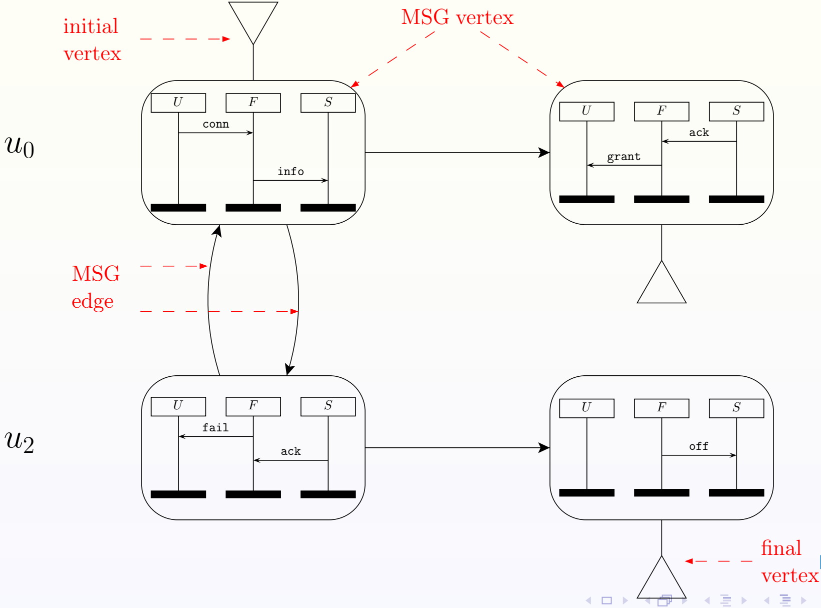
⇒ This yields Message Sequence Graphs

- Alternatives: ensembles of MSCs, high-level MSCs (MSC'96)

Message Sequence Graphs



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A Message Sequence Graph (MSG) $G = (V, \rightarrow, v_0, F, \lambda)$ with:

- $(V, \rightarrow)$ is a digraph with finite set V of vertices and $\rightarrow \subseteq V \times V$ a set of edges
- $v_0 \in V$ is the starting (or: initial) vertex
- $F \subseteq V$ is a set of final vertices
- $\lambda : V \rightarrow \mathbb{M}$ associates to each vertex $v \in V$, an MSC $\lambda(v)$

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Note:

An MSG can be considered as a non-deterministic finite-state automaton without input alphabet where states are MSCs. Obviously, every MSC is an MSG.

Example

Concatenation of MSCs: definition

Let $M_i = (\mathcal{P}_i, E_i, \mathcal{C}_i, l_i, m_i, \preceq_i)$ with $i \in \{1, 2\}$
be two MSCs with $E_1 \cap E_2 = \varnothing$

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$$l(e) = \begin{cases} l_1(e) & \text{if} \quad e \in E_1 \\ l_2(e) & \text{if} \quad e \in E_2 \end{cases} \qquad m(e) = \begin{cases} m_1(e) & \text{if} \quad e \in E_1 \\ m_2(e) & \text{if} \quad e \in E_2 \end{cases}$$

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$$\preceq \; = \left(\preceq_1 \cup \preceq_2 \cup \{(e, e') \mid \exists p \in \mathcal{P}.\, e \in E_1 \cap E_p \,,\; e' \in E_2 \cap E_p\} \right)^*$$

Concatenation of MSCs: observations

Ordering

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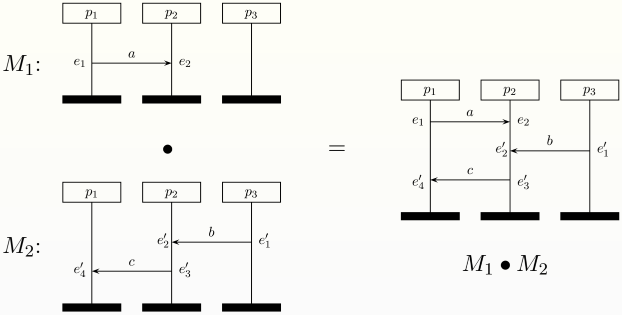
Observations

- events are ordered per process:

  every event at p in MSC M_1 precedes every event at p in MSC M_2
- events at distinct processes in M_1 and M_2 can be incomparable
- thus: a process may start with M_2 before other processes do pause
- this differs from: first complete M_1, then start with M_2

Example (1)



$M_1 \bullet M_2$

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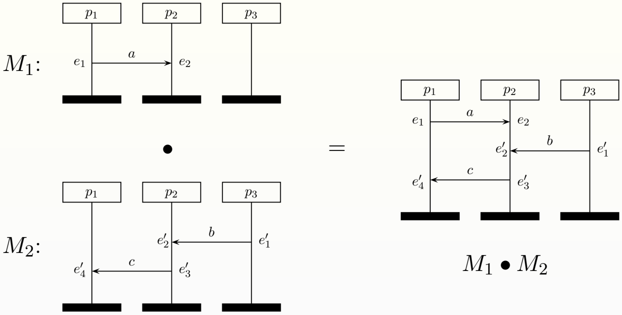
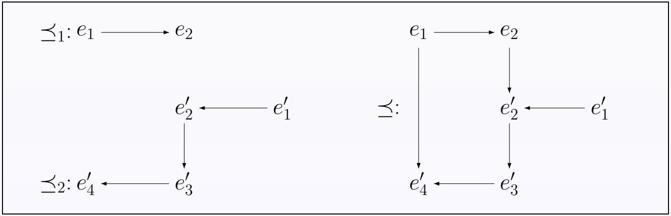
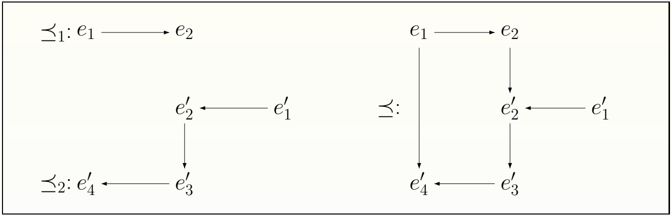




Example (2)



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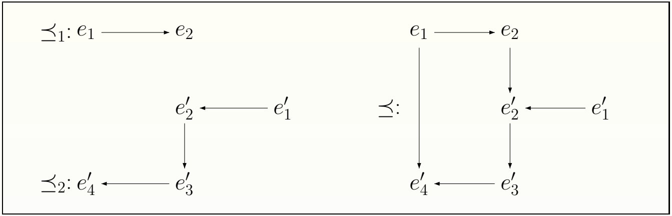

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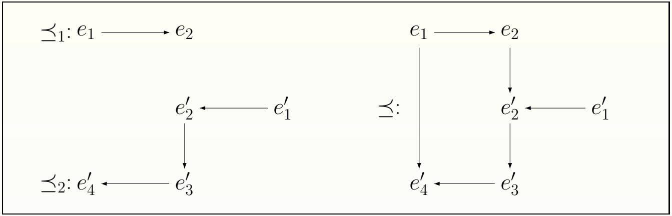

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Example linearizations:

$e_1 \quad e_2 \quad e'_1 \quad e'_2 \quad \ldots \in Lin(M_1 \bullet M_2)$

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3. Race-freeness, however, is not preserved

$$(M_1 \text{ is race-free } \wedge M_2 \text{ is race-free }) \quad \not\Rightarrow \quad M_1 \bullet M_2 \text{ is race-free}$$

Paths in MSGs

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A path through MSG G is a finite traversal through the graph G.

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A path π in MSG G is a finite sequence

$$\pi = u_0\, u_1\, \ldots\, u_n \text{ with } u_i \in V \;\; (0 \leq i \leq n) \text{ and } u_i \rightarrow u_{i+1} \;\; (0 \leq i < n)$$

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An accepting path through MSG G is a path starting in the initial vertex and ending in a final vertex.

Definition

Path $\pi = u_0 \, \ldots \, u_n$ is accepting if: $u_0 = v_0$ and $u_n \in F$.

Paths in an MSG represent MSCs

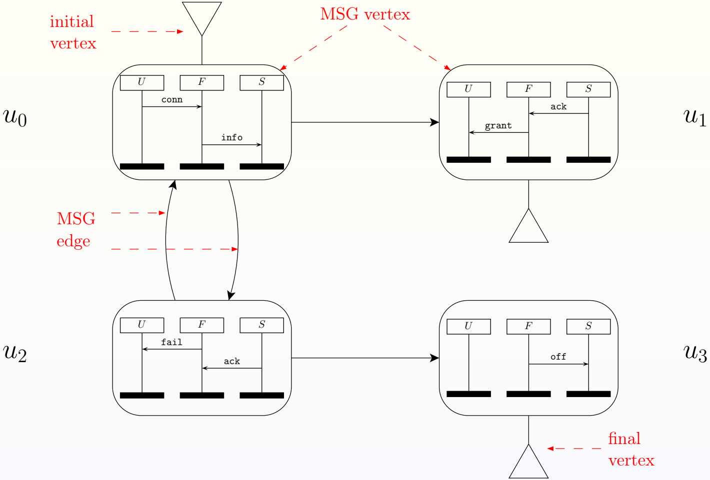
Let $G = (V, \rightarrow, v_0, F, \lambda)$ be an MSG.

Definition

The MSC of a path $\pi = u_0 \ \ldots \ u_n$ through MSG G is defined by:

$$M(\pi) \ = \ \underbrace{\lambda(u_0)}_{\text{MSC of } u_0} \bullet \underbrace{\lambda(u_1)}_{\text{MSC of } u_1} \bullet \ldots \bullet \underbrace{\lambda(u_n)}_{\text{MSC of } u_n}$$

Example paths



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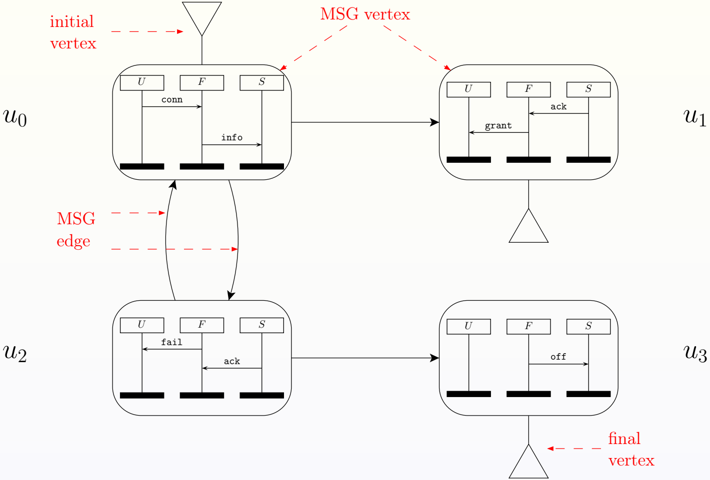

$u_0\ u_2\ u_0\ u_1$ is accepting; $u_0\ u_2\ u_0\ u_2$ is not accepting

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Definition

The word language of MSG G is defined by $Lin(L(G))$ where

$$Lin(\{M_1, \ldots, M_k\}) = \bigcup_{i=1}^{k} Lin(M_i).$$

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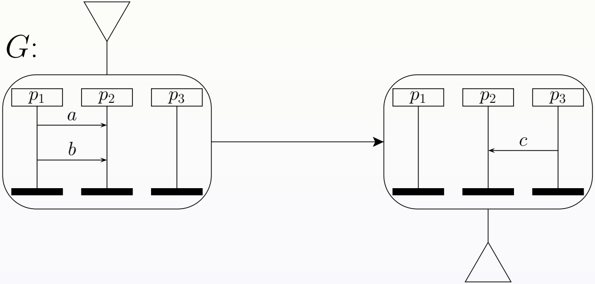

MSG G has a race.

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No undecidable problem can ever be solved by a computer or computer program of any kind.

Do MSGs have an MSC in common?

Theorem: undecidability of empty intersection

The decision problem:

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Proof: Reduction from Post's Correspondence Problem (PCP)

... black board ...