Overview

1. Lecture 3: Races

Theoretical Foundations of the UML

Lecture 3: Races

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moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

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Summary of Lecture #2

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1. A Message Sequence Chart is a partial order
   - between send and receive events
   - totally ordered per process — vertical ordering
   - receive events happen after their send events — horizontal ordering
   - respecting the first-in first out (FIFO) property

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   - all linearizations of an MSC are well-formed
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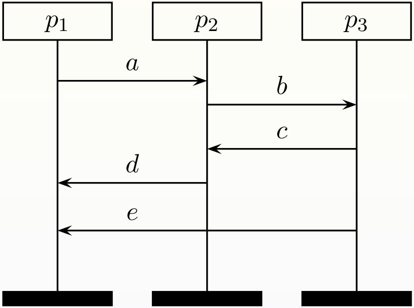
3. Every well-formed word can be transformed into an MSC
   - two linearizations of the same MSC yield isomorphic MSCs
4. So: there is a 1-to-1 relation between an MSC and its linearizations

Example



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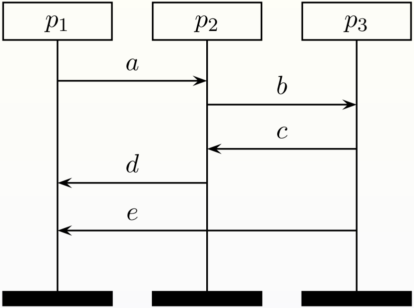

These pictures are formalized using partial orders.

Message Sequence Chart (MSC) (1)

Definition

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$$E = \biguplus_{p \in \mathcal{P}} E_p = E_? \,\dot{\cup}\, E_!$$

- $\mathcal{C}$, a finite set of message contents
- $l : E \to Act$, a labelling function defined by:

$$l(e) = \begin{cases} !(p,q,a) & \text{if} \quad e \in E_p \cap E_! \\ ?(p,q,a) & \text{if} \quad e \in E_p \cap E_? \end{cases} \text{, for } p \neq q \in \mathcal{P},\ a \in \mathcal{C}$$

Message Sequence Chart (MSC) (2)

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- $m : E_! \rightarrow E_?$ a bijection ("matching function"), satisfying:

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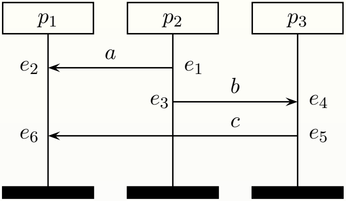
- $\preceq \subseteq E \times E$ is a partial order ("visual order") defined by:

$$\leqslant \; = \Big(\underbrace{\bigcup_{p \in \mathcal{P}} <_p}_{<_p \text{ is a total order = "top-to-bottom" order on process } p} \; \cup \; \underbrace{\{(e, m(e)) \mid e \in E_!\}}_{\text{communication order } <_c} \Big)^*$$

where for relation R, R^* denotes its reflexive and transitive closure.

Example

Visual order can be misleading



can e_6 occur before e_2?

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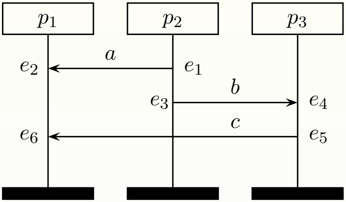

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If message b takes much shorter than message a, then c might arrive at p_1 before a.

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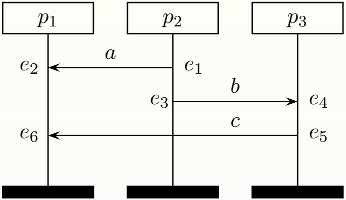

can e_6 occur before e_2?

If message b takes much shorter than message a, then c might arrive at p_1 before a.

In practice, e_6 might occur before e_2, but $e_2 <_{p_1} e_6$ and thus $e_2 \preceq e_6$.

This is misleading and called a race.

What is a race?

A race condition asserts a particular order of events will occur because of the visual ordering (i.e., the partial order $\preceq$) when, in practice, this order cannot be guaranteed to hold.

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Q: When are race conditions possible and how to detect them?

Causal order

Causal order

Main principles:

- Send events should happen before their matching receive events
- The ordering of events wrt. sends on same process is unaffected
- Receive events on a process sent from the same process are ordered as their sends

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$\ll^*$ is a partial order (referred to as causal order) in which events at the same process are not necessarily ordered.

Causal order: example

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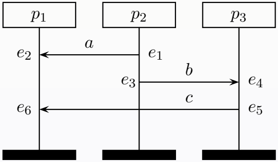

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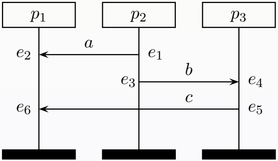

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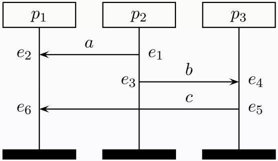

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$e_1 \ll e_2, \quad e_3 \ll e_4, \quad e_5 \ll e_6,$

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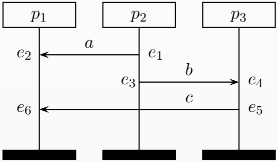

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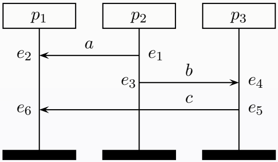


Example

$e_1 \ll e_2$, $e_3 \ll e_4$, $e_5 \ll e_6$, $e_1 \ll e_3$, $e_4 \ll e_5$, not $(e_2 \ll e_6)$

Races

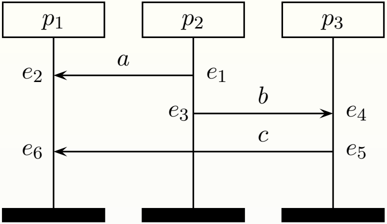
Definition

MSC M contains a race if for some $e, e' \in E_?$ and process p:

$$e <_p e' \text{ but not } (e \ll^* e')$$

where $\ll^* \subseteq E \times E$ is the reflexive and transitive closure of $\ll$.

Race: example



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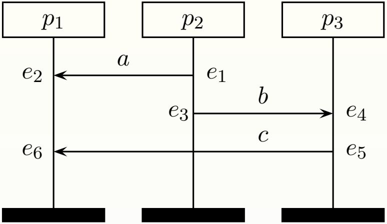

Visual order versus causal order

1. $e_1 \preceq e_2,\ e_3 \preceq e_4,\ e_5 \preceq e_6,\ e_1 \preceq e_3,\ e_4 \preceq e_5,\ e_2 \preceq e_6$
2. $e_1 \ll e_2,\ e_3 \ll e_4,\ e_5 \ll e_6,\ e_1 \ll e_3,\ e_4 \ll e_5,$ not $(e_2 \ll e_6)$

As $\ll^* \not\subseteq \preceq$, this MSC contains a race.

Other examples

On the black board.

Why are races problematic?

Recall: MSC M has a race if $\preceq \not\subseteq \ll^*$ or equivalently:

$$\exists e, e' \in E_? \;.\; (e <_p e' \text{ and } e \not\ll^* e')$$

Whenever $\preceq \not\subseteq \ll^*$, implementations based on $<_p$ may cause problems:

1. unspecified message reception
   - a process receives a message which by the MSC is not possible
2. deadlocks
   - a process blocking on receipt of an unexpected message may block others too
3. message loss
   - unexpectedly received messages may be ignored
4. exploiting wrong message content

Checking whether an MSC has a race

[1] for digraphs without negative cycles.

Checking whether an MSC has a race

- MSC M has a race if $\preceq \not\subseteq \ll^*$

- How to check whether MSC M has a race?

  compute $\ll^*$ *and check whether* $\preceq \subseteq \ll^*$

- transitive closure $\ll^*$ is computed using Floyd-Warshall's algorithm

  - algorithm for finding shortest paths in a weighted digraph with positive or negative edge weights[1]
  - easily adapted for computing the transitive closure of digraphs
  - worst-case time complexity $\mathcal{O}(|E|^3)$
  - by using some specifics of MSC, this is reduced to $\mathcal{O}(|E|^2)$

- So: race checking can be done quadratically in the number of events

[1] for digraphs without negative cycles.

Computing $\ll^*$: Warshall's algorithm

Algorithm

$$\underbrace{\text{compute } \ll^*}_{\text{Warshall's Algorithm}} \quad \text{and compare with } \preceq$$

Warshall's Algorithm: <u>input:</u> $R \subseteq X \times X$ where X is a set
<u>output:</u> R^*

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Idea:

Consider R and R^* as directed graphs

There is an edge $x \Rightarrow y$ in R^* iff there is a (possibly empty) sequence

$$x = x_0 \to x_1 \to x_2 \to \ldots \to x_n = y \text{ in } R$$

(our setting: $X = E, R = \ll, \ R^* = \ll^*$)

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- for $j \in \{1, \ldots, n{+}1\}$ define relation $\overset{j}{\Longrightarrow}$ as follows:

  $x \overset{j}{\Longrightarrow} y$ iff $\exists$ path in R from x to y such that all vertices on the path ($\neq x, y$) have a smaller number than j

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- Then:
  (1) $x \Longrightarrow y$ iff $x \overset{n+1}{\Longrightarrow} y$
  (2) $x \overset{1}{\Longrightarrow} y$ iff $x = y$ or $x \ll y$
  (3) $x \overset{y+1}{\Longrightarrow} z$ iff $x \overset{y}{\Longrightarrow} z$ or $x \overset{y}{\Longrightarrow} y \overset{y}{\Longrightarrow} z$

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- Store $\stackrel{i}{\Longrightarrow}$ in a boolean matrix C

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- Store $\overset{i}{\Longrightarrow}$ in a boolean matrix C
- Postcondition: $C[x, y] = \texttt{true}$ iff $(x, y) \in R^*$
- Precondition: $\forall x, y \in X \;.\; C[x, y] = \texttt{false}$

Warshall's algorithm

/* first compute $x \stackrel{1}{\Longrightarrow} y$ */
for $x := 1$ to n do
    for $y := 1$ to n do
        $C[x,y] := (x = y \text{ or } \underbrace{(x,y) \in R}_{x \ll y})$

/* loop invariant: after the j-th iteration of */
/* outermost loop it holds: $C[x,y] = \texttt{true}$ iff $x \stackrel{j+1}{\Longrightarrow} y$ */
for $y := 1$ to n do
    for $x := 1$ to n do
        if $C[x,y]$ then
            for $z := 1$ to n do
                if $C[y,z]$ then
                    $C[x,z] := \texttt{true}$

Correctness and complexity

Lemma: correctness

After j iterations: $x \overset{j+1}{\Longrightarrow} y$ iff $C[x, y] = \texttt{true}$.

Proof.

if: trivial; *only if*: by induction on j. $\square$

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Complexity

Worst-case time complexity of Warshall's algorithm : $\mathcal{O}(n^3)$ with $n = |X|$

Proof.

follows from the facts that there is a triple-nested loop of which each loop has at most n iterations. □

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Using some properties of $\ll$ the complexity can be improved.

Exploit that for $\ll$:

1. $\ll$ is acyclic (as it is a partial order)
2. number of immediate predecessors of $e \in E$ under $\ll$ is at most two (why?)

   Recall that e is an immediate predecessor of e' (under $\ll$) if:

$$e \ll e' \text{ and } \neg(\exists e'' \notin \{e, e'\}.\, e \ll e'' \wedge e'' \ll e')$$

Efficiency improvement [Alur et al. '96]

The main loop of Warshall's algorithm:

```
for e := 1 to n do
    for e' := 1 to n do
        if C[e', e] then
            for e'' := 1 to n do
                if C[e, e''] then
                    C[e', e''] := true
```

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The main loop of Alur *et. al.*'s algorithm for detecting races in MSCs:

for $e := 1$ to n do
  for $e' := e - 1$ downto 1 do
    if (not $C[e', e]$ and $e' \ll e$) then
      $C[e', e] := \texttt{true}$
      for $e'' := 1$ to $e' - 1$ do
        if $C[e'', e']$ then
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Detecting races in MSCs

Theorem

Let M be an MSC with set E of events and $n = |E|$. Checking whether M has a race can be done in $\mathcal{O}(n^2)$.

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of the triple-nested main loop is executed for (e, e') only if e' is an immediate predecessor of e under $\ll$. As for MSCs, an event can have at most two immediate predecessors, the innermost loop is executed at most $2 \cdot n$ times. This yields a total worst-case time complexity of $n^2{+}2{\cdot}n$. □

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