Theoretical Foundations of the UML

Lecture 17: Introduction to Statecharts

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moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

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Outline

Overview

1. Background
2. Ingredients of Statecharts
   - Mealy Machines
   - State Hierarchy
   - Orthogonality
   - Broadcast Communication
   - Some Small Examples
   - Other Features: Priority, Nondeterminism and Negated Events
3. Semantics of Statecharts
4. Formal Definition of UML Statecharts

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  - professor at Weizmann Institute (Israel); co-founder of I-Logix Inc.
- Extensively used in embedded systems, automotive and avionics
- Variants: UML Statecharts, Stateflow, hierarchical state machines
  - supported by Statemate toolset, and Matlab/Simulink

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- Orthogonality, i.e., concurrency
- Refinement, and
- Encouraging "zoom" capabilities for moving easily back and forth between levels of abstraction

What are Statecharts?

$$\begin{aligned} \text{Statecharts} := & \text{ Mealy machines} \\ & + \text{State hierarchy} \\ & + \text{Broadcast communication} \\ & + \text{Orthogonality} \end{aligned}$$

Mealy machines [Mealy, 1953]

Definition (Mealy machine)

A Mealy machine $\mathcal{A} = (Q, q_0, \Sigma, \Gamma, \delta, \omega)$ with:

- Q is a finite set of states with initial state $q_0 \in Q$
- Σ is the input alphabet
- Γ is the output alphabet
- $\delta : Q \times \Sigma \to Q$ is the deterministic (input) transition function, and
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A Mealy machine (or: finite-state transducer) is a finite-state automaton that produces output on a transition, based on current input and state.

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Moore machines

In a Moore machine $\omega : Q \to \Gamma$, output is purely state-based.

Mealy machines

Mealy machines

- No final (accepting) states
- Transitions produce output
- Deterministic input transition function

$\Rightarrow$ Acceptance of input words is not important, but the generation of output words from input words is important

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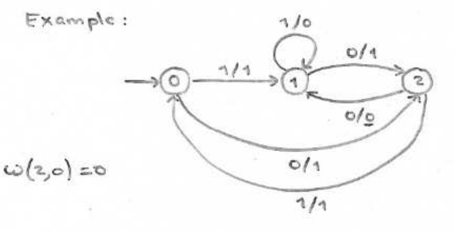
Example



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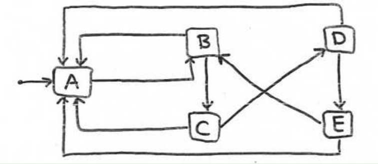
- No notion of concurrency
  - need for modeling independent components
- No notion of communication between automata.

Scalability

A bit unstructured Mealy machine



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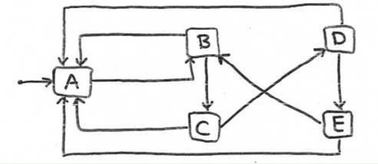
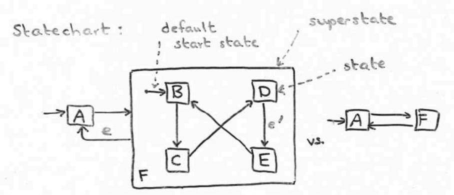


An equivalent statechart



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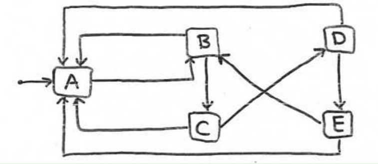

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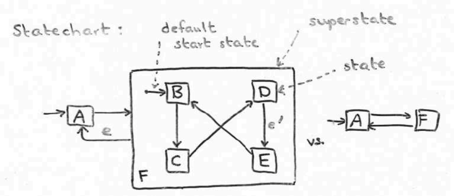
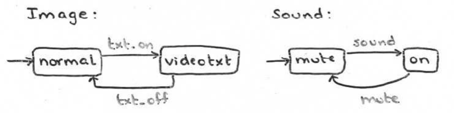


State hierarchy yields modular, hierarchical and structured models.

Orthogonality

Two independent components



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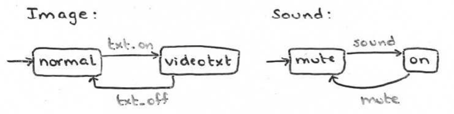
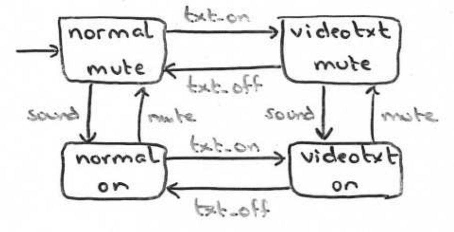


Mealy machine for $Image \parallel Sound$



Number of states is exponential in size of concurrent components

Orthogonality

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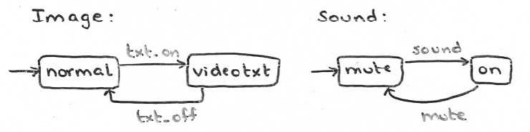

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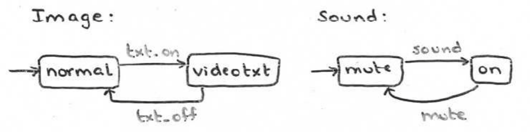
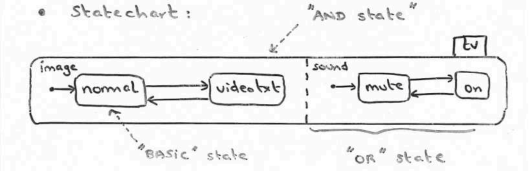


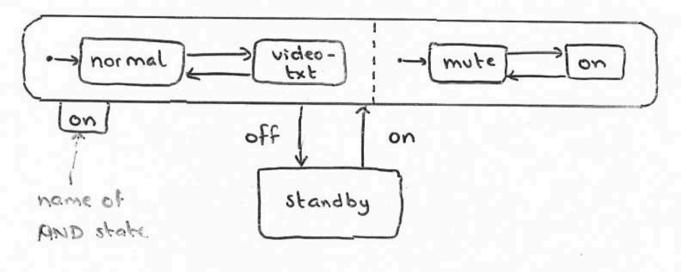
Statechart for *Image* ‖ *Sound*



Concurrency modeled by independence

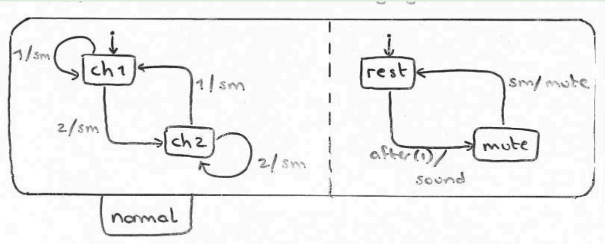
Combined with state hierarchy

Switching on and off the television



Broadcast

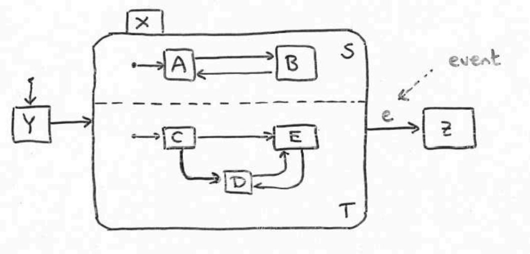
Turn off sound on switching a tv channe



- Output is broadcast that can be received by any other component
- When pushing button 1, channel switches to its state channel 1, while generating signal *sm* on which component *SM* switches off the sound.

Concurrency

Example concurrency in statecharts



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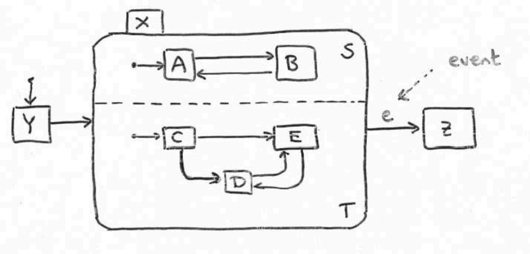

Active

- As long as node X is active, nodes S and T are active

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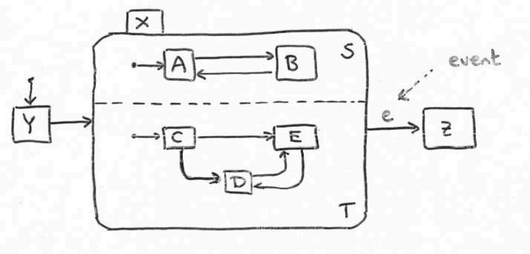

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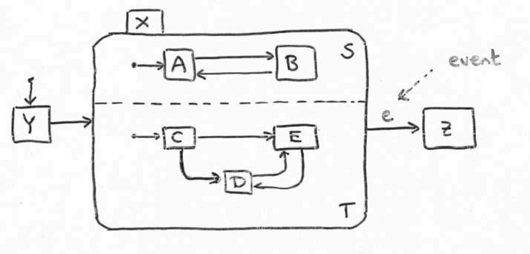

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- Node S is active when either node A or B is active
- Node T is active if one of C, D or E is active

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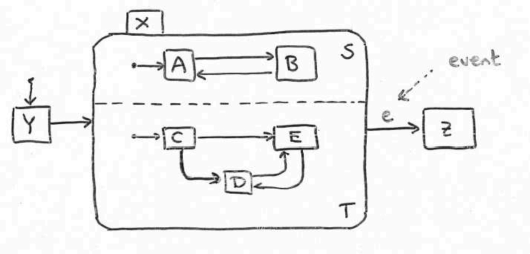

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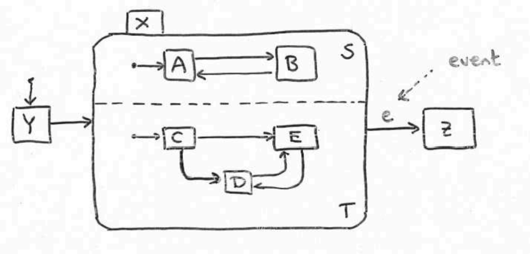

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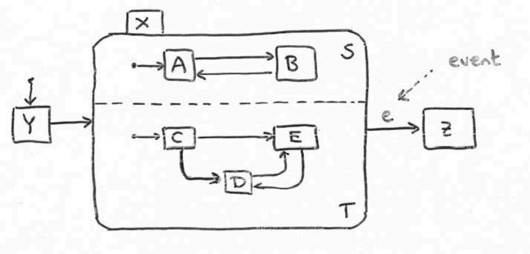

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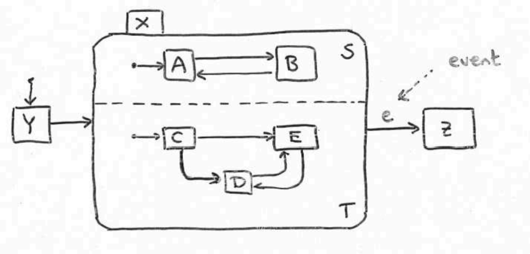
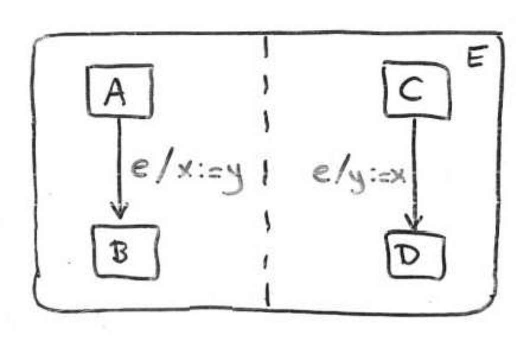


Exit behaviour

- When node X exits, both nodes S and T exit
- When Y exits, X starts, S starts in A, and T starts in C
- On the occurrence of event e, node X exits (regardless of current state in S or T)

Swapping two variables

Swapping the value of variables x and y



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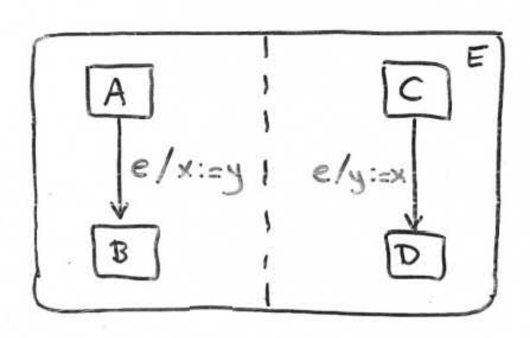

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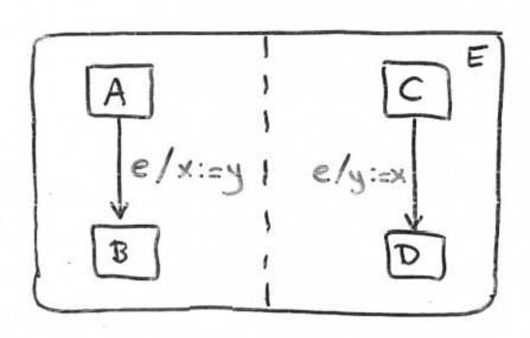

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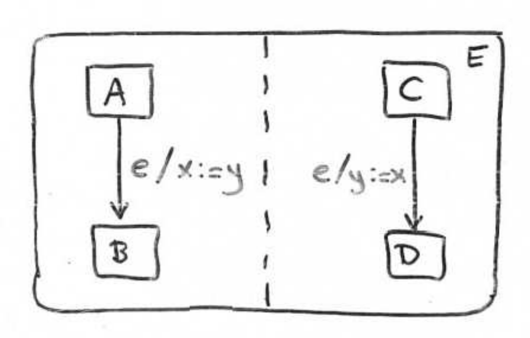
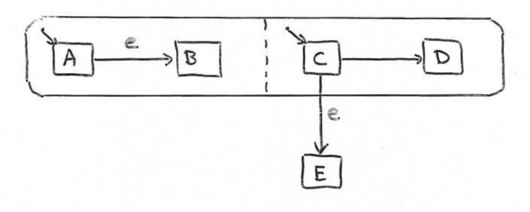


- If nodes A and C are active, assume $x = 1$, $y = 2$
- On occurrence of event e, B and D are active, and $x = 2$, $y = 1$

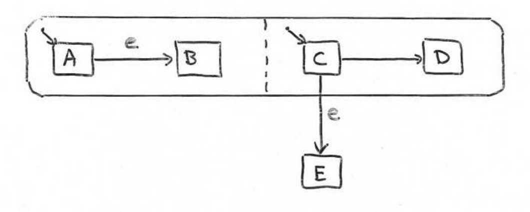
⇒ In Harel's statecharts, memory is shared, i.e., concurrent components have access to shared variables.

What if event e occurs when A and C are active?



Priority

What if event e occurs when A and C are active?



Solution:

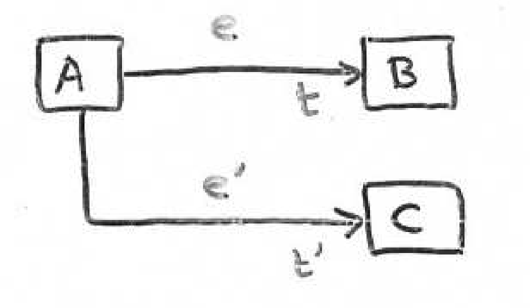
Add a priority mechanism that decides whether:

- inter-level transitions (such as $C \rightarrow E$), or
- intra-level transitions (such as $A \rightarrow B$)

prevail in case both are enabled.

Nondeterminism

What if event e and e' occur in A?



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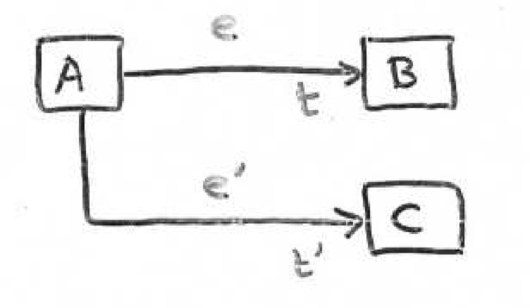
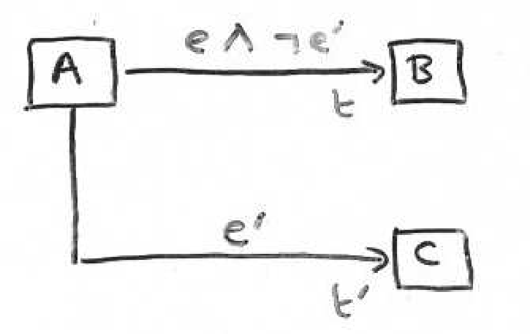


Solution:

Choice is resolved nondeterministically, i.e., the next state is either B or C, but not both.

Negation of events

Priority of events by negated events



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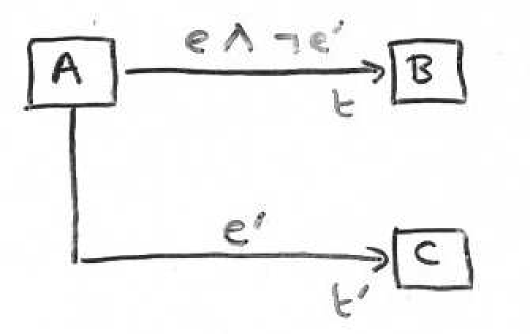

Note:

In UML statecharts, negated events do not occur

Overview

Semantic problems with Statecharts

- Synchrony hypothesis (or: zero response time)
- Self-triggering
- Negated trigger events
- Transition effect is contradicting its cause
- Interrupts

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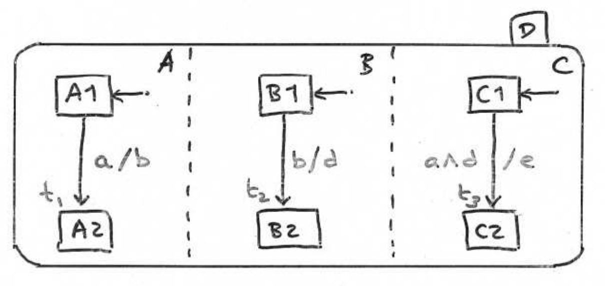
Note: [von der Beeck, 1994]

Due to all these problems, hundred(s) (!) of different semantics for Statecharts have been defined in the literature.

Synchrony hypothesis

Event may yield chain of reactions



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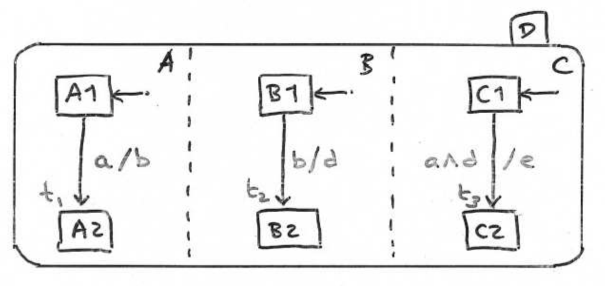

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- If $A1$, $B1$ and $C1$ are active and event a occurs, a chain of reactions occurs: transition t_1 triggers t_2, and t_2 triggers t_3

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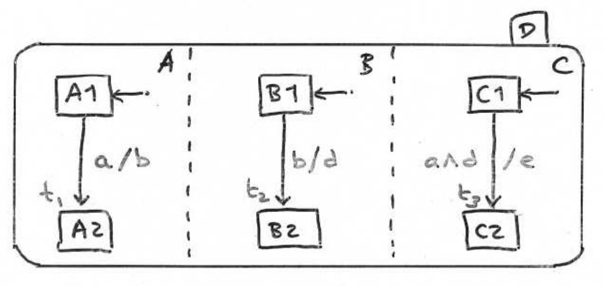

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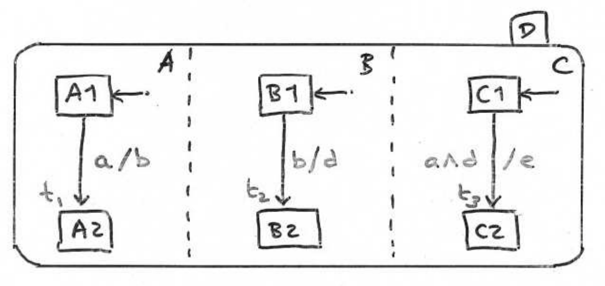

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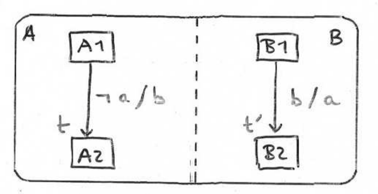
Paradox

Negated events and synchrony may yield paradox



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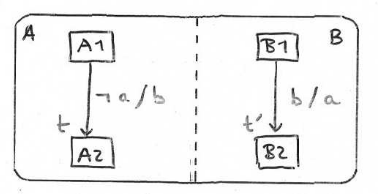

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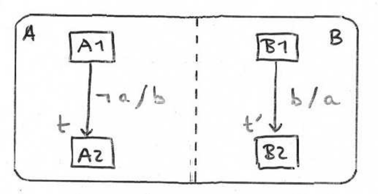

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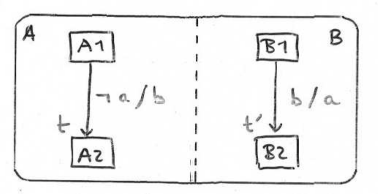

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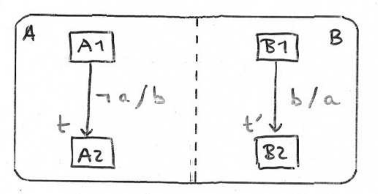

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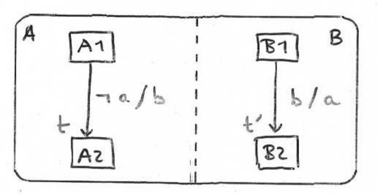

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- Transition t' can be taken, generating event a
- But then t should not have taken place as it is not enabled
- But then t' cannot be taken since b does not occur
- Hence, a does not occur and t cannot be taken

Simplifications in UML statecharts

1. No shared variables
2. No negated and no compound events (like $e \wedge e'$)
3. Two-party communication rather than broadcast
4. No synchrony hypothesis:
   - events generated in step i can only be consumed in step i+1,
   - and die otherwise, i.e., when they are not consumed in step i+1, events disappear

Overview

Definition (Statecharts)

A statechart SC is a triple $(N, E, Edges)$ with:

1. N is a set of nodes (or: states) structured in a tree
2. E is a set of events
    - pseudo-event $after(d)$ denotes a delay of $d \in \mathbb{R}_{\geqslant 0}$ time units
    - $\perp \notin E$ stands for "no event available"
3. $Edges$ is a set of (hyper-) edges, defined later on.

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Definition (System)

A system is described by a finite collection of statecharts $(SC_1, \ldots, SC_k)$.

Syntactic sugar

this is an elementary form; the UML allows more constructs
that can be defined in terms of these basic elements

- Deferred events — simulate by regeneration
- Parametrised events — simulate by set of parameter-less events
- Activities that take time — simulate by start and end event
- Dynamic choice points — simulate by intermediate state
- Synchronization states — use a hyperedge with a counter
- History states — (re)define an entry point

Tree structure

Function *children*

Nodes obey a tree structure defined by function $children : N \to 2^N$ where $x \in children(y)$ means that x is a child of y, or equivalently, y is the parent of x.

Partial order $\trianglelefteq$

The partial order $\trianglelefteq \subseteq N \times N$ is defined by:

- $\forall x \in N.\, x \trianglelefteq x$
- $\forall x, y \in N.\, x \trianglelefteq y$ if $x \in children(y)$
- $\forall x, y, z \in N.\, x \trianglelefteq y \wedge y \trianglelefteq z \Rightarrow x \trianglelefteq z$

$x \trianglelefteq y$ means that x is a descendant of y, or equivalently, y is an ancestor of x. If $x \trianglelefteq y$ or $y \trianglelefteq x$, nodes x and y are ancestrally related.

Root node

There is a unique root with no ancestors, and $\forall x \in N.\, x \trianglelefteq \text{root}$.

Functions on nodes

The type of nodes

Nodes are typed, $type(x) \in \{ \text{BASIC}, \text{AND}, \text{OR} \}$ such that for $x \in N$:

- $type(\text{root}) = \text{OR}$
- $type(x) = \text{BASIC}$ iff $children(x) = \varnothing$, i.e., x is a leaf
- $type(x) = \text{AND}$ implies $(\forall y \in children(x).\ type(y) = \text{OR})$

Default nodes

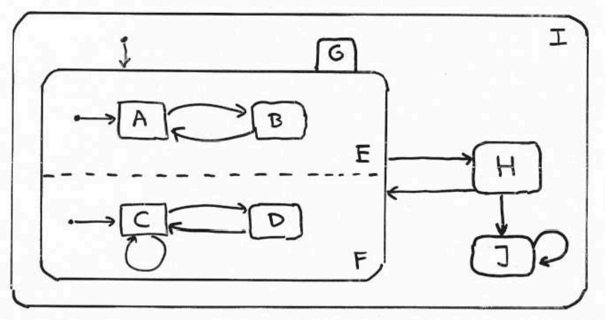
$default : N \to N$ is a partial function on domain $\{ x \in N \mid type(x) = \text{OR} \}$ such that

$$default(x) = y \quad \text{implies} \quad y \in children(x).$$

The function *default* assigns to each OR-node x one of its children as default node that becomes active once x becomes active.

Example

Example statechart



Edges

Definition (Edges)

An edge is a quintuple (X, e, g, A, Y), denoted $X \xrightarrow{e[g]/A} Y$ with:

- $X \subseteq N$ is a set of source nodes with $X \neq \varnothing$
- $e \in E \cup \{ \perp \}$ is the trigger event
- $A \subseteq Act$ is a set of actions
  - such as $v :=$ expr or local variable v and expression expr
  - or *send* $j.e$, i.e., send event e to statechart SC_j
- Guard g is a Boolean expression over all variables in $(SC_1, \ldots, SC_k)$
- $Y \subseteq N$ is a set of target nodes with $Y \neq \varnothing$

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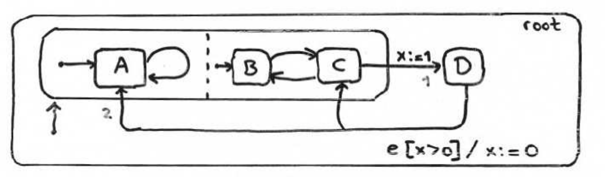
The sets X and Y may contain nodes at different depth in the node tree.

Example (1)

Example statechart

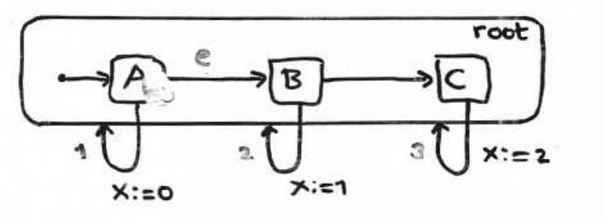


$$\text{edge 1: } \{ C \} \xrightarrow{\bot[true]/\{ x:=1 \}} \{ D \}$$

$$\text{edge 2: } \{ D \} \xrightarrow{e[x>0]/\{ x:=0 \}} \{ A, C \}$$

Example (2)

Example statechart

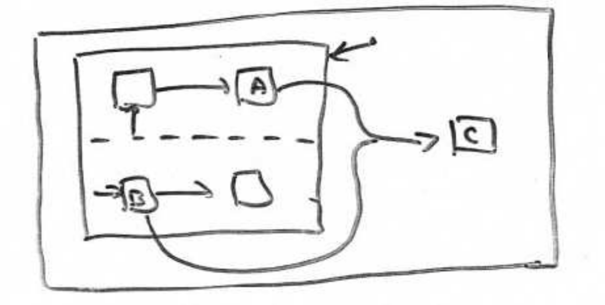


$$\text{edge 1: } \{ A \} \xrightarrow{e[true]/\varnothing} \{ B \}$$

$$\text{edge 2: } \{ B \} \xrightarrow{\bot[true]/\{ x{:=}1 \}} \{ \text{root} \}$$

Example (3)

Example statechart



$$\text{edge} : \{\, A, B \,\} \overset{\cdots}{\longrightarrow} \{\, C \,\}$$