Theoretical Foundations of the UML

Lecture 15+16: A Logic for MSCs

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moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

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RWTH AACHEN UNIVERSITY

Outline

Overview

1. Introduction
2. Local Formulas and Path Expressions
   - Syntax
   - Formal Semantics
3. PDL Formulas
4. Verification problems for PDL
   - Model checking MSCs
   - Model checking CFMs
   - Model checking MSGs
   - Satisfiability

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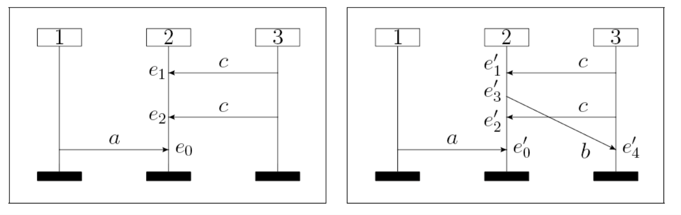
- Based on propositional dynamic logic (PDL) [Fischer & Ladner, 1979]
  - combines easy-to-grasp concepts such as regular expressions and Boolean operators
- Syntax, semantics, examples and various verification problems.

Some informal example properties



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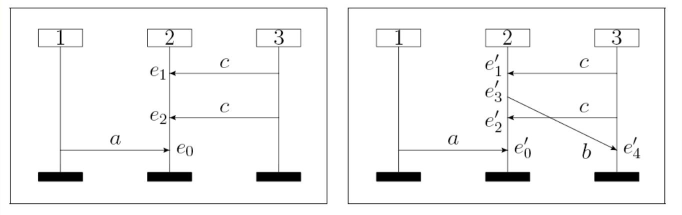

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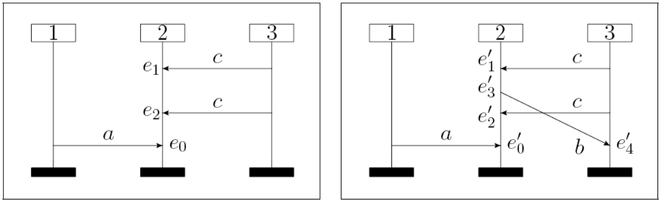

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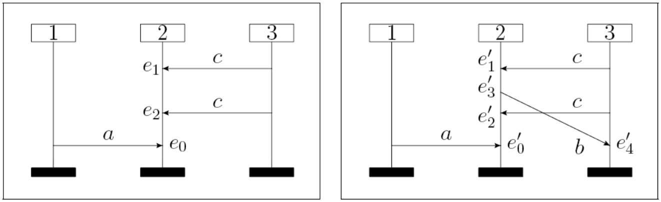

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4. The number of send events at process 3 is odd. No. No.

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- As formal language for properties we use logic.

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- Local formulas
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- PDL-formulas
  - Express properties about an entire MSC

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- $[\{\, \neg !(1,2,a)\, \}] \mathit{true}$ — An event is possible after any event different from $!(1,2,a)$

Local formulas

Definition (Syntax of local formulas)

For communication action $\sigma \in Act$ and path expression α, the grammar of local formulas is given by:

$$\varphi \; ::= \; true \; \mid \; \sigma \; \mid \; \neg\varphi \; \mid \; \varphi \vee \varphi \; \mid \; \langle\alpha\rangle\varphi \; \mid \; \langle\alpha\rangle^{-1}\varphi$$

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$$\begin{aligned}
false &:= \neg true \\
\varphi_1 \wedge \varphi_2 &:= \neg(\neg\varphi_1 \vee \neg\varphi_2) \\
\varphi_1 \rightarrow \varphi_2 &:= \neg\varphi_1 \vee \varphi_2 \\
[\alpha]\varphi &:= \neg\langle\alpha\rangle\neg\varphi \\
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\end{aligned}$$

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How are path expressions like α defined?

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$$\alpha \ ::= \ \{\varphi\} \ | \ \mathsf{proc} \ | \ \mathsf{msg} \ | \ \alpha;\alpha \ | \ \alpha+\alpha \ | \ \alpha^{*}$$

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  3. $(e, e') \models \mathsf{msg}$ iff e' is the matching event of e, i.e., $e' = m(e)$

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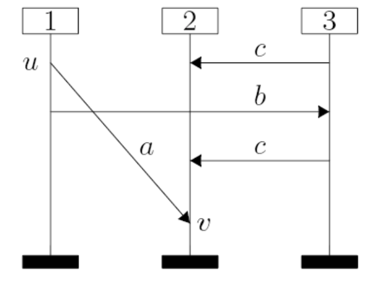
Example



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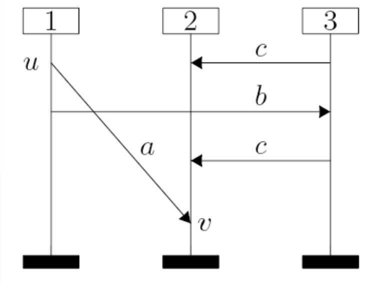

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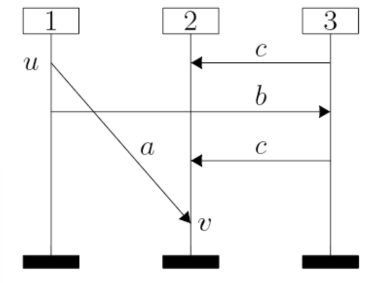

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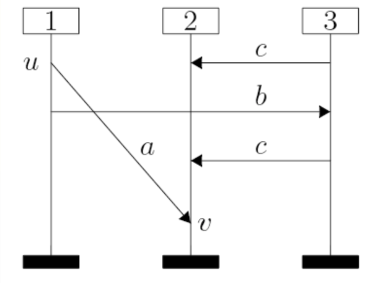

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4. $u \models \langle (\mathsf{proc} + \mathsf{msg})^* \rangle !(3,2,c)$ — event u happens before $!(3,2,c)$

Semantics of local formulas (1)

Definition (Syntax of local formulas)

For communication action $\sigma \in Act$ and path expression α:

$$\varphi \ ::= \ true \ \mid \ \sigma \ \mid \ \neg\varphi \ \mid \ \varphi \vee \varphi \ \mid \ \langle\alpha\rangle\varphi \ \mid \ \langle\alpha\rangle^{-1}\varphi$$

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Definition (Semantics of base local formulas)

Let $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ be an MSC and $e \in E$.

Binary relation $\models$ is defined such that $((M, e), \varphi) \in \models$ iff event e of MSC M satisfies local formula φ. We write $M, e \models \varphi$ for $((M, e), \varphi) \in \models$.

$$\begin{array}{rcl} M, e \models true & & \text{for all } e \in E \\ M, e \models \sigma & \text{iff} & l(e) = \sigma \\ M, e \models \neg\varphi & \text{iff} & \text{not } M, e \models \varphi \\ M, e \models \varphi_1 \vee \varphi_2 & \text{iff} & M, e \models \varphi_1 \text{ or } M, e \models \varphi_2 \end{array}$$

Semantics of local formulas (2)

Definition (Semantics of forward path formulas)

Let $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ be an MSC and $e \in E$.

$$
\begin{array}{rcl}
e \models \langle\{\psi\}\rangle\varphi & \text{iff} & e \models \psi \text{ and } e \models \varphi \\
e \models \langle\mathsf{proc}\rangle\varphi & \text{iff} & \exists e' \in E.\, e \lessdot_p e' \text{ and } e' \models \varphi \\
e \models \langle\mathsf{msg}\rangle\varphi & \text{iff} & \exists e' \in E.\, e' = m(e) \text{ and } e' \models \varphi \\
e \models \langle\alpha_1;\alpha_2\rangle\varphi & \text{iff} & e \models \langle\alpha_1\rangle\langle\alpha_2\rangle\varphi \\
e \models \langle\alpha_1 + \alpha_2\rangle\varphi & \text{iff} & e \models \langle\alpha_1\rangle\varphi \text{ or } e \models \langle\alpha_2\rangle\varphi \\
e \models \langle\alpha^*\rangle\varphi & \text{iff} & \exists n \in \mathbb{N}.\, e \models (\langle\alpha\rangle)^n\,\varphi
\end{array}
$$

Where $e \lessdot_p e'$ iff $e <_p e'$ and $\neg(\exists e''.\, e <_p e'' <_p e')$, i.e., e' is a direct successor of e under $<_p$.

Semantics of local formulas (3)

Definition (Semantics of backward path formulas)

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e \models \langle \mathsf{msg} \rangle^{-1} \varphi & \text{iff} & \exists e' \in E.\, e' = m^{-1}(e) \text{ and } e' \models \varphi \\
e \models \langle \alpha_1 ; \alpha_2 \rangle^{-1} \varphi & \text{iff} & e \models \langle \alpha_1 \rangle^{-1} \langle \alpha_2 \rangle^{-1} \varphi \\
e \models \langle \alpha_1 + \alpha_2 \rangle^{-1} \varphi & \text{iff} & e \models \langle \alpha_1 \rangle^{-1} \varphi \text{ or } e \models \langle \alpha_2 \rangle^{-1} \varphi \\
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\end{array}
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Overview

PDL formulas

Definition (Syntax of PDL formulas)

For local formula φ, the grammar of PDL formulas is given by:

$$\Phi \ ::= \ \exists \varphi \ \mid \ \forall \varphi \ \mid \ \Phi \wedge \Phi \ \mid \ \Phi \vee \Phi$$

Negation

Negation is absent. As existential and universal quantification, as well as conjunction and disjunction are present, PDF-formulas are closed under negation.

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- MSC M satisfies $\exists\langle\alpha\rangle\varphi$ if from some event e in M, there exists an α-labelled path from e to an event e', say, satisfying φ

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- MSC M satisfies $\exists[\alpha]\varphi$ if from some event e in M, every event that can be reached via an α-labelled path satisfies φ

Semantics of PDL formulas

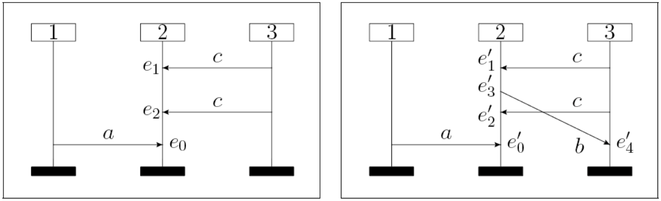
Definition (Semantics of PDL formulas)

Let $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ be an MSC.

$(M, \Phi) \in \models$ iff PDL formula Φ holds in MSC M.

$$
\begin{aligned}
M &\models \exists\varphi && \text{iff} && \exists e \in E.\, M, e \models \varphi \\
M &\models \forall\varphi && \text{iff} && \forall e \in E.\, M, e \models \varphi \\
M &\models \Phi_1 \wedge \Phi_2 && \text{iff} && M \models \Phi_1 \text{ and } M \models \Phi_2 \\
M &\models \Phi_1 \vee \Phi_2 && \text{iff} && M \models \Phi_1 \text{ or } M \models \Phi_2
\end{aligned}
$$

Example (1)



- The (unique) maximal event of M is labeled by $?(2, 1, a)$ Yes. No.

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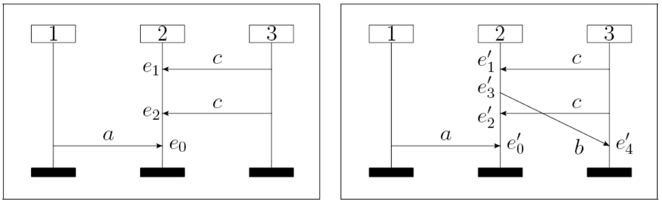
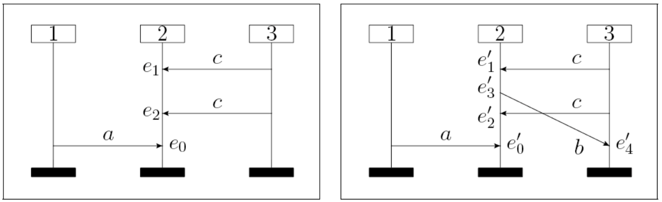
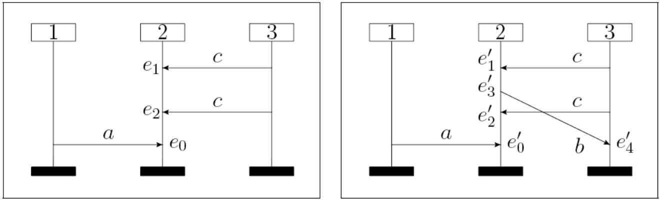


- The (unique) maximal event of M is labeled by $?(2, 1, a)$ Yes. No.
- $\forall\,(\langle(\mathsf{proc} + \mathsf{msg})^*\rangle([\mathsf{proc}]\,\mathit{false} \wedge ?(2, 1, a)))$ Yes. No.



- The maximal event on process 2 is labeled by $?(2, 1, a)$ Yes. Yes.

Example (2)



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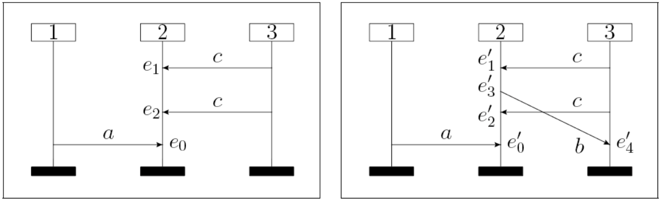
Example (3)



- No two consecutive events are labeled with $?(2, 3, c)$ No. Yes.

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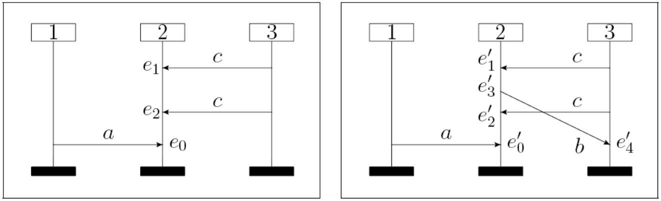
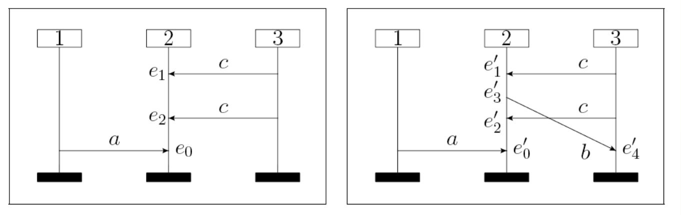


- No two consecutive events are labeled with $?(2, 3, c)$ No. Yes.
- $\forall\,([\{\,?(2,3,c)\,\}; \mathsf{proc}; \{\,?(2,3,c)\,\}]\,\mathit{false})$ No. Yes.

Example (4)



- The number of send events at process 3 is odd. No. No.

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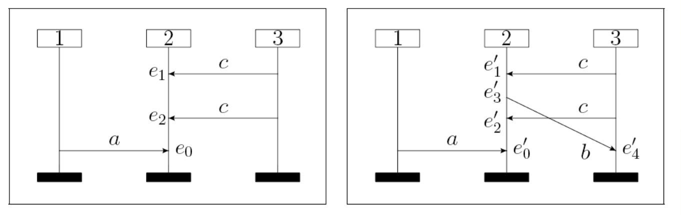

- The number of send events at process 3 is odd. No. No.

- See next slide

Example

MSC M has an even number of messages sent from process 1 to 2:

$$\forall\Big(\underbrace{[\mathsf{proc}]^{-1} \mathit{false} \wedge P_1}_{\text{minimal event on process 1}} \rightarrow \langle\alpha\rangle \underbrace{[\mathsf{proc}]\, \mathit{false}}_{\text{maximal event on process}} \Big)$$

where $P_1 = \bigvee_{j \in \mathcal{P}, j \neq 1} (!_{1,j} \vee ?_{1,j})$ with $!_{1,j} = \bigvee_{a \in \mathcal{C}} !(1, j, a)$ and $?_{1,j}$ is defined in a similar way, i.e., $e \models P_1$ iff e occurs at process 1.

Path expression α is defined by:

$$\alpha \;=\; ((\{\neg !_1\}; \mathsf{proc})^*; \{!_1\}; \mathsf{proc}; (\{\neg !_1\}; \mathsf{proc})^*; \{!_1\}; \mathsf{proc}; (\{\neg !_1\}; \mathsf{proc})^*)^*$$

and where $!_1$ abbreviates $\bigvee_{a \in \mathcal{C}} !(1, 2, a)$

Overview

Model checking MSCs

Model checking MSCs versus PDL [Kern, 2009]

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The following model-checking problem is decidable in polynomial time:

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PDL model checking algorithm for MSCs (1)

LOCAL FORMULA CHECK:

```
V = {0, .. , n-1}

boolean[] Sat(LocalFormula f) {
    boolean[] sat = new boolean[n];
    switch(f) {
    case Not(f1):
        boolean[] sat1 = Sat(f1);
        for (int i = 0; i < n; i++)
            sat[i] = !sat1[i];
        break;
    case Or(f1, f2):
        boolean[] sat1 = Sat(f1);
        boolean[] sat2 = Sat(f2);
        for (int i = 0; i < n; i++)
            sat[i] = sat1[i] || sat2[i];
        break;
    case Event(..):
        for (int i = 0; i < n; i++)
            sat[i] = (V[i].event.equals(f));
        break;
```

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PDL model checking algorithm for MSCs (2)

```
    case <p1> f2:
        boolean[][] trans1 = Trans(p1);
        boolean[] sat2 = Sat(f2);
        for (int i = 0; i < n; i++) {
            sat[i] = false;
            for (int j = 0; j < n; j++)
                if(trans[i][j])
                sat[i] = sat2[j];
        }
        break;
    case <p1>^-1 f2:
        boolean[][] trans1 = TransBack(p1);
        boolean[] sat2 = Sat(f2);
        for (int i = 0; i < n; i++) {
            sat[i] = false;
            for (int j = 0; j < n; j++)
                if(trans[i][j])
                sat[i] = sat2[j];
        }
        break;
    }
}
```

PDL model checking algorithm for MSCs (3)

FORWARD PATH EXPRESSION CHECK:

```
boolean[][] Trans(PathFormula p) {
    boolean[][] trans = new boolean[n][n];
    switch(p) {
    case (p1; p2):
        boolean[][] trans1 = Trans(p1);
        boolean[][] trans2 = Trans(p2);
        for (int i = 0; i < n; i++)
            for (int k = 0; k < n; k++) {
                trans[i][k] = false;
                for (int j = 0; j < n; j++)
                    if(trans1[i][j] && trans1[j][k])
                        trans[i][k] = true;
            }
        break;
    case p1 + p2:
        boolean[][] trans1 = Trans(p1);
        boolean[][] trans2 = Trans(p2);
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                trans[i][j] = trans1[i][j] || trans2[i][j];
        break;
```

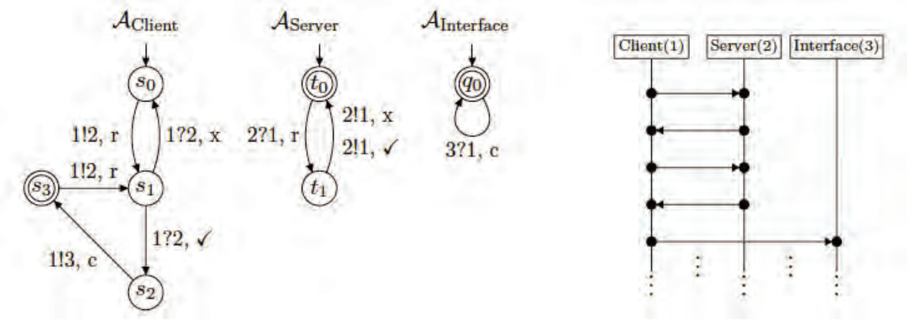
PDL model checking algorithm for MSCs (4)

```
    case p1*:
        boolean[][] trans1 = Trans(p1);
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                star[i][j] = (i==j);
        while (true) {
            for (int i = 0; i < n; i++)
                for (int j = 0; j < n; j++)
                    if (trans1[i][j])
                        for (int k = 0; k < n; k++)
                            if (!trans[i][k] && trans1[j][k]) {
                                trans[i][k] = true;
                                continue;
                            }
            break;
        }
        break;
    }
}
```

Communication finite-state machines

Let a CFM now be accepting if all its processes have reached a local accepting state and either halt there or visit a local accepting state infinitely often.

An example CFM and an infinite MSC accepted by it



Client-server interaction to get access to an interface. Accepting state is (s_3, t_0, q_0).

PDL formulas on CFMs

A CFM is accepting if all its processes have reached a local accepting state and reside their ad infinitum.

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CFM versus PDL

A CFM $\mathcal{A}$ satisfies PDL-formula Φ, denoted $\mathcal{A} \models \Phi$, whenever for all MSCs M it holds: $M \in \mathcal{L}(\mathcal{A})$ if and only if $M \models \Phi$.

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The example CFM satisfies $\forall \, (P_1 \rightarrow (\langle \mathsf{proc}^*; \mathsf{msg}; \mathsf{proc}^*; \mathsf{msg} \rangle \, P_3)$ where for $i \in \mathcal{P}$, formula $P_i = \bigvee_{j \in \mathcal{P}, j \neq i} (!_{i,j} \vee ?_{i,j})$, i.e., $M, e \models P_i$ iff e occurs at process i. The PDL formula asserts that process 3 (Interface) can be "reached" from 1 (Client) by exactly two messages using an intermediate process in between.

PDL model checking problem

Model checking CFMs versus PDL

The following model-checking problem is undecidable:

INPUT: a CFM $\mathcal{A}$, PDL-formula Φ

OUTPUT: is there an MSC $M \in \mathcal{L}(\mathcal{A})$ with $M \models \Phi$?

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Follows immediately from the fact that the emptiness problem for CFMs is undecidable. By using the formula *true*, the above problem encodes the emptiness problem. □

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To obtain decidable model-checking problems, we consider B-bounded MSCs.

Model checking CFMs versus PDL

Model checking CFMs versus PDL [Bollig *et. al*, 2011]

The following model-checking problem is PSPACE-complete:

INPUT: a CFM $\mathcal{A}$ and $B \in \mathbb{N}_{>0}$, PDL-formula Φ

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(Sketch). Every PDL formula Φ can effectively be translated into a CFM $\mathcal{A}_\Phi$ such that $\mathcal{A}_\Phi \models \Phi$.

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Construct a finite automaton $\mathcal{A}_G$ that accepts a linearization for every $M \in \mathcal{L}(G)$, and vice versa, each word accepted by $\mathcal{A}_G$ is a linearization of some $M \in \mathcal{L}(G)$.

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(Sketch.) For every vertex v, we can determine a linearization of the MSC $\lambda(v)$. Construct a finite automaton $\mathcal{A}_G$ that accepts a linearization for every $M \in \mathcal{L}(G)$, and vice versa, each word accepted by $\mathcal{A}_G$ is a linearization of some $M \in \mathcal{L}(G)$. The size of $\mathcal{A}_G$ is linear in the size of G.

Model checking MSGs versus PDL

Model checking MSGs versus PDL [Bollig et. al, 2011]

The following model-checking problem is PSPACE-complete:

INPUT: a MSG G and PDL-formula Φ

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Satisfiability problem for MSCs

Model checking MSCs versus PDL [Kern, 2009]

The following model-checking problem is decidable in polynomial time:

INPUT: MSC M, PDL-formula Φ

OUTPUT: does $M \models \Phi$?

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OUTPUT: does $M \models \Phi$?

MSC satisfiability for PDL [Bollig *et. al*, 2011]

The following satisfiability problem is undecidable:

INPUT: PDL-formula Φ

OUTPUT: is there an MSC M with $M \models \Phi$?

Other PDL decision problems

Theorem: [Alur *et al.*, 2001, Bollig *et al.*, 2007]

Let Φ be a PDL formula. Then:

1. The decision problem "does there exist a CFM $\mathcal{A}$ such that for any MSC $M \in \mathcal{L}(\mathcal{A})$ we have $M \models \Phi$" is undecidable.

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3. The decision problem "for MSG G, is there an MSC $M \in \mathcal{L}(G)$ such that $M \models \Phi$" is NP-complete.