Theoretical Foundations of the UML

Lecture 14: Realising Local Choice MSGs

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moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

20. Juni 2016

Outline

1. Introduction
2. Local Choice MSGs
3. Regular Expressions over MSCs
4. A Realisation Algorithm for MSGs

Overview

1. Introduction
2. Local Choice MSGs
3. Regular Expressions over MSCs
4. A Realisation Algorithm for MSGs

Safe realisability

Definition (Realisability of MSGs)

1. MSG G is realisable whenever $\mathcal{L}(G) = \mathcal{L}(\mathcal{A})$ for some CFM $\mathcal{A}$.
2. MSG G is safely realisable whenever $\mathcal{L}(G) = \mathcal{L}(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.

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3. Regular MSGs are (safely) realisable by ∀-bounded CFMs.
4. Checking regularity of MSGs is undecidable.
5. Communication-closedness implies regularity; its check is co-NP complete.
6. Local communication-closedness implies regularity, and can be checked in P.

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- What happens if we allow synchronisation messages?
  - recall that weak CFMs do not involve synchronisation messages
- How do we obtain a CFM realising an MSG algorithmically?
  - in particular, for non-local choice MSGs

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Results:

1. Realisability for constrained regular expressions of local-choice MSGs.
2. An algorithm that generates a CFM from such local-choice MSG.

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Non-local choice

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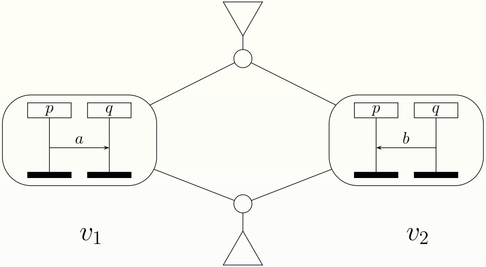

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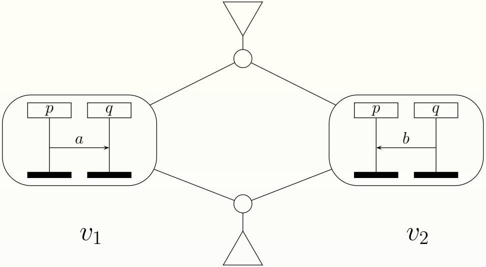

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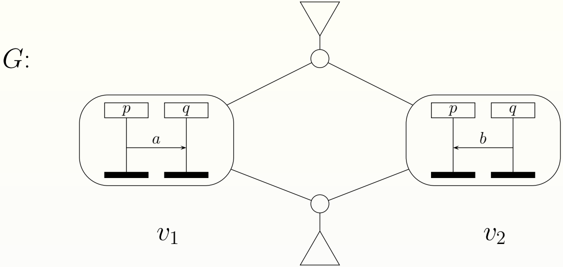

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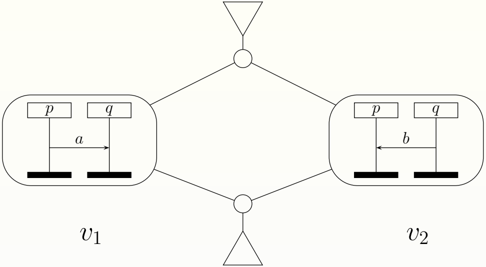

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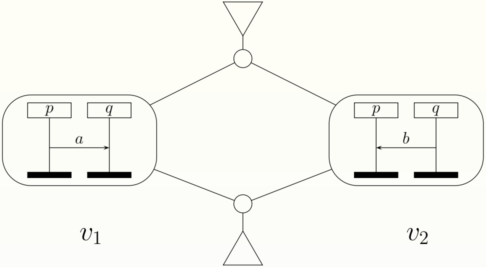

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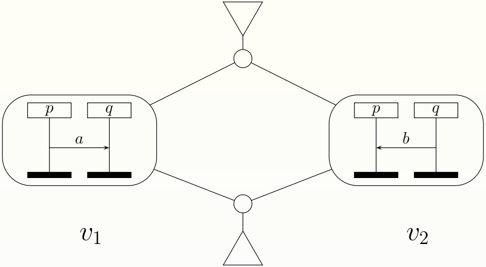

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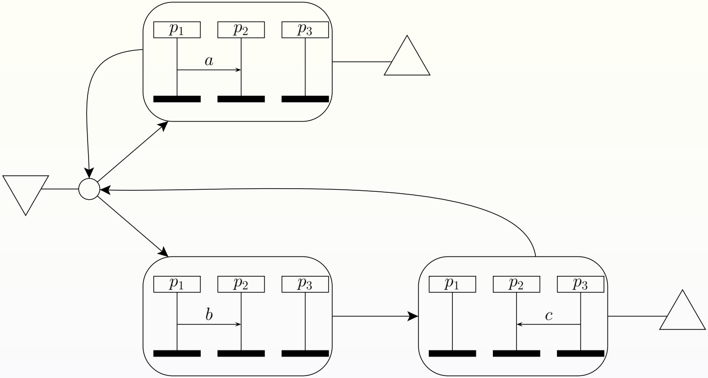
A (more involved) non-local choice



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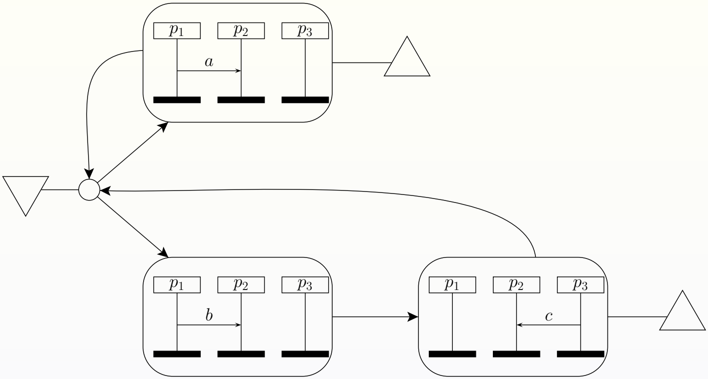

Problem:

Inconsistency if p_1 decides to send a and p_3 decides to send c.
Which branch to take in the initial vertex?

Definition (Minimal event)

Let $(E, \preceq)$ be a poset. Event $e \in E$ is a minimal event wrt. $\preceq$ if $\neg(\exists e' \neq e.\ e' \preceq e)$.

Preliminaries

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Definition (Partial order of a path)

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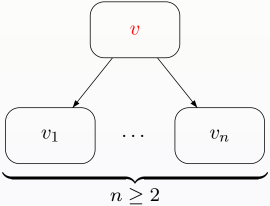
Let $\min(\pi)$ be the set of minimal events wrt. $<_{M(\pi)}$ along finite path π.

Branching vertices

A branching vertex in MSG G either has at least two successors, or is a final vertex with at least one successor.

Pictorially, vertex v is branching if either:



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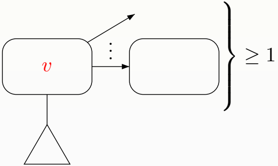

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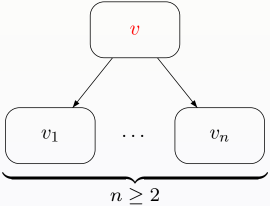

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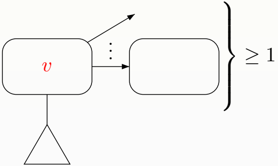

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Deciding whether MSG G is local choice or not is in P.

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Local choice

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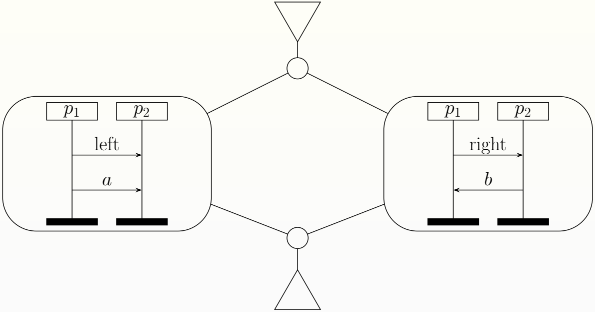

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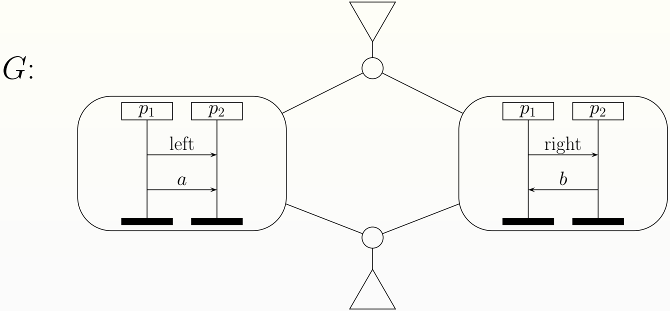

How to resolve a non-local choice?

Amend your MSG and add control messages (cf. above example)

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Regular expressions over MSCs

Definition (Asynchronous iteration)

For $\mathcal{M}_1, \mathcal{M}_2 \subseteq \mathbb{M}$ sets of MSCs, let:

$$\mathcal{M}_1 \bullet \mathcal{M}_2 \;=\; \{\, M_1 \bullet M_2 \mid M_1 \in \mathcal{M}_1, M_2 \in \mathcal{M}_2 \,\}$$

For $\mathcal{M} \subseteq \mathbb{M}$ let

$$\mathcal{M}^i \;=\; \begin{cases} \{M_\epsilon\} & \text{if } i{=}0\text{, where } M_\epsilon \text{ denotes the empty MSC} \\ \mathcal{M} \bullet \mathcal{M}^{i-1} & \text{if } i > 0 \end{cases}$$

The asynchronous iteration of $\mathcal{M}$ is now defined by:

$$\mathcal{M}^* = \bigcup_{i \geqslant 0} \mathcal{M}^i.$$

Regular expressions over MSCs

Definition (Regular expressions over MSCs)

The set $\text{REX}_{\mathbb{M}}$ of regular expressions over $\mathbb{M}$ is given by the grammar:

$$\alpha ::= \varnothing \mid M \mid \alpha_1 \cdot \alpha_2 \mid \alpha_1 + \alpha_2 \mid \alpha^*$$

where MSC $M \in \mathbb{M}$.

Definition (Semantics of regular expressions, $\mathcal{L}(.) : \text{REX}_{\mathbb{M}} \to 2^{\mathbb{M}}$)

- $\mathcal{L}(\varnothing) = \varnothing$
- $\mathcal{L}(M) = \{ M \}$
- $\mathcal{L}(\alpha_1 \cdot \alpha_2) = \mathcal{L}(\alpha_1) \bullet \mathcal{L}(\alpha_2)$
- $\mathcal{L}(\alpha_1 + \alpha_2) = \mathcal{L}(\alpha_1) \cup \mathcal{L}(\alpha_2)$
- $\mathcal{L}(\alpha^*) = \mathcal{L}(\alpha)^*$

Locally accepting CFMs

Definition (Locally accepting CFM)

CFM $\mathcal{A} = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F)$ is locally accepting (la, for short) if

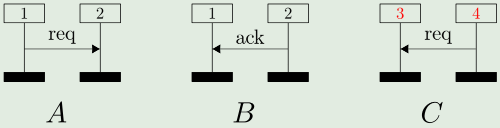
$$F \;=\; \prod_{p \in \mathcal{P}} F_p \quad \text{where} \quad F_p \subseteq S_p.$$

Thus: every combination of local accept states is a global accept state of the CFM.

Regular expressions for MSCs

Let $\mathcal{P} = \{1, 2, 3, 4\}$ and $\mathcal{C} = \{\text{req}, \text{ack}\}$.

Example



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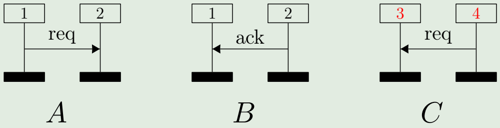

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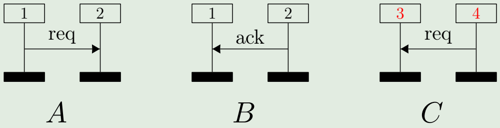

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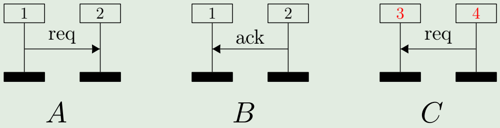

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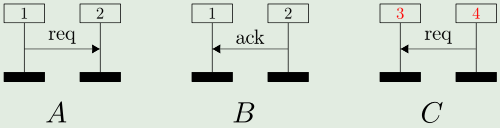

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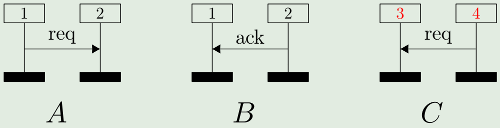

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Realising local-choice expressions by deadlock-free CFMs

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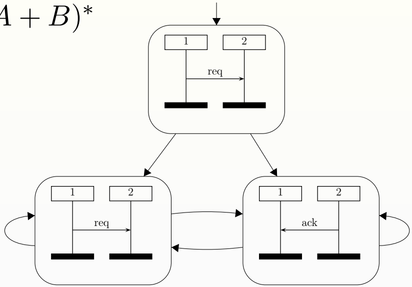

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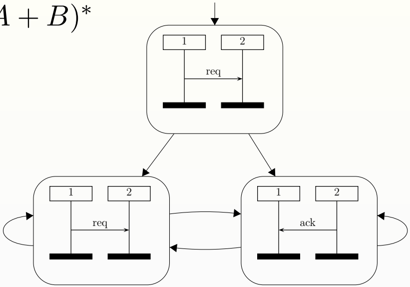

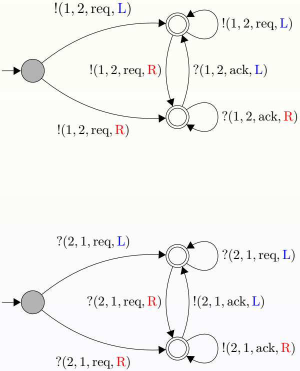

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Realising local-choice expressions by deadlock-free CFMs

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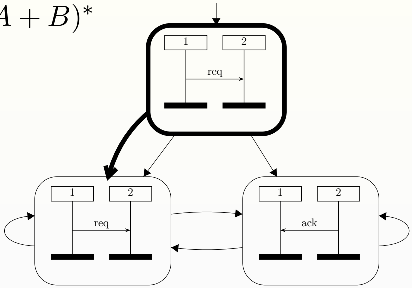

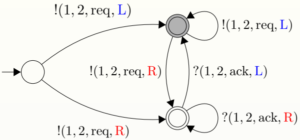

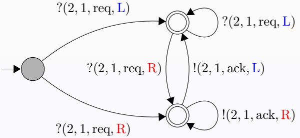

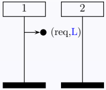

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Realising local-choice expressions by deadlock-free CFMs

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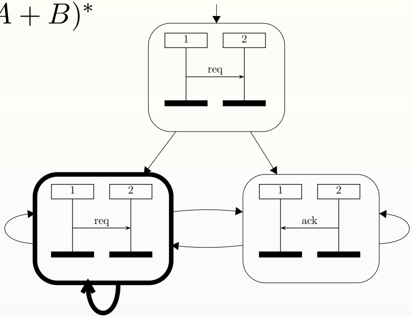

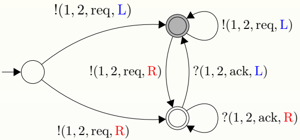

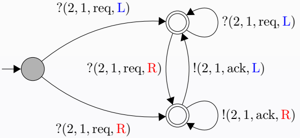

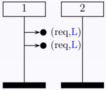

$1 \to 2$: (req,L) (req,L)
$2 \to 1$:

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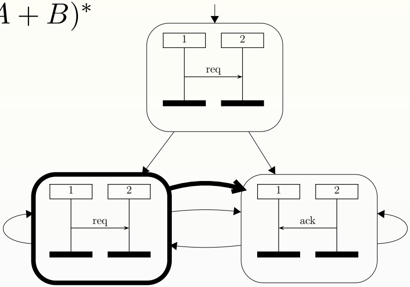

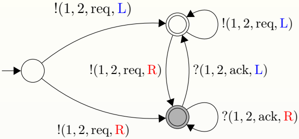

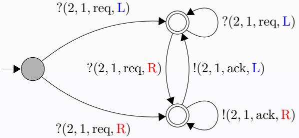

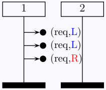

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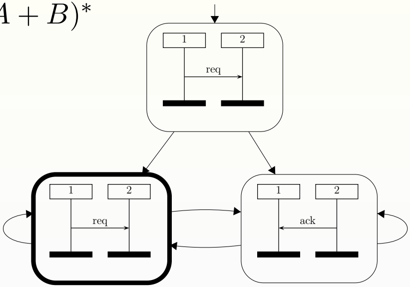

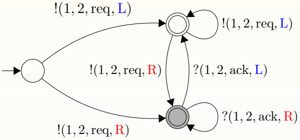

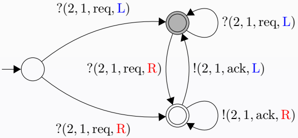

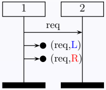

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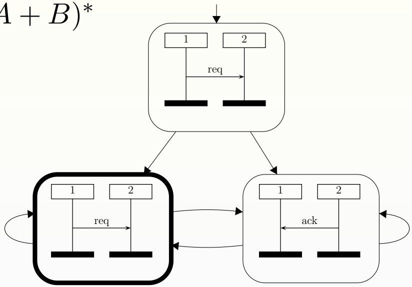

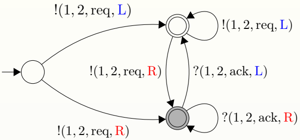

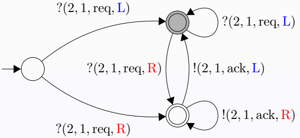

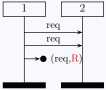

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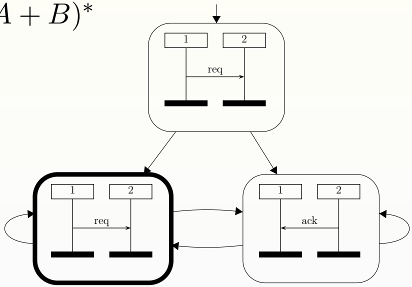

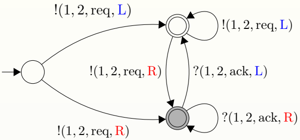

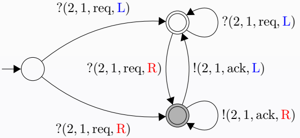

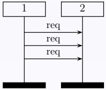

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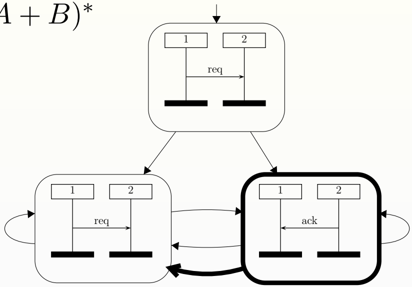

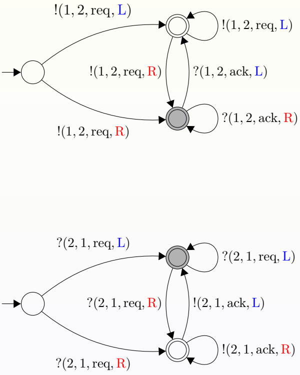

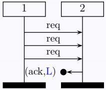

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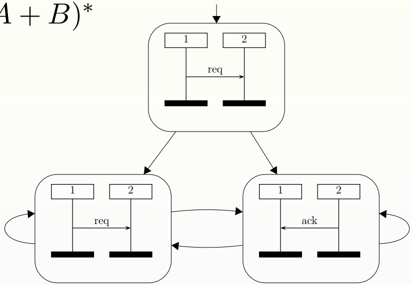

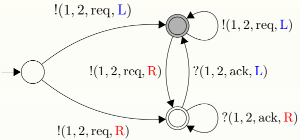

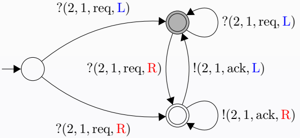

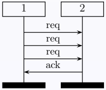

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Star-connected regular expressions

Definition (Connected MSC)

An MSC $M = (\mathcal{P}, E, \mathcal{C}, l, m, <) \in \mathbb{M}$ is connected if:

$$\forall e, e' \in E. \, (e, e') \in (< \cup <^{-1})^*$$

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Examples on the black board.

Regular expressions vs. CFMs

Definition (Finitely generated)

Set of MSCs $\mathcal{M} \subseteq \mathbb{M}$ is finitely generated if there is a finite set of MSCs $\widehat{\mathcal{M}} \subseteq \mathbb{M}$ such that $\mathcal{M} \subseteq \widehat{\mathcal{M}}^*$.

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Theorem [Morin 2002]

Let $\mathcal{M}$ be finitely generated. Then:

$\mathcal{M}$ is realisable

iff

there exists a star-connected regular expression α with $\mathcal{L}(\alpha) = \mathcal{M}$.

Overview

1. Introduction
2. Local Choice MSGs
3. Regular Expressions over MSCs
4. A Realisation Algorithm for MSGs

Local choice MSGs

An example local-choice MSG on black board.

Realising local choice (C)MSGs

Theorem [Genest *et al.*, 2005]

Any local-choice MSG G is safely realisable by a CFM with additional synchronisation data (which is of size linear in G).

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2. Process $p(v)$ informs all other processes about its decision by adding synchronisation data to the exchanged messages.
3. Synchronisation data is the path (in G) from v to the next branching vertex along the direction chosen by $p(v)$.

Maximal non-branching paths

Definition (Maximal non-branching paths)

For MSG $G = (V, \rightarrow, v_0, F, \lambda)$, let $nbp : V \rightarrow V^*$ be defined by:

$$nbp(v) = \begin{cases} v & \text{if } v \in F \text{ or } v \text{ is a branching vertex} \\ v_1 \ldots v_n & \text{otherwise} \end{cases}$$

where $v_1 \ldots v_n \in V^*$ is a maximal path (i.e., a path that cannot be prolonged) satisfying:

1. $v_i = v$ for some i, $0 < i \leqslant n$, and
2. $v_n \in F$ or is a branching vertex, and
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Intuition

$nbp(v)$ is the maximal non-branching path to which v belongs.

Structure of the CFM of local choice MSG G

Let MSG $G = (V, \rightarrow, v_0, F, \lambda)$ be local choice.

Define the CFM $\mathcal{A}_G = (((S_p, \Delta_p))_{p \in \mathcal{P}}, \mathbb{D}, s_{init}, F')$ with:

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3. $s_{init} = \{ \, (v_0, \varnothing) \, \}^n$ where $n = |\mathcal{P}|$
   each local automaton $\mathcal{A}_p$ starts in initial state $(v_0, \varnothing)$, i.e., in initial vertex v_0 while no events of p have been performed

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4. $\overline{s} \in F'$ iff for all $p \in \mathcal{P}$, local state $\overline{s}[p] = (v, E)$ with $E \subseteq E_p$ and:
   1. $v \in F$ and E contains a maximal event wrt. $<_p$ in MSC $\lambda(v)$, or
   2. $v \notin F$ and $\pi = v \ldots w$ is a path in G with $w \in F$ and E contains a maximal event wrt. $<_p$ in MSC $\lambda(\pi)$.

State space of local automaton $\mathcal{A}_p$

- $S_p = V \times E_p$ such that for any $s = (v, E) \in S_p$:

$$\forall e, e' \in \lambda(v). \left(e <_p e' \text{ and } e' \in E \quad \text{implies} \quad e \in E\right)$$

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- Initial state of $\mathcal{A}_p$ is $(v_0, \varnothing)$

Transition relation of local automaton $\mathcal{A}_p$

- Executing events within a vertex of the MSG G:

$$\frac{e \in E_p \cap \lambda(v) \text{ and } e \notin E}{(v, E) \xrightarrow{l(e), nbp(v)}_p (v, E \cup \{ e \})}$$

Note: since $E \cup \{e\}$ is downward-closed wrt. $<_p$, e is enabled

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- Taking an edge (possibly a self-loop) of the MSG G:

$$\frac{E = E_p \cap \lambda(v) \text{ and } e \in E_p \cap \lambda(w) \text{ and} \quad v u_0 \ldots u_n w \in V^* \text{ with } p \text{ not active in } u_0 \ldots u_n}{(v, E) \xrightarrow{l(e), nbp(w)}_p (w, \{e\})}$$

Note: vertex w is the first successor vertex of v on which p is active

Examples

A couple of examples on the black board.