# Theoretical Foundations of the UML

## Lecture 11: Safe Realisability


Joost-Pieter Katoen

Lehrstuhl für Informatik 2
Software Modeling and Verification Group


moves.rwth-aachen.de/teaching/ss-16/theoretical-foundations-of-the-uml/

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# Outline

# Overview

1. Safe realisability
2. Closure and inference revisited
3. Characterisation and complexity of safe realisability

# From requirements to implementation

## Realisability problem

INPUT: a set of MSCs

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2. If so, how complex is it to obtain such CFM?
3. If so, how do such algorithms work?

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## Results:

1. Conditions for realisability of a finite set of MSCs by a deadlock-free weak CFM.
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   (Realisability for weak CFMs that may deadlock is co-NP complete.)

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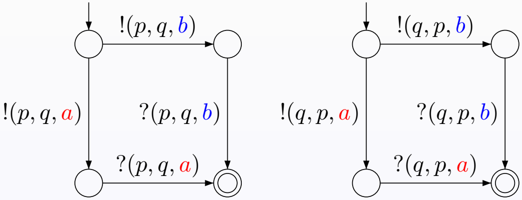


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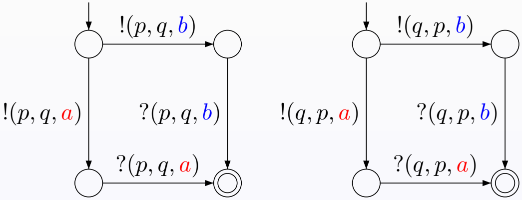


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Deadlock occurs when, e.g., $p$ sends $a$ and $q$ sends $b$

# Safe realisability

## Definition (Safe realisability)

1. MSC $M$ is safely realisable whenever $\{M\} = \mathcal{L}(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.
2. A finite set $\{M_1, \ldots, M_n\}$ of MSCs is safely realisable whenever $\{M_1, \ldots, M_n\} = \mathcal{L}(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.
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## Phrased using linearisations

$L \subseteq Act^*$ is safely realisable if $L = Lin(\mathcal{A})$ for some deadlock-free CFM $\mathcal{A}$.

## Note:

Safe realisability implies realisability, but the converse does not hold.

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**Definition (Inference relation and closure)**

For well-formed $L \subseteq Act^*$, and well-formed word $w \in Act^*$, let:

$$L \models w \quad \text{iff} \quad (\forall p \in \mathcal{P}.\, \exists v \in L.\, w \restriction p = v \restriction p)$$

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## Definition (Weak closure)

Language $L$ is weakly closed under $\models$ whenever for every well-formed prefix $w$ of some word in $L$, it holds $L \models w$ implies $w \in L$.

Weak closure thus restricts closure under $\models$ to well-formed prefixes in $L$ only. So far, closure was required for all $w \in Act^*$.

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## Definition ((Deadlock-free) Inference relation)

For well-formed $L \subseteq \mathit{Act}^*$, and proper word $w \in \mathit{Act}^*$, i.e., $w$ is a prefix of a well-formed word, let:

$$L \models^{\mathit{df}} w \quad \text{iff} \quad (\forall p \in \mathcal{P}.\, \exists v \in \mathit{pref}(L).\, w \upharpoonright p \text{ is a prefix of } v \upharpoonright p)$$

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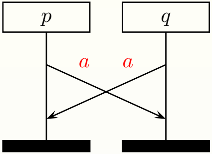
## Intuition

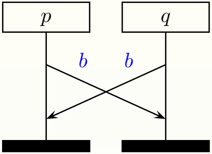
The closure condition asserts that the set of partial MSCs (i.e., prefixes of $L$) can be constructed from the projections of the MSCs in $L$ onto individual processes.

# Example

MSC $M_1$:




MSC $M_2$:



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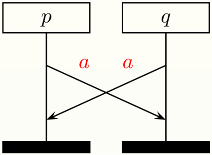


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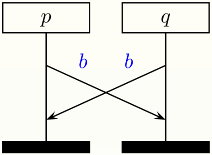


## Example

$L = Lin(\{M_1, M_2\})$ is not closed under $\models^{df}$:

$$w \ = !(p, q, a)!(q, p, b) \notin \ pref(L)$$

But: $L \models^{df} w$ since $w$ is a proper prefix of a well-formed word, and

- for process $p$, there exists $u \in L$ with $w \upharpoonright p = !(p, q, a) \in pref(\{u \upharpoonright p\})$, and
- for process $q$, there exists $v \in L$ with $w \upharpoonright q = !(q, p, b) \in pref(\{v \upharpoonright q\})$.

Note that $L$ is closed under $\models$. So this shows that closure under $\models$ does not imply closure under $\models^{df}$.

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**Lemma:**

For every deadlock-free weak CFM $\mathcal{A}$, $Lin(\mathcal{A})$ is closed under $\models^{df}$.

**Proof.**

Similar proof strategy as for the closure of weak CFMs under $\models$ (see previous lecture).

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# Characterisation of safe realisability

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**Corollary**

The finite set of MSCs $\{M_1, \ldots, M_n\}$ is safely realisable iff $\bigcup_{i=1}^{n} Lin(M_i)$ is closed under $\models$ and $\models^{df}$.

# Characterisation of safe realisability

**Theorem**

For any well-formed $L \subseteq Act^*$:

$$L \text{ is regular and closed under } \models$$
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**Proof**

1. For a given finite set of MSCs, safe realisability can be checked in time $\mathcal{O}((n^2 + r) \cdot k)$ where $k$ is the number of processes, $n$ the number of MSCs, and $r$ the number of events in all MSCs together.
2. If the MSCs are not safely realisable, the algorithm returns an MSC which is implied, but not included in the input set of MSCs.

(We skip the details in this lecture.)