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Theoretical Foundations of the UML SS 2016
— Series 1 —

Hand in until April 28 before the exercise class.

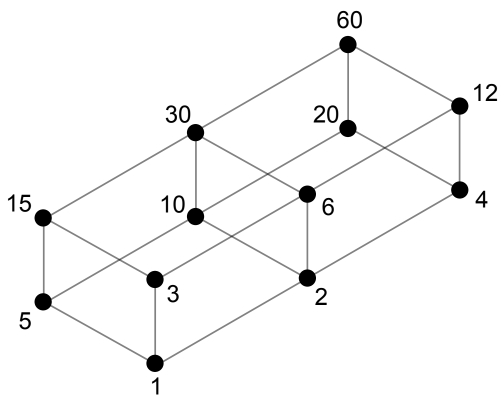
Exercise 1 (Partial Orders) (1+1 Points)

Let $\mathcal{P}(M)$ denote the powerset of a set M, i.e. $\mathcal{P}(M) = \{N \mid N \subseteq M\}$, and let $\mathbb{N}$ denote the set of natural numbers, i.e. $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$. Prove or disprove whether the following are partial orders:

1. $(\mathcal{P}(\mathbb{N}), \subseteq)$
2. $(\mathcal{P}(\mathbb{N}), \subsetneq)$
3. $(\mathbb{N}, \neq)$
4. $(\mathbb{N}, \mathcal{R})$ with $\mathcal{R} = \{(x, y) \mid x \text{ and } y \text{ are both even or both odd}\}$

Exercise 2 (Hasse Diagrams) (1.5+1.5 Points)

1. Draw the Hasse diagram for the partial order $(\mathcal{P}(\{1, 2, 3\}), \subseteq)$.
2. Formally provide the partial order for the following Hasse diagram[1]:



Exercise 3 (Message Sequence Charts) (2+3 Points)

1. Consider the following formal definition of an MSC: $M_1 = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$, with

$$\mathcal{P} = \{p_1, p_2, p_3\},$$

$$E = E_1 \cup E_2 \cup E_3 = E_? \cup E_!, \text{ with}$$

$$E_1 = \{e_1, e_6\},$$

$$E_2 = \{e_2, e_3\},$$

$$E_3 = \{e_4, e_5\},$$

$$E_? = \{e_2, e_4, e_6\}, \text{ and}$$

[1] Source: wikipedia.org

$E_! = \{e_1, e_3, e_5\}$,

$\mathcal{C} = \{a, b, c\}$,

$l\colon E \to Act$, with

$l(e_1) = !(p_1, p_2, a)$,

$l(e_2) = ?(p_2, p_1, a)$,

$l(e_3) = !(p_2, p_3, b)$,

$l(e_4) = ?(p_3, p_2, b)$,

$l(e_5) = !(p_3, p_1, c)$, and

$l(e_6) = ?(p_1, p_3, c)$,

$m\colon E_! \to E_?$, with

$m(e_1) = e_2$,

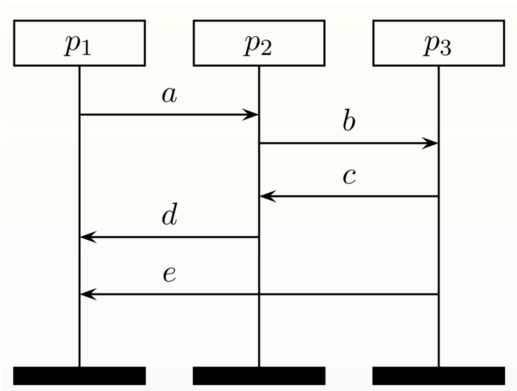
$m(e_3) = e_4$, and

$m(e_5) = e_6$, and

$\preceq \;= \big\{(e_1, e_6), (e_1, e_2), (e_2, e_3), (e_3, e_4), (e_4, e_5), (e_5, e_6)\big\}^*$.

Draw the visual representation of M_1, then provide $Lin(M_1)$.

2. Reconsider the following visual representation of an MSC M_2 from Lecture 2:



First provide names for the events, then provide the formal definition of M_2 in the form of $M_2 = (\mathcal{P}, E, \mathcal{C}, l, m, \preceq)$, with ...