Semantics and Verification of Software

Summer Semester 2015

Lecture 1: Introduction

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http://moves.rwth-aachen.de/teaching/ss-15/sv-sw/

Outline of Lecture 1

Preliminaries

Introduction

The Imperative Model Language WHILE

Preliminaries

Staff

- Lectures: Thomas Noll
  - Lehrstuhl für Informatik 2, Room 4211
  - E-mail `noll@cs.rwth-aachen.de`
- Exercise classes:
  - Christoph Matheja (`matheja@cs.rwth-aachen.de`)
  - Federico Olmedo (`federico.olmedo@cs.rwth-aachen.de`)
- Student assistants:
  - Frederick Prinz

Target Audience

- MSc Informatik:
  - Theoretische Informatik
- MSc Software Systems Engineering:
  - Theoretical Foundations of SSE

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- MSc Informatik:
  - Theoretische Informatik
- MSc Software Systems Engineering:
  - Theoretical Foundations of SSE
- In general:
  - interest in formal models for programming languages
  - application of mathematical reasoning methods
- Expected: basic knowledge in
  - essential concepts of imperative programming languages
  - formal languages and automata theory
  - mathematical logic

Organisation

- Schedule:
  - Lecture Tue 14:15–15:45 AH 2 (starting 14 April)
  - Lecture Thu 11:15–12:45 AH 2 (starting 9 April)
  - Exercise class Wed 15:00–16:30 AH 6 (starting 22 April)
- Irregular lecture dates – checkout web page!

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  - submission by 22 April
  - presentation on 29 April
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- Work on assignments in groups of three
- Examination (6 ECTS credits):
  - oral or written (depending on number of participants)
  - date to be fixed
- Admission requires at least 50% of the points in the exercises
- Written material in English, lecture and exercise classes "on demand", rest up to you

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Aspects of Programming Languages

Syntax: "How does a program look like?"

- hierarchical composition of programs from structural components

$\Rightarrow$ *Compiler Construction*

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Pragmatics:
- length and understandability of programs
- learnability of programming language
- appropriateness for specific applications, ...

$\Rightarrow$ *Software Engineering*

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Historic development:

- Formal syntax since 1960s (scanners, LL/LR parsers); semantics defined by compiler/interpreter
- Formal semantics since 1970s (operational/denotational/axiomatic)

Why Semantics?

Idea: compiler = ultimate semantics!

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But:

- Compilers are highly complicated software systems
  - code optimisations
  - memory management
  - interaction with runtime system
  - ...
- Most languages have more than one compiler (with different outputs)
- Most compilers have bugs

$\Rightarrow$ Does not help with formal reasoning about programming language or individual programs

The Semantics of “Semantics”

Originally: study of meaning of symbols (linguistics)

Semantics of a program: meaning of a concrete program

- mapping input $\rightarrow$ output values
- interaction behaviour (shared variables, communication, ...)
- ...

Semantics of a programming language: mapping of each (syntactically correct) program of a concrete programming language to its meaning

Semantics of software: various techniques for defining the semantics of diverse programming languages

- operational
- denotational
- axiomatic
- ...

Motivation for Rigorous Formal Treatment I

Example 1.1

1. How often will the following loop be traversed?

```
for i := 2 to 1 do ...
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FORTRAN IV: once
PASCAL: never

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2. What if `p = nil` in the following program?

```
while p <> nil and p^.key < val do ...
```

   Pascal: strict boolean operations $\lightning$
   Modula: non-strict boolean operations ✓

Motivation for Rigorous Formal Treatment II

- Support for development of
  - new programming languages: missing details, ambiguities and inconsistencies can be recognized
  - compilers: automatic compiler generation from appropriately defined semantics
  - programs: exact understanding of semantics avoids uncertainties in the implementation of algorithms

Motivation for Rigorous Formal Treatment II

- Support for development of
  - new programming languages: missing details, ambiguities and inconsistencies can be recognized
  - compilers: automatic compiler generation from appropriately defined semantics
  - programs: exact understanding of semantics avoids uncertainties in the implementation of algorithms
- Support for correctness proofs of
  - programs: comparison of program semantics with desired behavior (e.g., termination properties, absence of deadlocks, ...)
  - compilers:

$$\begin{tikzcd} \text{programming language} \arrow[r, "\text{compiler}"] \arrow[d, "\text{semantics}"'] & \text{machine code} \arrow[d, "\text{(simple) semantics}"] \\ \text{meaning} \arrow[r, phantom, "\stackrel{?}{=}"] & \text{meaning} \end{tikzcd}$$

  - optimizing transformations:

$$\begin{tikzcd} \text{code} \arrow[r, "\text{optimization}"] \arrow[d, "\text{semantics}"'] & \text{code} \arrow[d, "\text{semantics}"] \\ \text{meaning} \arrow[r, phantom, "\stackrel{?}{=}"] & \text{meaning} \end{tikzcd}$$

(Complementary) Kinds of Formal Semantics

Operational semantics: describes computation of the program on some (very) abstract machine (G. Plotkin)

- example: $\text{(seq)}\dfrac{\langle c_1, \sigma\rangle \to \sigma' \quad \langle c_2, \sigma'\rangle \to \sigma''}{\langle c_1 ; c_2, \sigma\rangle \to \sigma''}$
- application: implementation of programming languages (compilers, interpreters, ...)

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Denotational semantics: mathematical definition of input/output relation of the program by induction on its syntactic structure (D. Scott, C. Strachey)

- example: $\mathfrak{C}[\![.]\!] : \mathit{Cmd} \to (\Sigma \dashrightarrow \Sigma)$
  $\mathfrak{C}[\![c_1 ; c_2]\!] := \mathfrak{C}[\![c_2]\!] \circ \mathfrak{C}[\![c_1]\!]$
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- example: $\mathfrak{C}[\![.]\!] : Cmd \to (\Sigma \dashrightarrow \Sigma)$
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Axiomatic semantics: formalization of special properties of programs by logical formulae (assertions/proof rules; R. Floyd, T. Hoare)

- example: $\text{(seq)}\dfrac{\{A\}\, c_1\, \{C\} \quad \{C\}\, c_2\, \{B\}}{\{A\}\, c_1 ; c_2\, \{B\}}$
- application: program verification

Overview of the Course

1. The imperative model language WHILE
2. Operational semantics of WHILE
3. Denotational semantics of WHILE
4. Equivalence of operational and denotational semantics
5. Axiomatic semantics of WHILE
6. Extensions: procedures and dynamic data structures
7. Applications: compiler correctness etc.

Literature

(also see the collection [“Handapparat”] at the CS Library)

- Formal semantics
  - G. Winskel: *The Formal Semantics of Programming Languages*, The MIT Press, 1996
- Compiler correctness
  - H.R. Nielson, F. Nielson: *Semantics with Applications: An Appetizer*, Springer Undergraduate Topics in Computer Science, 2007

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Syntactic Categories

WHILE: simple imperative programming language without procedures or advanced data structures

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Syntactic categories:

Category	Domain	Meta variable
Numbers	$\mathbb{Z} = \{0, 1, -1, \ldots\}$	z
Truth values	$\mathbb{B} = \{\mathsf{true}, \mathsf{false}\}$	t
Variables	$Var = \{\mathrm{x}, \mathrm{y}, \ldots\}$	x
Arithmetic expressions	*AExp* (next slide)	a
Boolean expressions	*BExp* (next slide)	b
Commands (statements)	*Cmd* (next slide)	c

Syntax of WHILE Programs

Definition 1.2 (Syntax of WHILE)

The syntax of WHILE Programs is defined by the following context-free grammar:

$$a ::= z \mid x \mid a_1 \mathtt{+} a_2 \mid a_1 \mathtt{-} a_2 \mid a_1 \mathtt{*} a_2 \in AExp$$
$$b ::= t \mid a_1 \mathtt{=} a_2 \mid a_1 \mathtt{>} a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in BExp$$
$$c ::= \mathtt{skip} \mid x \mathtt{:=} a \mid c_1 \mathtt{;} c_2 \mid \mathtt{if}\ b\ \mathtt{then}\ c_1\ \mathtt{else}\ c_2\ \mathtt{end} \mid \mathtt{while}\ b\ \mathtt{do}\ c\ \mathtt{end} \in Cmd$$

Syntax of WHILE Programs

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The syntax of WHILE Programs is defined by the following context-free grammar:

$$a ::= z \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \in AExp$$
$$b ::= t \mid a_1 = a_2 \mid a_1 > a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in BExp$$
$$c ::= \texttt{skip} \mid x := a \mid c_1 ; c_2 \mid \texttt{if } b \texttt{ then } c_1 \texttt{ else } c_2 \texttt{ end} \mid \texttt{while } b \texttt{ do } c \texttt{ end} \in Cmd$$

Remarks: we assume that

- the syntax of numbers, truth values and variables is predefined (i.e., no “lexical analysis”)
- the syntactic interpretation of ambiguous constructs (expressions) is uniquely determined (by brackets or priorities)

A WHILE Program

Example 1.3

```
x := 6;
y := 7;
z := 0;
while x > 0 do
  x := x - 1;
  v := y;
  while v > 0 do
    v := v - 1;
    z := z + 1
  end
end
```

A WHILE Program and Its Flow Diagram

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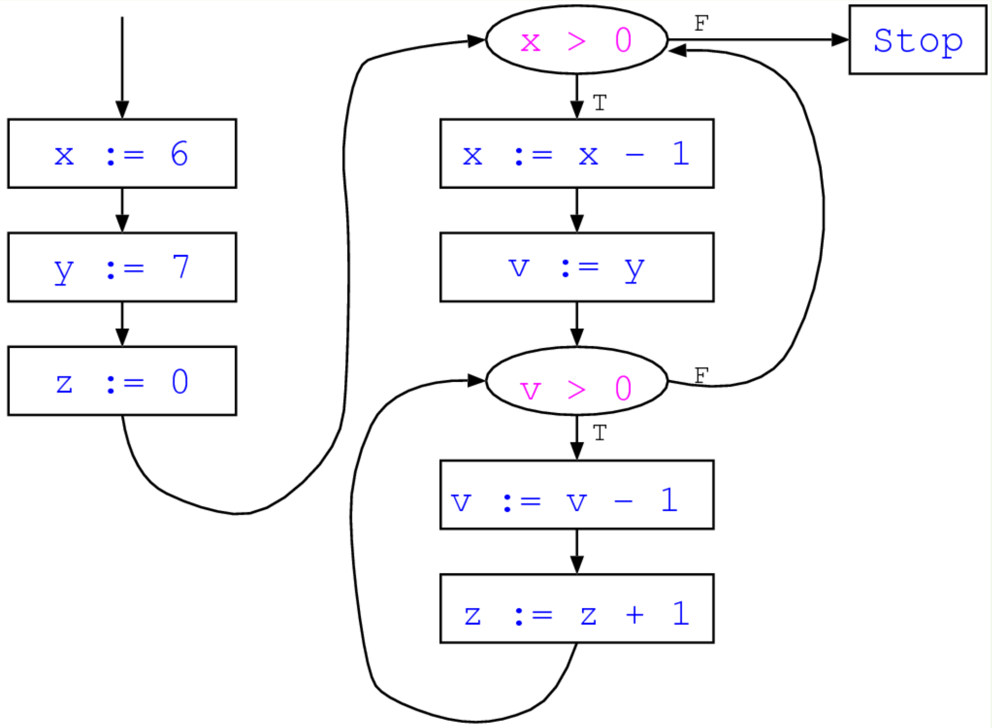

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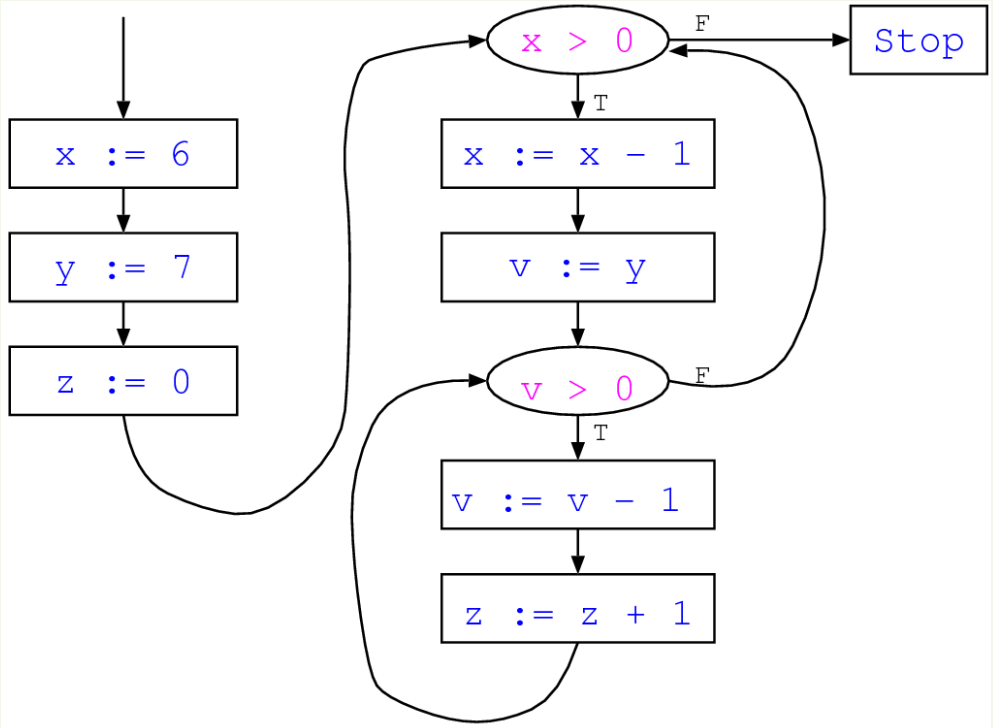

Effect: `z := x * y = 42`