Introduction to Model Checking 2015: Exercise 5.

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Exercise 1 *(1 Points)*

Consider the GNBA $G = (Q, \Sigma, \delta, Q_0, \mathcal{F})$:
$Q = q_0, q, q_2$, $\Sigma = \{A, B\}$, $\delta = \{(q_0, A, q_1), (q_1, B, q_1), (q_1, B, q_2), (q_2, B, q_0), (q_1, B, q_0)\}$, $Q_0 = \{q_0\}$ *and* $\mathcal{F} = \{\{q_1\}, \{q_2\}\}$. *Construct an equivalent NBA.*

Exercise 2 *(2+2+1 points)*

Recall the topological closure defined in the Exercise sheet 3.

$$\mathsf{cl} : \mathcal{P}(\Sigma^\omega) \to \mathcal{P}(\Sigma^\omega), \text{ where } \mathsf{cl}(A) = \{t \mid \forall x \prec t.\ \exists t' :\ x{\cdot}t' \in A\}.$$

Show the following:

1. *If L is ω-regular then show that $\mathsf{cl}(L)$ is also ω-regular.*
   Hint: Consider the Büchi automaton for L and construct the Büchi automaton for $\mathsf{cl}(L)$.
2. *For a ω-regular language L, construct the ω-regular language $\overline{\mathsf{cl}(A)}$ (complement of $\mathsf{cl}(L)$).*
   Remark: You can argue that Büchi automata can be complemented, but complementation of Büchi automata is highly non-trivial. Fortunately, there is a very simple way to construct $\overline{\mathsf{cl}(L)}$. Can you find it?
3. *Prove the* decomposition theorem *for ω-regular languages. That is, every ω-regular set can be decomposed into two ω-regular sets one of which is* Safe *and the other is* Live.

Exercise 3 *(2 points)*

Let e be a boolean function (acyclic) with n input bits and n output bits, and f be a boolean formula with $n+m$ variables. For any number i, let $i_\flat$ be the binary encoding of i. This defines a transition system $T_{e,f} = (S, \to, AP, L)$, where the set of states $S = \{v_0, v_1, \ldots, v_{2^n-1}\}$ and $AP = \{a_0, \ldots, a_{2^m-1}\}$, $(v_i, v_j) \in\to$ iff $e(i_\flat) = j_\flat$, and $a_i \in L(v_j)$ iff $f(j_\flat, i_\flat)$ is true. Graph represented thus, are called succinct graphs.

Consider the following succinct graph $T_{e,f} = (S, \to, AP, L)$, (where $S, \to, AP, L$ are as defined before) for which e and f has the following properties:

$$e(i_\flat) = i_\flat + 1 \quad \text{the `+' is a boolean addition on } n \text{ bits, particularly } (1\ldots1)\ +\ 1 = (0\ldots0). \tag{1}$$

$$\forall v_j \in S,\quad \forall a_i \in AP:\quad f(j_\flat, i_\flat) \implies f(j_\flat, i'_\flat) \text{ for } i' \geq i. \tag{2}$$

Assume e and f are defined by polynomial number of gates. I.e., the number of gates in e and f is $(n+m)^k$ for some constant k.

1. *Given $T_{e,f}$ (e, f with properties 1 and 2) and $a \in AP$, find an algorithm to check whether $T_{e,f}$ satisfies "infinitely often a".*
2. *The algorithm runs in $O(n{\cdot}(n+m)^k)$ time.*

Hint: Checking weather a boolean formula is satisfiable for a given input can be done in polynomial (linear) time. Observe that the transition system has 2^n states, so the usual DFS will not yield the desired complexity bounds.

Exercise 4 *(2 points)*

Which of the following statements are correct? Prove the statement or give a counter example.

1. $\Diamond\Box a \to \Box\Diamond b \ \equiv\ \Box(a\mathsf{U}(\sim a \vee b))$
2. $\Box\Diamond a \to \Box\Diamond b \ \equiv\ \Box(a \to \Diamond b)$
3. $(a\mathsf{U}b)\mathsf{U}c \ \equiv\ a\mathsf{U}(b\mathsf{U}c)$
4. $(\Box a \to a) \wedge (\Box a \to \Box\Box a)$.

Exercise 5 *(Bonus 10 points)*

Show that for any succinctly represented TS, $T_{e,f}$ (not restricted to properties 1,2), finding whether $T_{e,f}$ satisfies the property "infinitely often a" can be done in PSPACE.