Exercise 1  \hspace{1cm} (1 Points)
Consider the GNBA \( G = (Q, \Sigma, \delta, Q_0, \mathcal{F}) \):
\[ Q = q_0, q_2, \Sigma = \{ A, B \}, \delta = \{(q_0, A, q_1), (q_1, B, q_1), (q_1, B, q_2), (q_2, B, q_0), (q_1, B, q_0)\}, \quad Q_0 = \{q_0\} \text{ and } \mathcal{F} = \{\{q_1\}, \{q_2\}\} \]. Construct an equivalent NBA.

Exercise 2  \hspace{1cm} (2+2+1 points)
Recall the topological closure defined in the Exercise sheet 3.
\[ cl: \mathcal{P}(\Sigma^\omega) \to \mathcal{P}(\Sigma^\omega), \text{ where } cl(A) = \{t \mid \forall x \prec t. \exists t' : x.t' \in A\}. \]
Show the following:

1. If \( L \) is \( \omega \)-regular then show that \( cl(L) \) is also \( \omega \)-regular.
   Hint: Consider the Büchi automaton for \( L \) and construct the Büchi automaton for \( cl(L) \).
2. For a \( \omega \)-regular language \( L \), construct the \( \omega \)-regular language \( cl(A) \) (complement of \( cl(L) \)).
   Remark: You can argue that Büchi automata can be complemented, but complementation of Büchi automata is highly non-trivial. Fortunately, there is a very simple way to construct \( cl(L) \). Can you find it?
3. Prove the decomposition theorem for \( \omega \)-regular languages. That is, every \( \omega \)-regular set can be decomposed into two \( \omega \)-regular sets one of which is Safe and the other is Live.

Exercise 3  \hspace{1cm} (2 points)
Let \( e \) be a boolean function (acyclic) with \( n \) input bits and \( m \) output bits, and \( f \) be a boolean formula with \( n + m \) variables. For any number \( i \), let \( i \) be the binary encoding of \( i \). This defines a transition system \( T_{e,f} = (S, \to, AP, L) \), where the set of states \( S = \{v_0, v_1, \ldots, v_{2^n-1}\} \) and \( AP = \{a_0, \ldots, a_{2^m-1}\} \), \((v_i, v_j) \in \to \iff e(v_i) = j_i, \text{ and } a_i \in L(v_j) \iff f(j_i, i) \) is true. Graph represented thus, are called succinct graphs.
Consider the following succinct graph \( T_{e,f} = (S, \to, AP, L) \), (where \( S, \to, AP, L \) are as defined before) for which \( e \) and \( f \) has the following properties:
\[ e(i_0) = i_0 + 1 \text{ the ‘+’ is a boolean addition on } n \text{ bits, particularly } (1 \ldots 1) + 1 = (0 \ldots 0). \]  
\[ \forall v_j \in S, \forall a_i \in AP: \quad f(j_i, i) \implies f(j_i', i') \quad \text{for } i' \geq i. \]
Assume \( e \) and \( f \) are defined by polynomial number of gates. i.e., the number of gates in \( e \) and \( f \) is \((n + m)^k\) for some constant \( k \).

1. Given \( T_{e,f} \) (\( e, f \) with properties 1 and 2) and \( a \in AP \), find an algorithm to check whether \( T_{e,f} \) satisfies “infinitely often a ”.
2. The algorithm runs in \( O(n.(n + m)^k) \) time.
Hint: Checking weather a boolean formula is satisfiable for a given input can be done in polynomial (linear) time. Observe that the transition system has \( 2^n \) states, so the usual DFS will not yield the desired complexity bounds.

Exercise 4  \hspace{1cm} (2 points)
Which of the following statements are correct? Prove the statement or give a counter example.

1. \( \square a \to \square \Diamond b \equiv \square(a \land \Diamond b) \)
2. \( \Box \Diamond a \to \Box \Diamond b \equiv \Box(a \rightarrow \Diamond b) \)
3. \( (a \land b)Uc \equiv a(U(b \land c)) \)
4. \( (\Box a \to a) \land (\Box a \to \Box \Box a) \).

Exercise 5  \hspace{1cm} (Bonus 10 points)
Show that for any succinctly represented TS, \( T_{e,f} \) (not restricted to properties 1,2), finding whether \( T_{e,f} \) satisfies the property “infinitely often a ” can be done in PSPACE.