Introduction to Model Checking 2015: Exercise 6.

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Exercise 1 *The linear time temporal logic (LTL) that we have seen in the lecture has the syntax:*

$$\varphi ::= a \mid\ \sim \varphi \mid \varphi \wedge \varphi \mid \mathsf{X}\varphi \mid \varphi \mathsf{U} \varphi$$

where a is an atomic proposition, and the tense operator X *represent the* neXttime *and* U *represents the* Until. *Convert the following English sentences to appropriate LTL formulas. Please take them seriously!*

1. *"You will be awarded no marks for the exercise if you do not hand in your transcript before the due date".*
   – S.C.
2. *"Things will get worse before it get any better,unless it doesn't".*
3. *"Fool me once shame on you, fool me twice shame on me".*
4. *"Begin at the beginning," the King said, very gravely, "and go on till you come to the end: then stop."*
   – L.C.
5. *"I become insane, with long intervals of horrible sanity." – E.A.P.*

Remark: You can make up atomic proposition of your own choosing.

Exercise 2 *Consider the linear temporal logic with the following syntax:*

$$\varphi ::= a \mid\ \sim \varphi \mid \varphi \wedge \varphi \mid \mathsf{X}\varphi \mid \varphi \mathsf{U} \varphi \mid \mathsf{G}\varphi$$

where the tense operator G *denotes the construct* Generally. *In this exercise we will study the axiomatization of the logic. The axioms are as follows:*

$$\begin{array}{lllll}
\mathsf{Prop}: & \textit{propositional Tautologies} & & \\
\mathsf{K_X}: & \mathsf{X}(\varphi \to \psi) & \to & \mathsf{X}\varphi \to \mathsf{X}\psi \\
\mathsf{Func_X}: & \sim \mathsf{X}\varphi & \leftrightarrow & \mathsf{X} \sim \varphi \\
\mathsf{FP_G}: & \mathsf{G}\varphi & \leftrightarrow & \varphi \ \wedge \ \mathsf{XG}\varphi \\
\mathsf{FP_U}: & \varphi\mathsf{U}\psi & \leftrightarrow & \psi \ \vee \ (\varphi \ \wedge \ \mathsf{X}(\varphi\mathsf{U}\psi))
\end{array}$$

The induction rules are as follows:

$$\begin{array}{llll}
I_{\mathsf{G}}: & \textit{If} \vdash \ \psi \ \to \ \varphi \wedge \mathsf{X}\psi & \textit{then} \vdash \ \psi \ \to \ \mathsf{G}\varphi \\
I_{\mathsf{U}}: & \textit{If} \vdash \ \psi \vee (\varphi \wedge X\theta) \ \to \ \theta & \textit{then} \vdash \ \varphi\mathsf{U}\psi \ \to \ \theta
\end{array}$$

And two deduction rules: Modus Ponens: If $\vdash \psi$ and $\vdash \psi \to \varphi$ then $\vdash \psi$, and Generalization: if $\vdash \varphi$ then $\vdash \mathsf{G}\varphi$. A theorem can be proved using the axioms and induction rules. For example: Theorem $4:\ \mathsf{G}\varphi \to \mathsf{GG}\varphi$ *can be proven as follows:*

$$\begin{array}{llll}
\vdash & \mathsf{G}\varphi \ \to \ \varphi \wedge \mathsf{XG}\varphi & \mathsf{FP_G} \\
\vdash & \mathsf{G}\varphi \ \to \ \mathsf{XG}\varphi & \wedge \textit{ Elimination} \\
\vdash & \mathsf{G}\varphi \ \to \ \mathsf{G}\varphi \wedge \mathsf{XG}\varphi & \\
\vdash & \mathsf{G}\varphi \ \to \ \mathsf{GG}\varphi & \textit{Induction rule } I_{\mathsf{G}}
\end{array}$$

Now you prove the following theorems: (You will do well to forget the intuitive meaning of the operators and just follow the axioms and rules).

1. *Distributive Law* X*:* $\mathsf{X}(a \wedge b) \ \to \ \mathsf{X}a \wedge \mathsf{X}b$.
2. *Distributive Law* G*:* $\mathsf{G}(a \wedge b) \ \to \ \mathsf{G}a \wedge \mathsf{G}b$.
3. $\mathsf{K_G}$*:* $\mathsf{G}(a \to b) \ \to \ \mathsf{G}a \to \mathsf{G}b$.
4. *Commutative Law* XG*:* $\mathsf{XG}a \leftrightarrow \mathsf{GX}a$.
5. $a\mathsf{U}(b \wedge \mathsf{G}(c \to a)) \ \to \ \mathsf{G}(c \to a) \wedge c\mathsf{U}b$.