Exercise 1 The linear time temporal logic (LTL) that we have seen in the lecture has the syntax:

\[ \varphi ::= a \mid \neg \varphi \mid \varphi \land \varphi \mid X\varphi \mid \varphi U \varphi \]

where \( a \) is an atomic proposition, and the tense operator \( X \) represents the next time and \( U \) represents the until. Convert the following English sentences to appropriate LTL formulas. Please take them seriously!

1. “You will be awarded no marks for the exercise if you do not hand in your transcript before the due date”.
   - S.C.
2. “Things will get worse before it get any better, unless it doesn’t”.
3. “Fool me once shame on you, fool me twice shame on me”.
4. “Begin at the beginning,” the King said, very gravely, “and go on till you come to the end: then stop.”
   - L.C.
5. “I become insane, with long intervals of horrible sanity.” – E.A.P.

Remark: You can make up atomic proposition of your own choosing.

Exercise 2 Consider the linear temporal logic with the following syntax:

\[ \varphi ::= a \mid \neg \varphi \mid \varphi \land \varphi \mid X\varphi \mid \varphi U \varphi \mid G\varphi \]

where the tense operator \( G \) denotes the construct Generally. In this exercise we will study the axiomatization of the logic. The axioms are as follows:

- **Prop:** propositional Tautologies
- **K \( X \):** \( X(\varphi \to \psi) \to X\varphi \to X\psi \)
- **Func \( X \):** \( \neg X\varphi \leftrightarrow X \neg \varphi \)
- **FP \( G \):** \( G\varphi \leftrightarrow \varphi \land XG\varphi \)
- **FP \( U \):** \( \varphi U \psi \leftrightarrow \psi \lor (\varphi \land X(\varphi U \psi)) \)

The induction rules are as follows:

- **I \( G \):** If \( \vdash \psi \to \varphi \land X\psi \) then \( \vdash \psi \to G\varphi \)
- **I \( U \):** If \( \vdash \psi \lor (\varphi \land X\theta) \to \theta \) then \( \vdash \varphi U \psi \to \theta \)

And two deduction rules: Modus Ponens: If \( \vdash \psi \) and \( \vdash \psi \to \varphi \) then \( \vdash \psi \), and Generalization: if \( \vdash \varphi \) then \( \vdash G\varphi \). A theorem can be proved using the axioms and induction rules. For example: Theorem 4: \( G\varphi \to GG\varphi \) can be proven as follows:

\[ \begin{align*}
\vdash & G\varphi \to \varphi \land XG\varphi \\
\vdash & G\varphi \to XG\varphi & \text{FP \( G \)} \\
\vdash & G\varphi \to G\varphi \land XG\varphi & \land \text{ Elimination} \\
\vdash & G\varphi \to GG\varphi & \text{Induction rule I \( G \)}
\end{align*} \]

Now you prove the following theorems: (You will do well to forget the intuitive meaning of the operators and just follow the axioms and rules).

1. **Distributive Law X:** \( X(a \land b) \to Xa \land Xb \).
2. **Distributive Law G:** \( G(a \land b) \to Ga \land Gb \).
3. **K \( G \):** \( G(a \to b) \to Ga \to Gb \).
4. **Commutative Law XG:** \( XGa \leftrightarrow GXa \).
5. \( aU(b \land G(c \to a)) \to G(c \to a) \land cUb \).