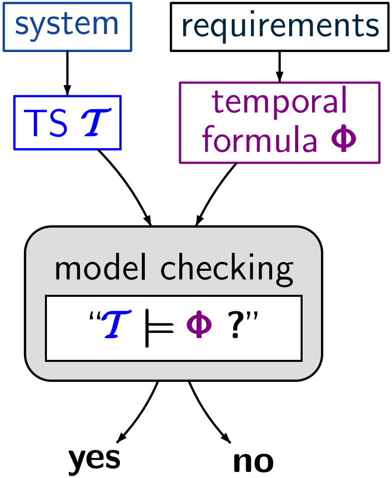
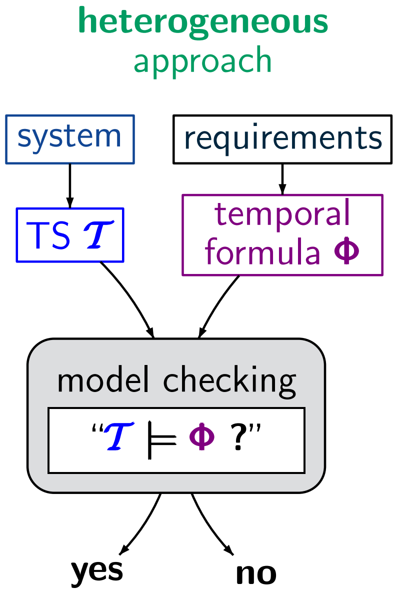
Overview

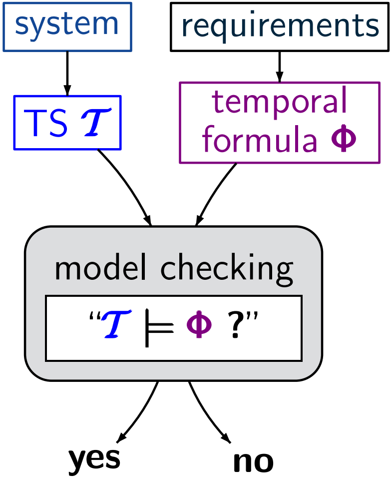
- Introduction
- Modelling parallel systems
- Linear Time Properties
- Regular Properties
- Linear Temporal Logic (LTL)
- Computation-Tree Logic
- **Equivalences and Abstraction**

Model checking

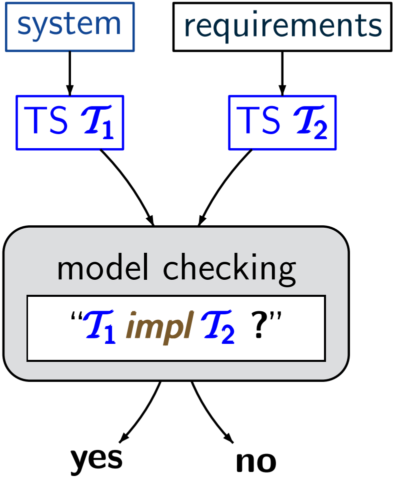



heterogeneous
approach
system
requirements
TS $\mathcal{T}$
temporal
formula Φ
model checking
"$\mathcal{T} \models \Phi$?"
yes
no

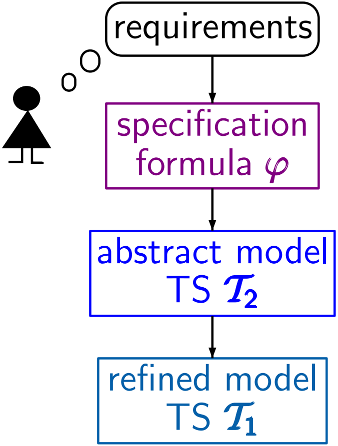
heterogeneous approach

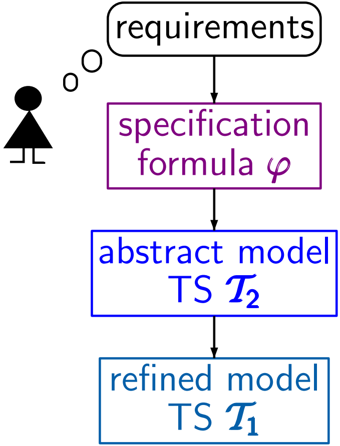


homogeneous approach



Implementation relations





implementation relation ***impl***

$\mathcal{T}_1$ ***impl*** $\mathcal{T}_2$ iff "$\mathcal{T}_1$ is a correct implementation of $\mathcal{T}_2$"

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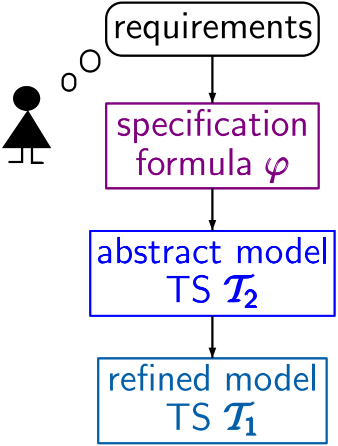

trace inclusion:

$\mathcal{T}_1$ *impl* $\mathcal{T}_2$ iff
$\quad Traces(\mathcal{T}_1) \subseteq Traces(\mathcal{T}_2)$

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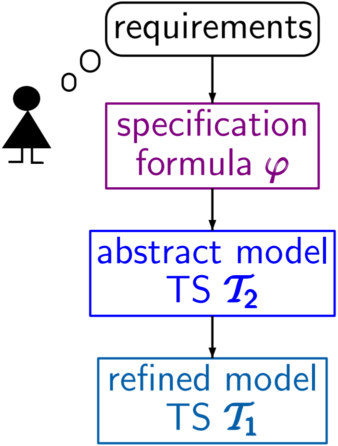

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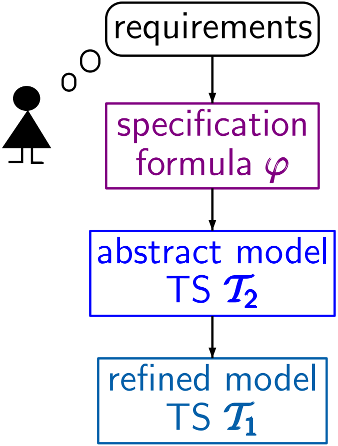

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i.e., if $\mathcal{T}_2 \models \varphi$ then $\mathcal{T}_1 \models \varphi$

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Trace equivalence

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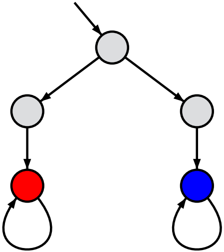
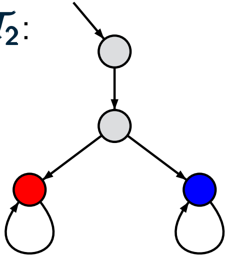


$\mathcal{T}_2$:

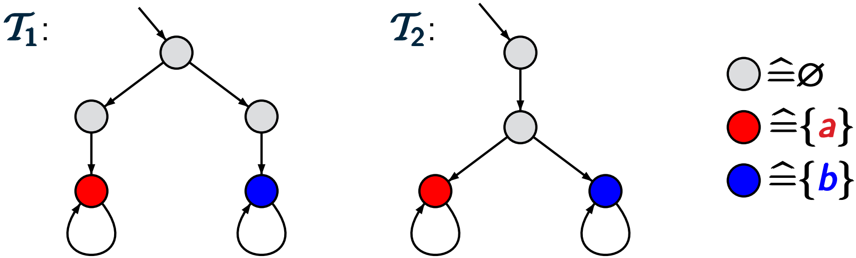


grey $\widehat{=} \varnothing$

red $\widehat{=} \{a\}$

blue $\widehat{=} \{b\}$

Trace equivalence



$$Traces(\mathcal{T}_1) = \{ \varnothing\, \varnothing\, a^{\omega},\ \varnothing\, \varnothing\, b^{\omega} \} = Traces(\mathcal{T}_1)$$

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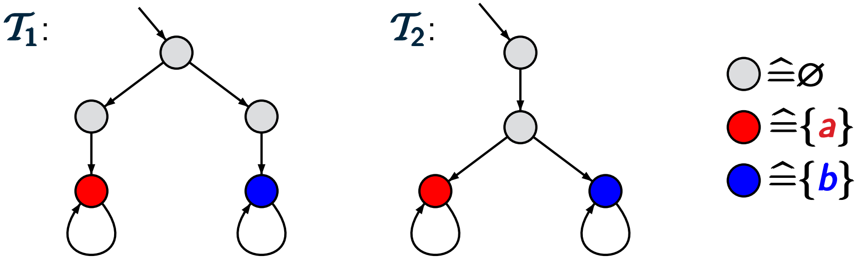

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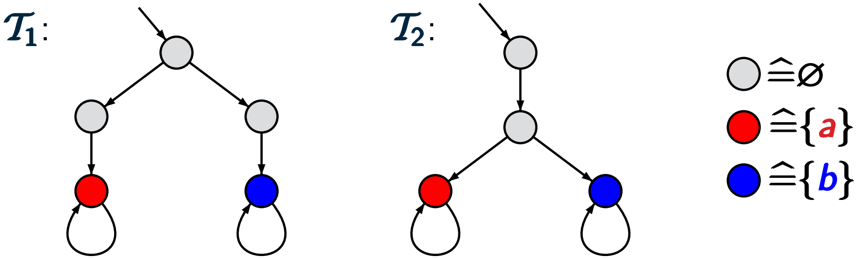

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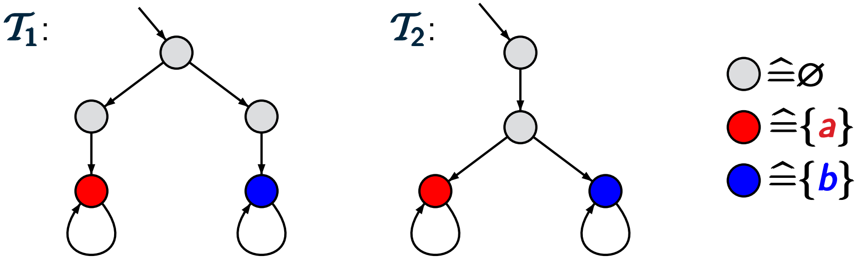
Trace equivalence is not compatible with CTL



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Implementation relations

- for the design of complex systems

  $\rightsquigarrow$ comparison of 2 transition systems

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  - $\rightsquigarrow$ comparison of **2** transition systems
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  - $\leadsto$ comparison of **2** transition systems
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use equivalence relation $\sim$ for the states of a single transition system $\mathcal{T}$ and analyze the quotient $\mathcal{T}/\!\sim$

Implementation relations

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  - $\leadsto$ comparison of **2** transition systems

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  - $\leadsto$ homogeneous model checking approach
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use equivalence relation $\sim$ for the states of a single transition system $\mathcal{T}$ and analyze the quotient $\mathcal{T}/\sim$

goal: define the equivalence $\sim$ in such a way that

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \mathcal{T}/\sim \models \Phi$$

for all "relevant" properties Φ

Linear-time implementation relations

Linear-time implementation relations

finite trace inclusion and equivalence:

e.g., $\mathit{Tracesfin}(\mathcal{T}_1) \subseteq \mathit{Tracesfin}(\mathcal{T}_2)$

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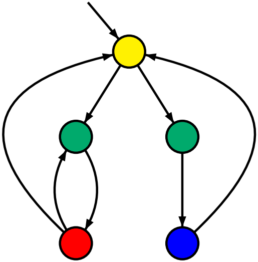
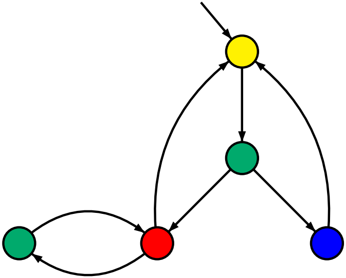
* **minimization ???**

Minimization w.r.t. trace equivalence?

$\mathcal{T}_1$:



$\mathcal{T}_2$:



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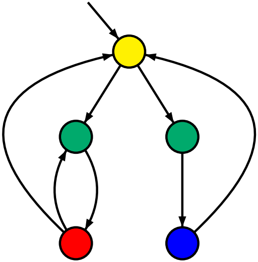

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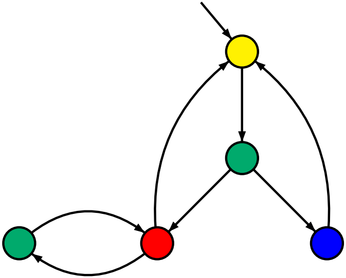

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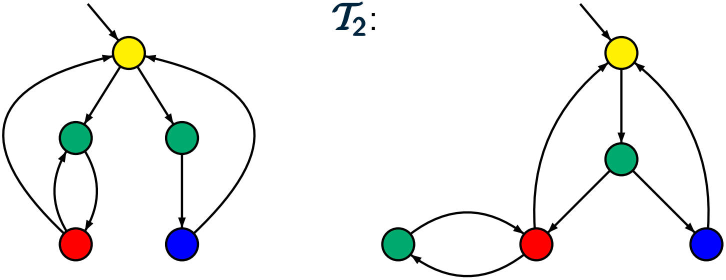

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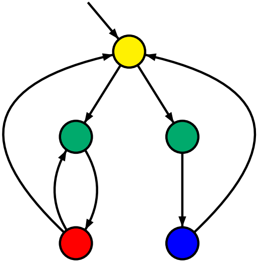

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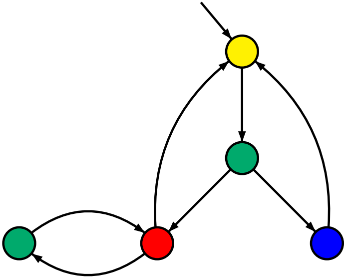

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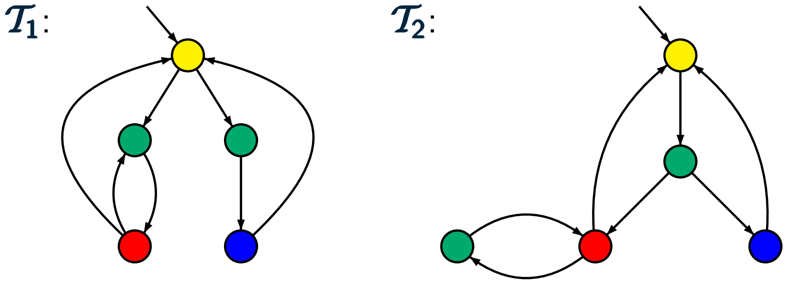

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- there is no smaller TS that is trace-equivalent to $\mathcal{T}_i$

Classification of implementation relations

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- linear vs. branching time
  - * linear time: trace relations
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- strong vs. weak relations
  - * strong: reasoning about all transitions
  - * weak: abstraction from stutter steps

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation ⟵
- CTL, CTL*-equivalence
- computing the bisimulation quotient
- abstraction stutter steps
- simulation relations

Bisimulation for two transition systems BSEQOR5.1-DEF-BIS-2TS

Bisimulation for two transition systems

let $\mathcal{T}_1 = (S_1, Act_1, \rightarrow_1, S_{0,1}, AP, L_1)$,

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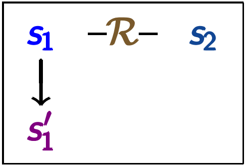

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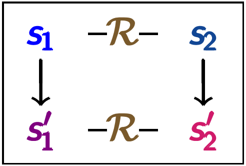

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and such that the following initial condition holds:

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bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$: relation $\mathcal{R} \subseteq S_1 \times S_2$ s.t.

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(2) } mutual stepwise simulation
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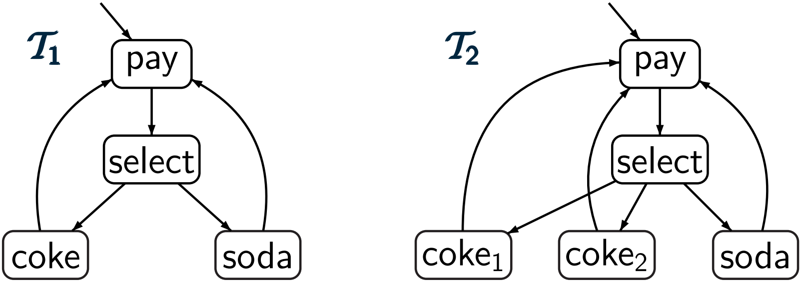
for state s_1 of $\mathcal{T}_1$ and state s_2 of $\mathcal{T}_2$:

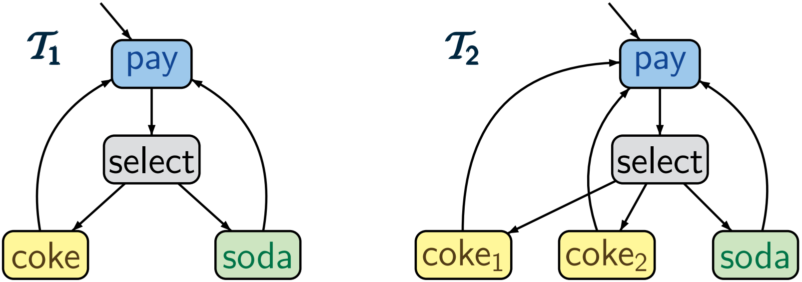
$s_1 \sim s_2$ iff there exists a bisimulation $\mathcal{R}$ for $(\mathcal{T}_1, \mathcal{T}_2)$ such that $(s_1, s_2) \in \mathcal{R}$

Two beverage machines



$$AP = \{pay, coke, soda\}$$

Two beverage machines



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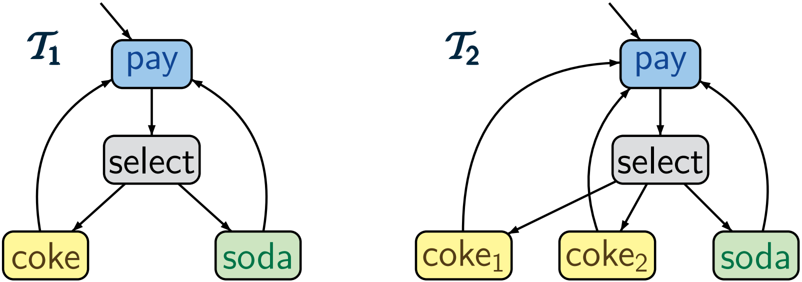
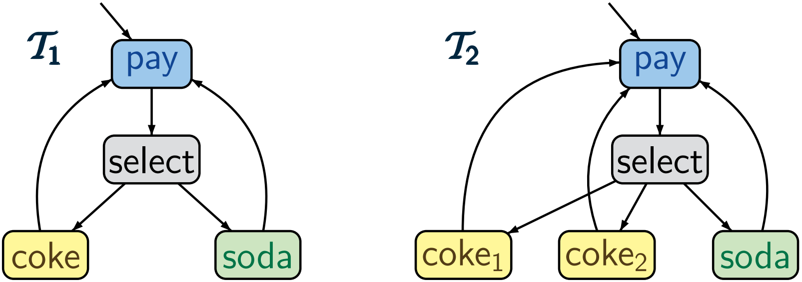


$AP = \{pay, coke, soda\}$

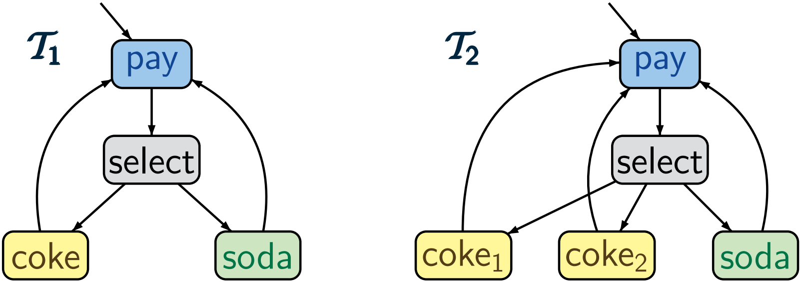
$\mathcal{T}_1 \sim \mathcal{T}_2$

Two beverage machines



$AP = \{pay, coke, soda\}$

$\mathcal{T}_1 \sim \mathcal{T}_2$ as there is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$:

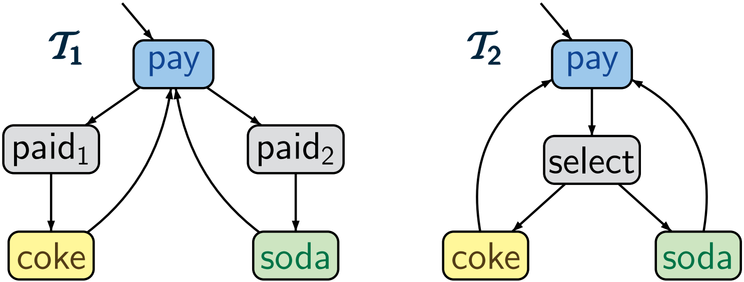


$AP = \{pay, coke, soda\}$

$\mathcal{T}_1 \sim \mathcal{T}_2$ as there is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$:

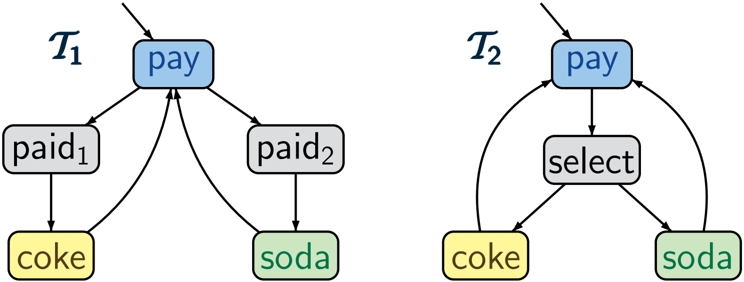
{ (pay,pay), (select,select), (soda,soda)
(coke,coke_1), (coke,coke_2) }

Two beverage machines



$$AP = \{pay, coke, soda\}$$

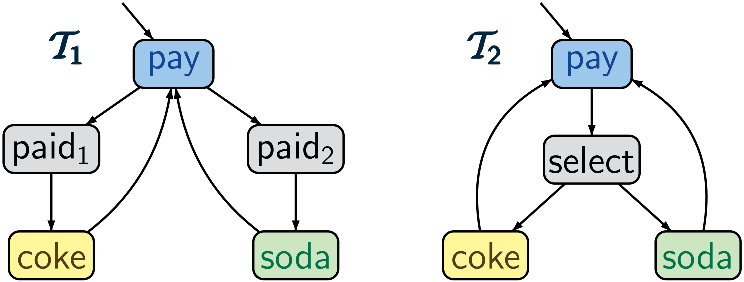
Two beverage machines



$AP = \{pay, coke, soda\}$

$\mathcal{T}_1 \not\sim \mathcal{T}_2$

Two beverage machines



$AP = \{pay, coke, soda\}$

$\mathcal{T}_1 \not\sim \mathcal{T}_2$

because there is no state in $\mathcal{T}_1$ that has both

- a successor labeled with coke and
- a successor labeled with soda

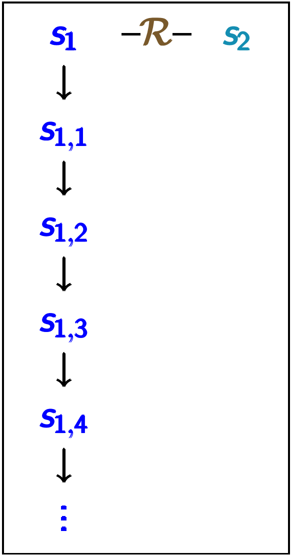
Simulation condition of bisimulations

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \\ s_1' & & \end{array}$$

can be completed to

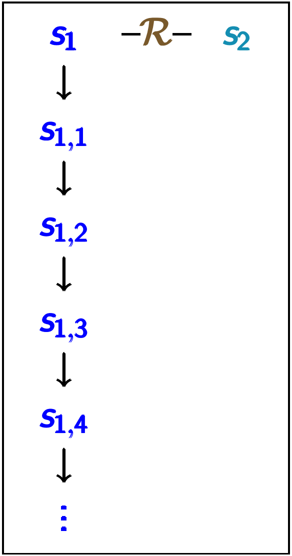
$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \downarrow \\ s_1' & -\mathcal{R}- & s_2' \end{array}$$

Path lifting for bisimulation $\mathcal{R}$



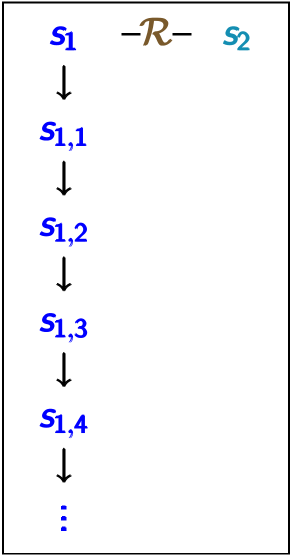
Path lifting for bisimulation $\mathcal{R}$

BSEQOR5.1-9-BIS

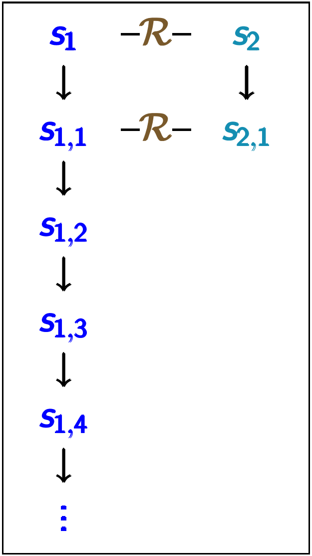


can be
completed to

Path lifting for bisimulation $\mathcal{R}$



can be completed to



Path lifting for bisimulation $\mathcal{R}$

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \\ s_{1,1} & & \\ \downarrow & & \\ s_{1,2} & & \\ \downarrow & & \\ s_{1,3} & & \\ \downarrow & & \\ s_{1,4} & & \\ \downarrow & & \\ \vdots & & \end{array}$$

can be completed to

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \downarrow \\ s_{1,1} & -\mathcal{R}- & s_{2,1} \\ \downarrow & & \downarrow \\ s_{1,2} & -\mathcal{R}- & s_{2,2} \\ \downarrow & & \\ s_{1,3} & & \\ \downarrow & & \\ s_{1,4} & & \\ \downarrow & & \\ \vdots & & \end{array}$$

Path lifting for bisimulation $\mathcal{R}$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \downarrow \\
s_{1,3} & -\mathcal{R}- & s_{2,3} \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

Path lifting for bisimulation $\mathcal{R}$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \downarrow \\
s_{1,3} & -\mathcal{R}- & s_{2,3} \\
\downarrow & & \downarrow \\
s_{1,4} & -\mathcal{R}- & s_{2,4} \\
\downarrow & & \downarrow \\
\vdots & & \vdots
\end{array}
$$

Properties of bisimulation equivalence

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- reflexivity: $\mathcal{T} \sim \mathcal{T}$ for all transition systems $\mathcal{T}$

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$\uparrow$

If S is the state space of $\mathcal{T}$ then

$$\mathcal{R} = \{(s, s) : s \in S\}$$

is a bisimulation for $(\mathcal{T}, \mathcal{T})$

Properties of bisimulation equivalence

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- symmetry: $\mathcal{T}_1 \sim \mathcal{T}_2$ implies $\mathcal{T}_2 \sim \mathcal{T}_1$

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↑

If $\mathcal{R}$ is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$ then

$$\mathcal{R}^{-1} = \{(s_2, s_1) : (s_1, s_2) \in \mathcal{R}\}$$

is a bisimulation for $(\mathcal{T}_2, \mathcal{T}_1)$

Properties of bisimulation equivalence

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- reflexivity: $\mathcal{T} \sim \mathcal{T}$ for all transition systems $\mathcal{T}$
- symmetry: $\mathcal{T}_1 \sim \mathcal{T}_2$ implies $\mathcal{T}_2 \sim \mathcal{T}_1$
- transitivity: if $\mathcal{T}_1 \sim \mathcal{T}_2$ and $\mathcal{T}_2 \sim \mathcal{T}_3$ then $\mathcal{T}_1 \sim \mathcal{T}_3$

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Let $\mathcal{R}_{1,2}$ be a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$,
$\mathcal{R}_{2,3}$ be a bisimulation for $(\mathcal{T}_2, \mathcal{T}_3)$.

Properties of bisimulation equivalence

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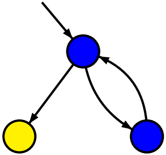
Let $\mathcal{R}_{1,2}$ be a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$,
$\mathcal{R}_{2,3}$ be a bisimulation for $(\mathcal{T}_2, \mathcal{T}_3)$.

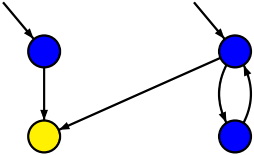
$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_3) : \exists s_2 \text{ s.t. } (s_1, s_2) \in \mathcal{R}_{1,2} \text{ and } (s_2, s_3) \in \mathcal{R}_{2,3} \}$$

is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_3)$

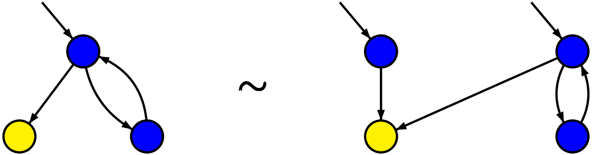
Correct or wrong?



$\sim$

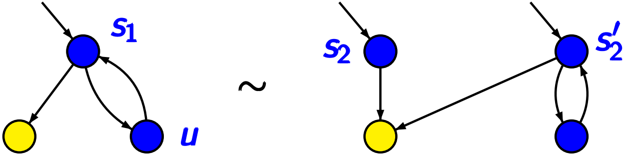


Correct or wrong?



wrong

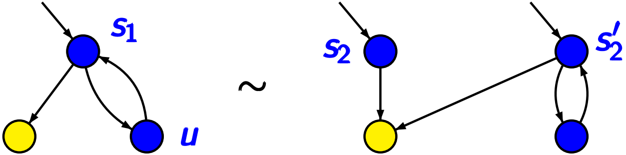
Correct or wrong?



wrong

$s_1 \longrightarrow u$, but $s_2 \not\longrightarrow$ *blue* (thus $s_1 \not\sim s_2$)

Correct or wrong?



wrong

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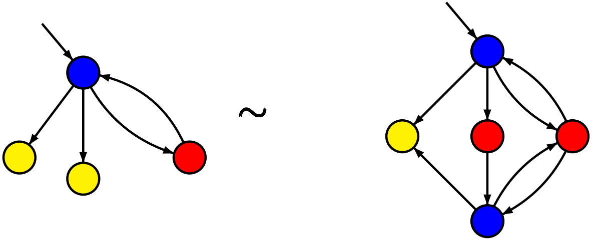
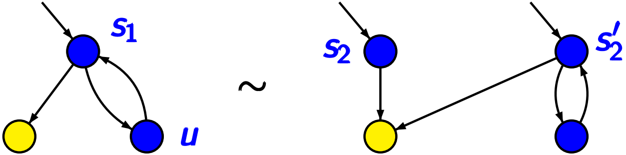


Correct or wrong?



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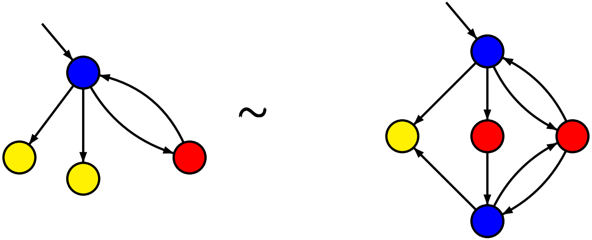
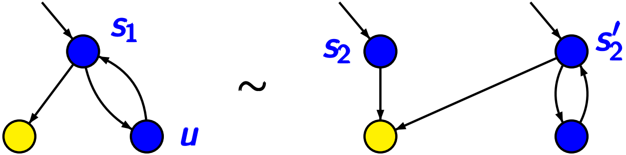


correct

Correct or wrong?



wrong

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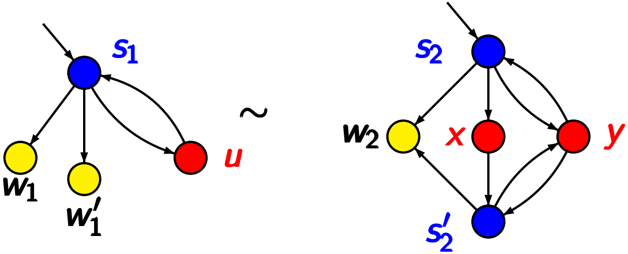


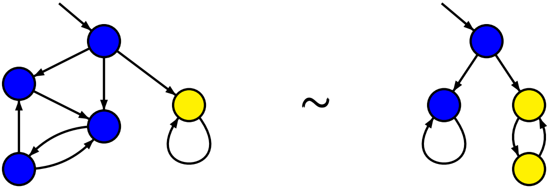
correct

bisimulation:

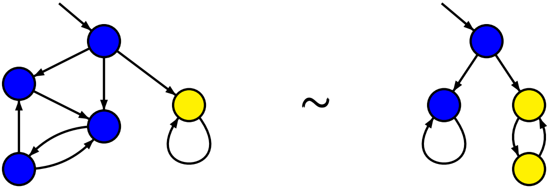
$\{(w_1, w_2), (w'_1, w_2), (s_1, s_2), (s_1, s'_2), (u, x), (u, y)\}$

Correct or wrong?

BSEQOR5.1-20

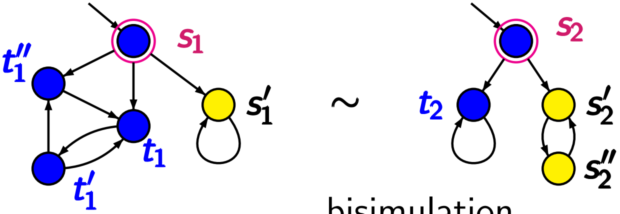


Correct or wrong?



correct

Correct or wrong?

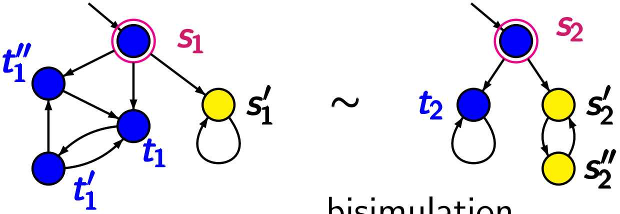


correct

bisimulation

$$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$$

Correct or wrong?



correct

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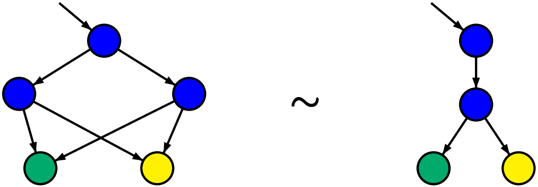
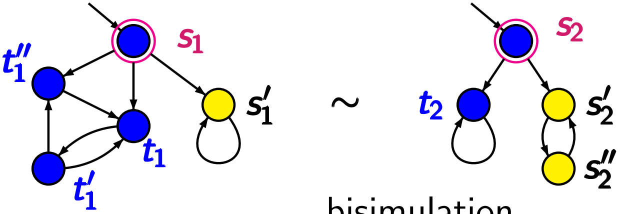


Correct or wrong?



correct

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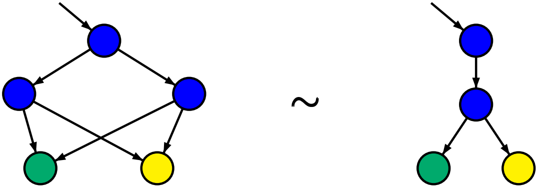
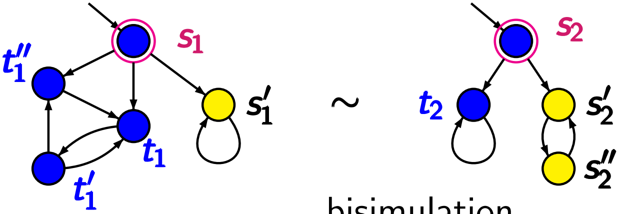


correct

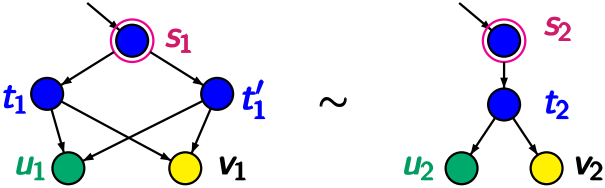
Correct or wrong?



correct

bisimulation

$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$



correct

bisimulation: $\{(s_1, s_2), (t_1, t_2), (t_1', t_2), (u_1, u_2), (v_1, v_2)\}$

Bisimulation vs. trace equivalence

Bisimulation vs. trace equivalence

$$\mathcal{T}_1 \sim \mathcal{T}_2 \implies \mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$$

Bisimulation vs. trace equivalence

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proof: ... path fragment lifting ...

Bisimulation vs. trace equivalence

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proof: ... path fragment lifting ...

$$\mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2) \not\Longrightarrow \mathcal{T}_1 \sim \mathcal{T}_2$$

Bisimulation vs. trace equivalence

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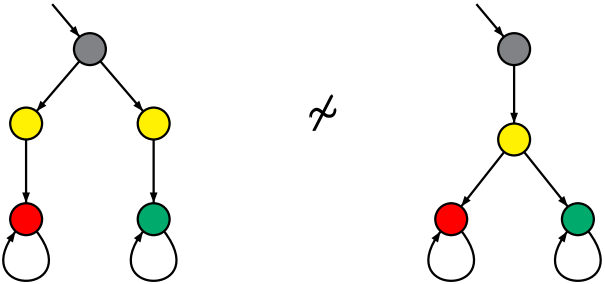

trace equivalent, but not bisimulation equivalent

Bisimulation vs. trace equivalence

$$\mathcal{T}_1 \sim \mathcal{T}_2 \implies \mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$$

proof: ... path fragment lifting ...

$$\mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2) \not\Longrightarrow \mathcal{T}_1 \sim \mathcal{T}_2$$

Trace equivalence is strictly coarser than bisimulation equivalence.

Bisimulation vs. trace equivalence

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Trace equivalence is strictly coarser than bisimulation equivalence.

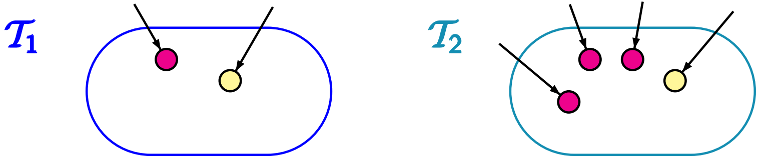
Bisimulation equivalent transition systems satisfy the same **LT properties** (e.g., **LTL** formulas).

Bisimulation equivalence ...

- as a relation that compares 2 transition systems

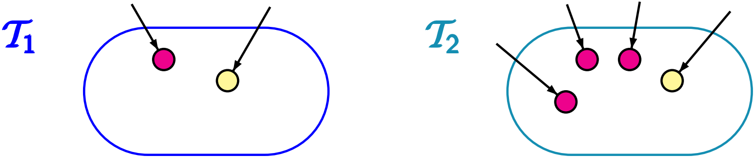
Bisimulation equivalence ...

- as a relation that compares **2** transition systems



Bisimulation equivalence ...

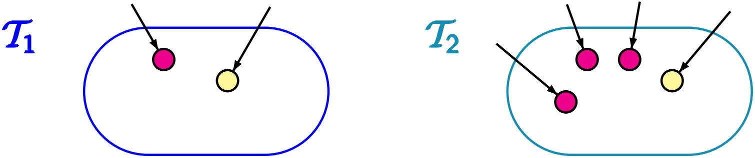
- as a relation that compares **2** transition systems



- as a relation on the states of **1** transition system

Bisimulation equivalence ...

- as a relation that compares **2** transition systems



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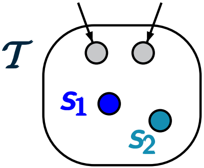

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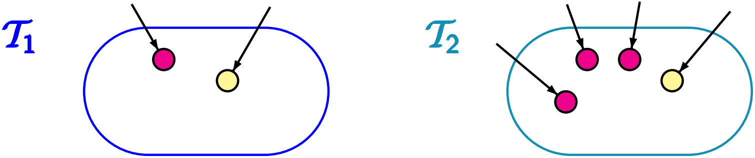

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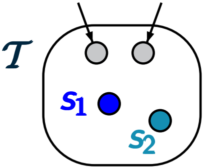
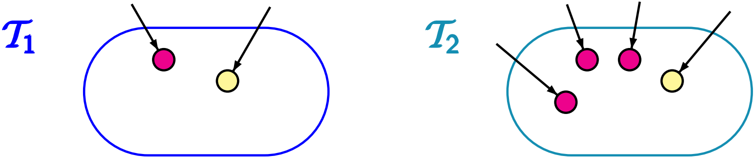


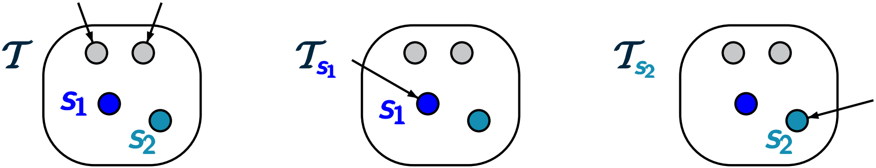
$s_1 \sim s_2$ iff $\mathcal{T}_{s_1} \sim \mathcal{T}_{s_2}$

Bisimulation equivalence ...

- as a relation that compares **2** transition systems



- as a relation on the states of **1** transition system



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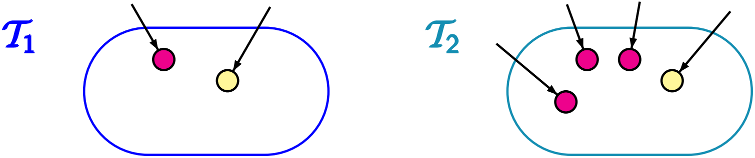
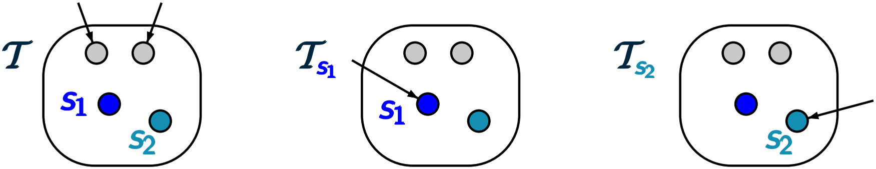
Bisimulation equivalence ...

- as a relation that compares **2** transition systems



- as a relation on the states of **1** transition system



$s_1 \sim s_2$ iff $\mathcal{T}_{s_1} \sim \mathcal{T}_{s_2}$ iff
there exists a bisimulation $\mathcal{R}$ for $\mathcal{T}$ s.t. $(s_1, s_2) \in \mathcal{R}$

Bisimulations on a single TS

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(1) $L(s_1) = L(s_2)$

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Bisimulation equivalence $\sim_{\mathcal{T}}$ on a single TS

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Two variants of bisimulation equivalence

$\sim$ relation that compares **2** transition systems

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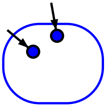
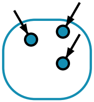
Derivation of $\sim$ from $\sim_{\mathcal{T}}$

given two transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$

$\mathcal{T}_1$ with state space S_1



$\mathcal{T}_2$ with state space S_2



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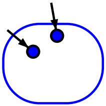

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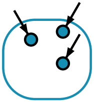

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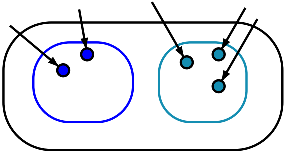

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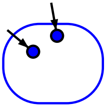

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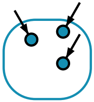

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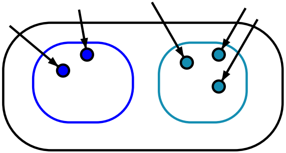

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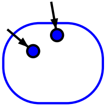

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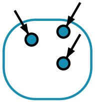

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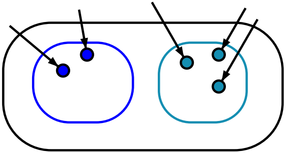

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Bisimulation quotient

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bisimulation quotient $\mathcal{T}/\sim$ arises from $\mathcal{T}$
by collapsing bisimulation equivalent states

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- state space: $S' = S/\!\sim_{\mathcal{T}}$

↑

set of bisimulation equivalence classes

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well-defined

by the labeling condition of bisimulations

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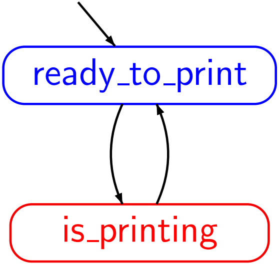
Example: interleaving of n printers

parallel system $\mathcal{T} = \underbrace{\textit{Printer} \mid\mid\mid \textit{Printer} \mid\mid\mid \ldots \mid\mid\mid \textit{Printer}}_{n \text{ printer}}$

Example: interleaving of n printers

parallel system $\mathcal{T} = \underbrace{\textit{Printer} \,|||\, \textit{Printer} \,|||\, \ldots \,|||\, \textit{Printer}}_{n \text{ printer}}$

transition system for each printer



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$AP = \{0, 1, \ldots, n\}$ "number of available printers"

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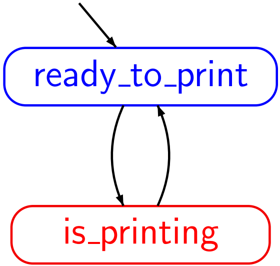

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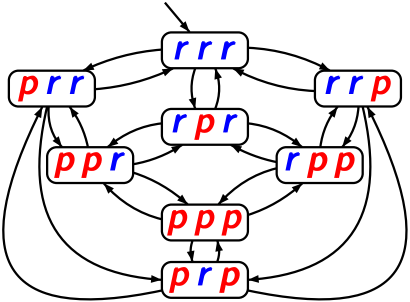

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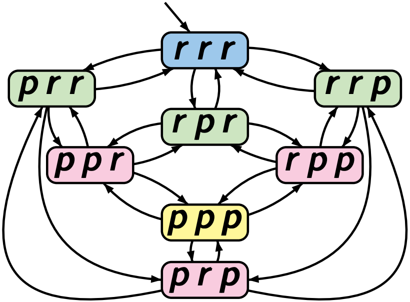

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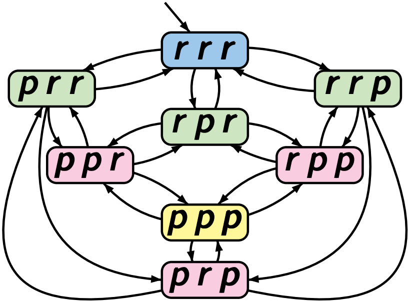

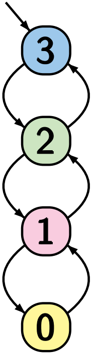

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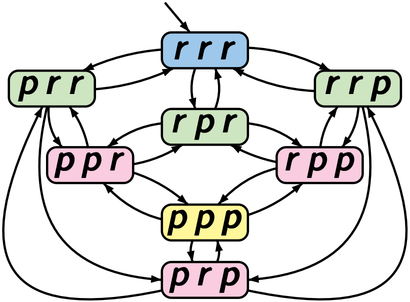

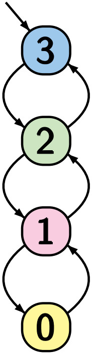

2^n states

$n+1$ states

Mutual exclusion

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solutions for mutual exclusion problems:

- semaphore
- Peterson's algorithm

Mutual exclusion: Bakery algorithm

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  $x_1 = x_2$: cannot happen

Bakery algorithm

protocol for P_1:

```
LOOP FOREVER
    noncritical actions
    x1 := x2+1
    AWAIT (x1 < x2) ∨ (x2=0);
    critical section;
    x1 := 0
END LOOP
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symmetric protocol for P_2

Bakery algorithm

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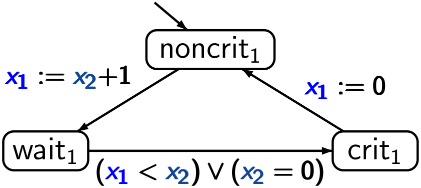
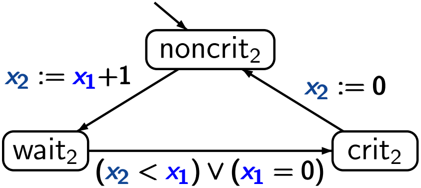
initially:
$x_1 = x_2 = 0$

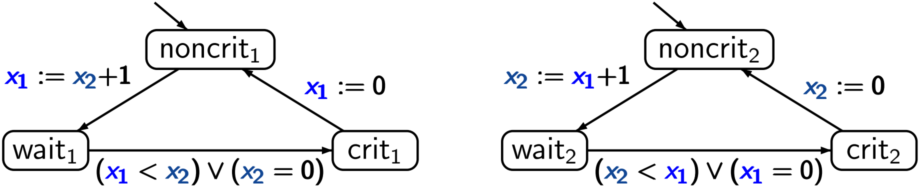
symmetric protocol for P_2

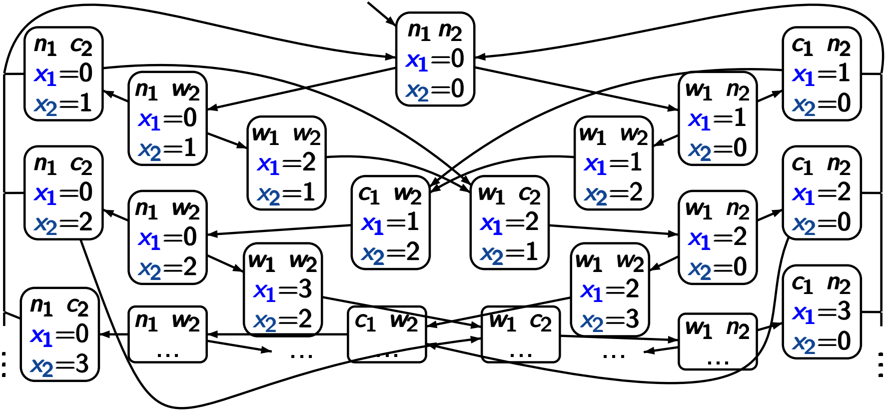
Program graphs for the Bakery algorithm





Transition system for the Bakery algorithm





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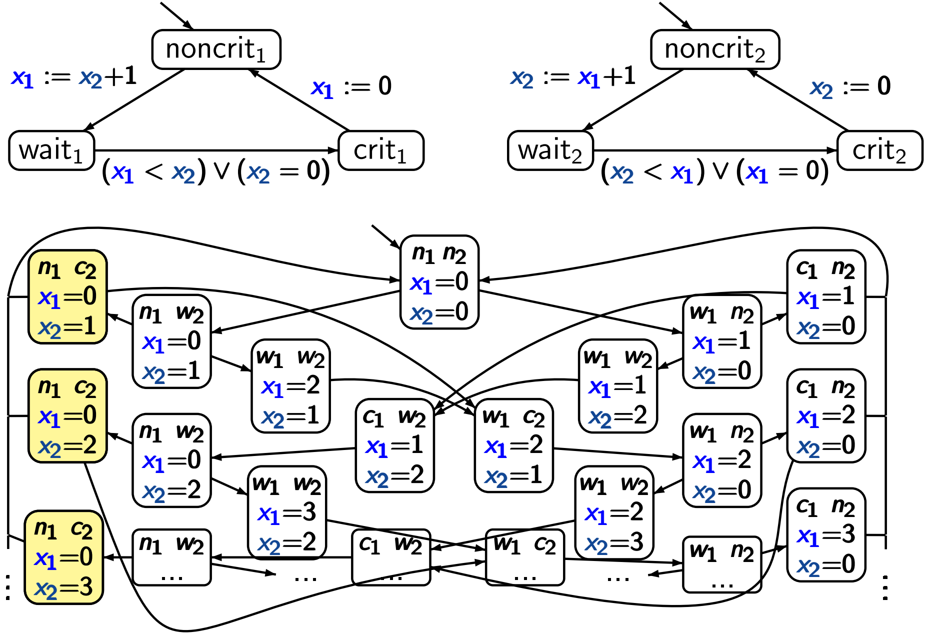
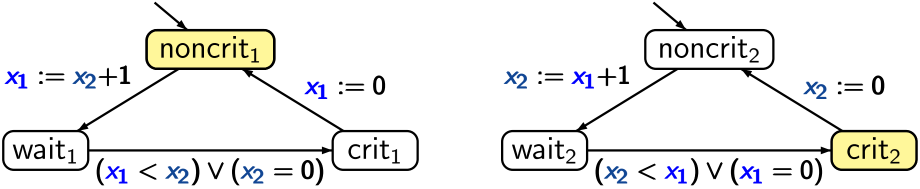
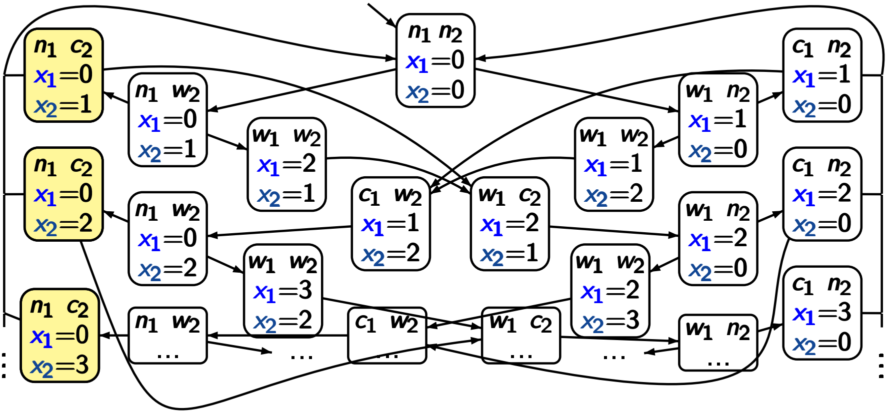
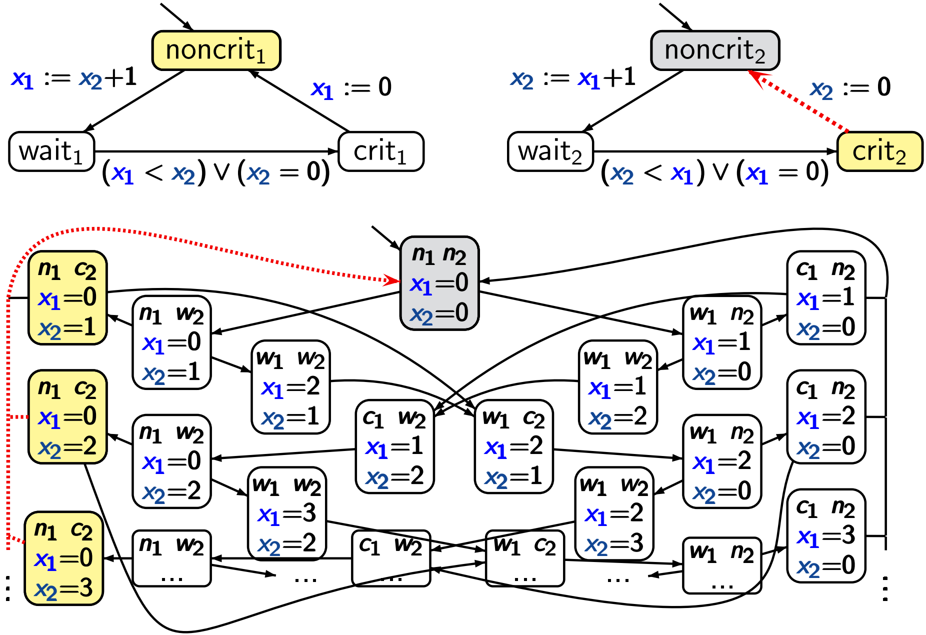


Transition system for the Bakery algorithm

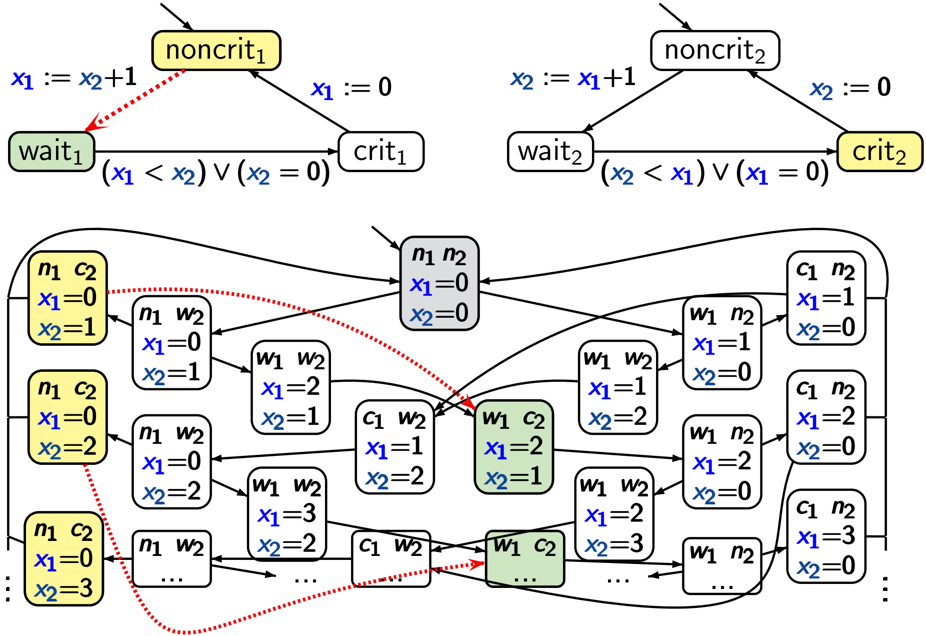




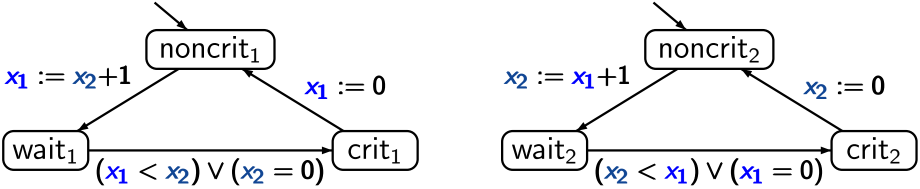
Transition system for the Bakery algorithm

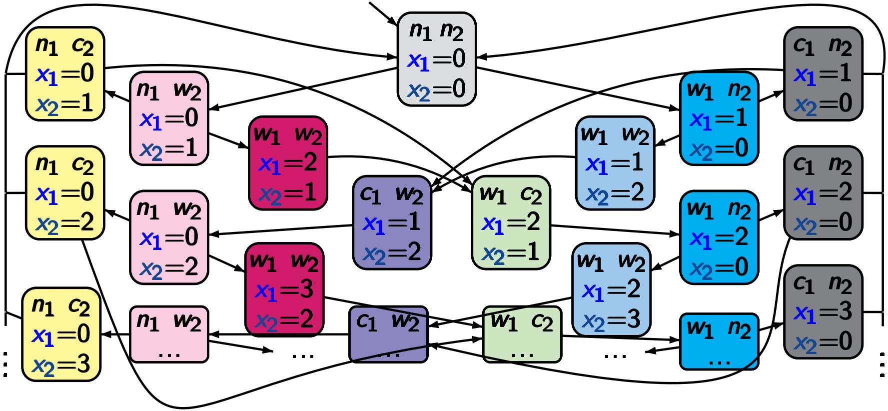


Transition system for the Bakery algorithm

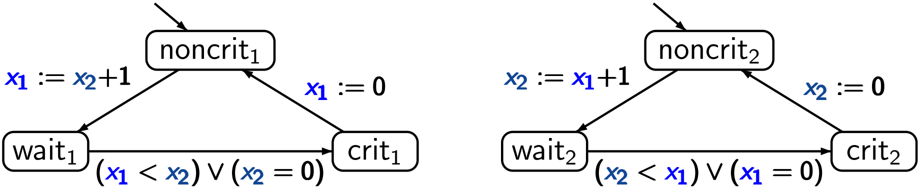


Transition system for the Bakery algorithm



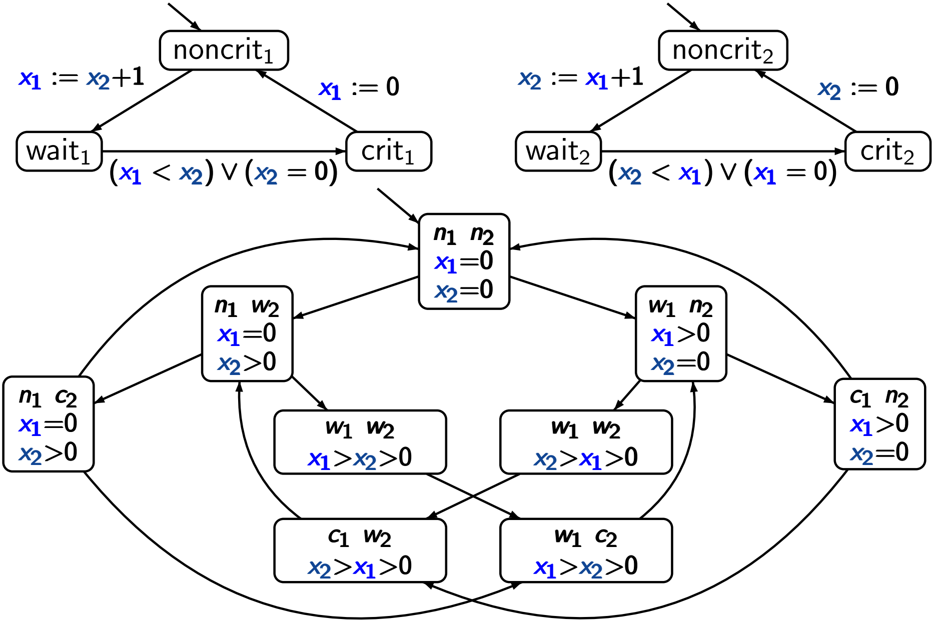


Bakery algorithm: bisimulation quotient

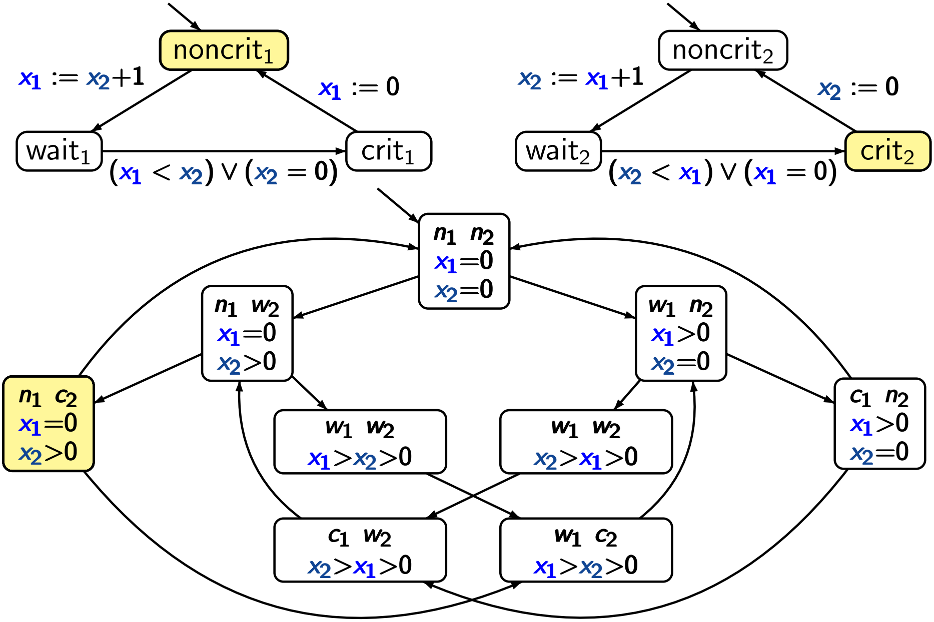


infinite transition system with a
finite bisimulation quotient

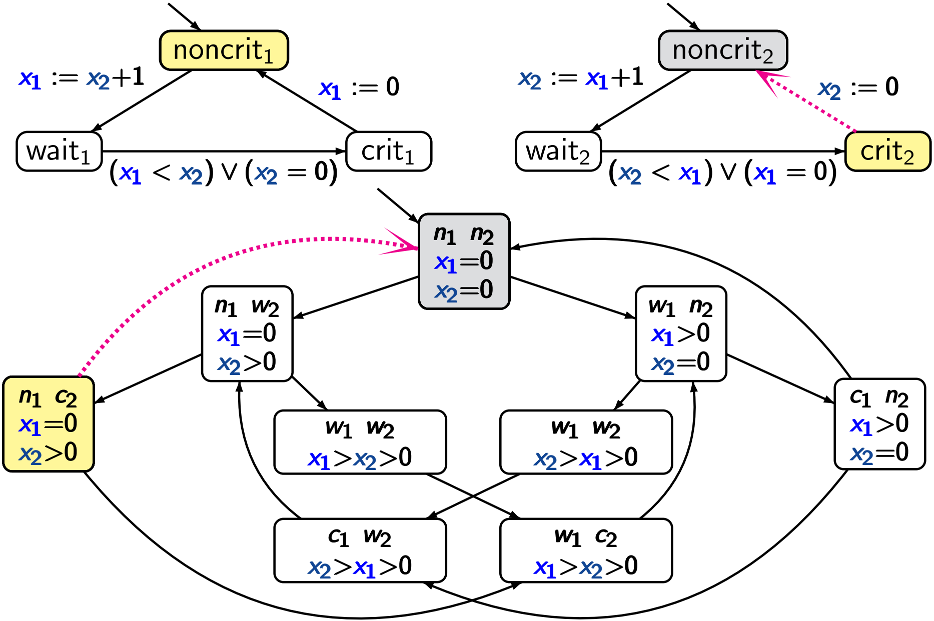
Bakery algorithm: bisimulation quotient



Bakery algorithm: bisimulation quotient



Bakery algorithm: bisimulation quotient



Bakery algorithm: bisimulation quotient