Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

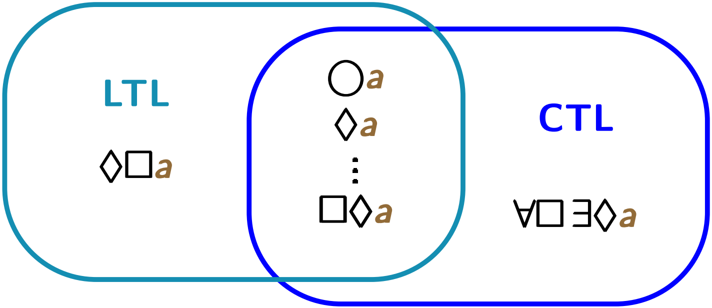
Linear Temporal Logic (LTL)

Computation Tree Logic

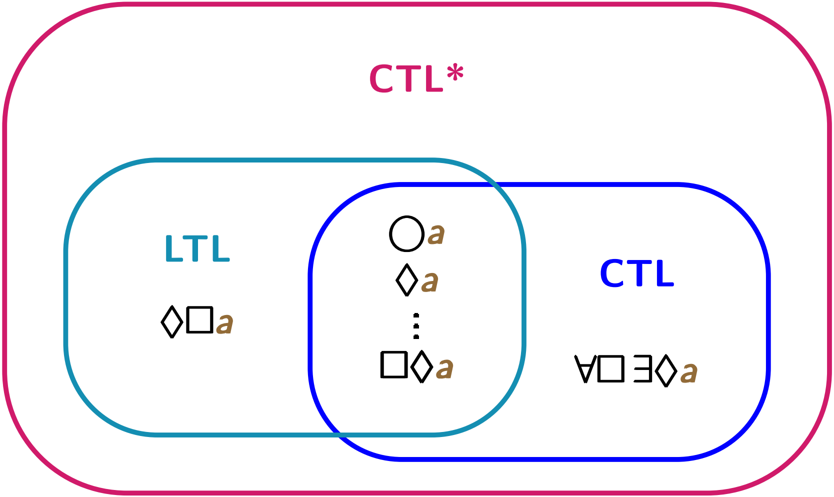
- syntax and semantics of CTL
- expressiveness of CTL and LTL
- CTL model checking
- fairness, counterexamples/witnesses
- CTL^+ and CTL^* ⟵

Equivalences and Abstraction

CTL and LTL



Combining LTL and CTL $\rightsquigarrow$ CTL*



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Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system without terminal states.

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define by structural induction:

- a satisfaction relation $\models$ for states $s \in S$ and **CTL*** state formulas
- a satisfaction relation $\models$ for infinite path fragments π in $\mathcal{T}$ and **CTL*** path formulas

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$s \models true$		
$s \models a$	iff	$a \in L(s)$
$s \models \neg\Phi$	iff	$s \not\models \Phi$
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$\pi \models \bigcirc\varphi$	iff	$\mathit{suffix}(\pi, 1) \models \varphi$
$\pi \models \varphi_1 \, \mathsf{U} \, \varphi_2$	iff	there exists $j \geq 0$ such that $\mathit{suffix}(\pi, j) \models \varphi_2$ $\mathit{suffix}(\pi, i) \models \varphi_1$ for $0 \leq i < j$

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CTL* formulas with existential quantification, e.g., Hamilton path problem (for fixed initial state)

$$\exists\Big(\bigwedge_{v\in V}\big(\Diamond v \wedge \Box(v \rightarrow \bigcirc\Box\neg v)\big)\Big)$$

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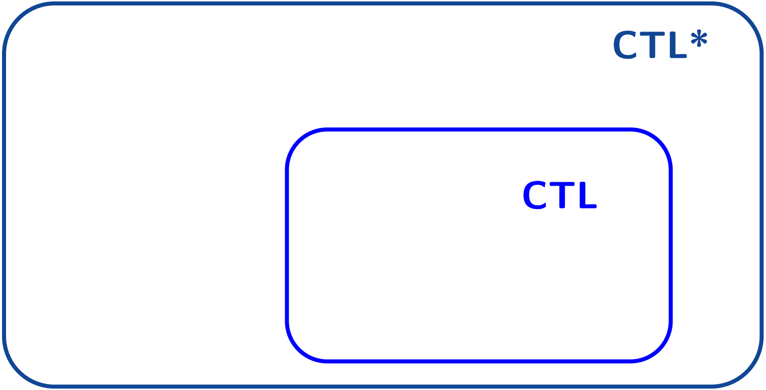

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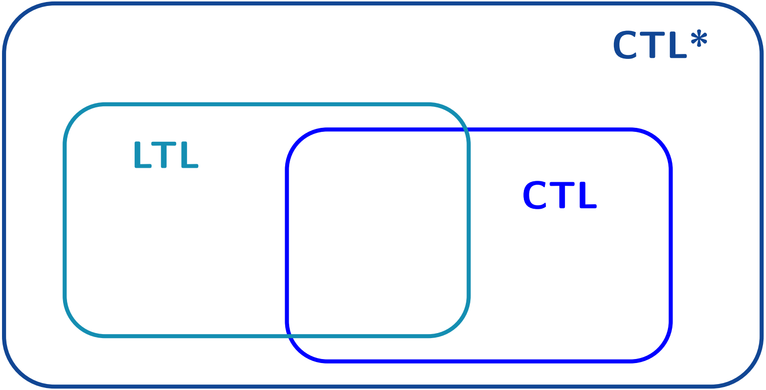

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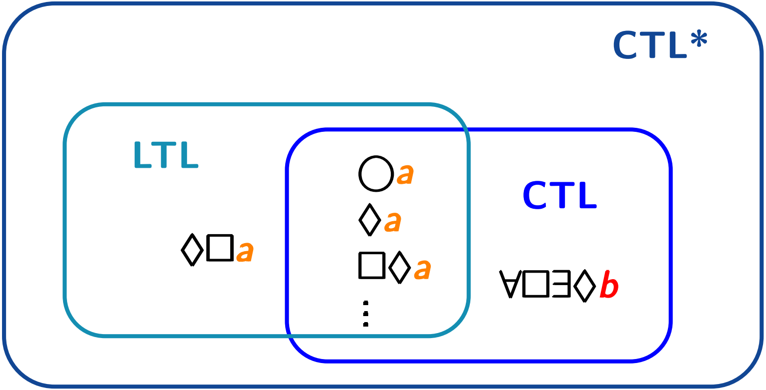

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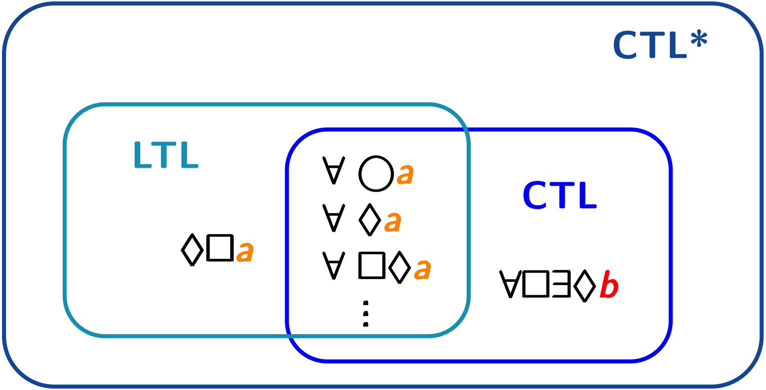

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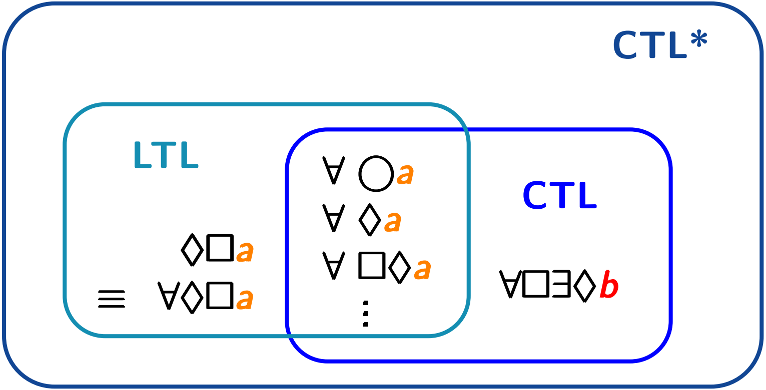
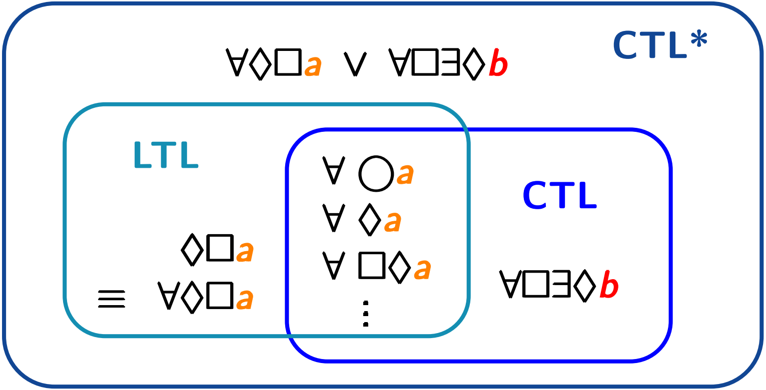


Expressiveness of CTL, LTL and CTL*

- **CTL** is a sublogic of **CTL***
- **LTL** is a sublogic of **CTL***
- **CTL*** is more expressive than **LTL** and **CTL**



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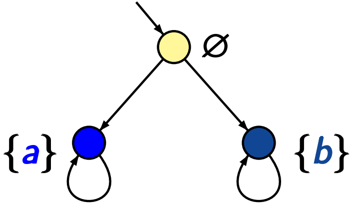

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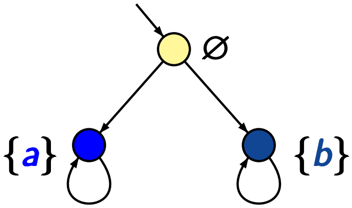

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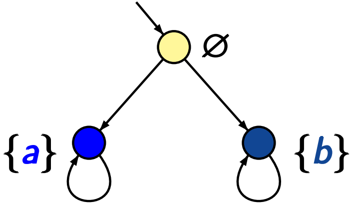

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correct. Both formulas assert that an a-state is reachable from the current state within one or more steps.

Combinations of □ and ◊ in CTL*

Combinations of $\Box$ and $\Diamond$ in CTL*

we already saw:

$$\forall\Box\,\forall\Diamond a \;\equiv\; \forall\Box\Diamond a$$

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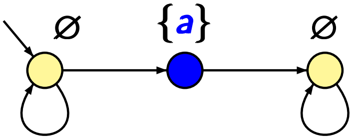

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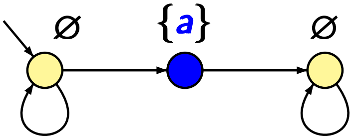
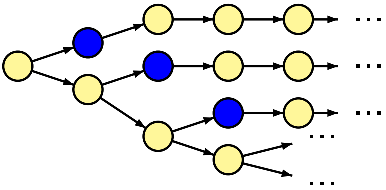


computation tree:



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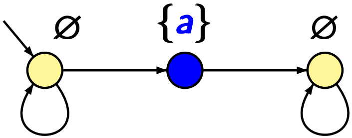

$\mathcal{T} \not\models \exists\Box\Diamond a$

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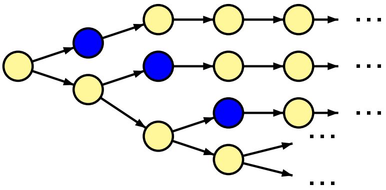

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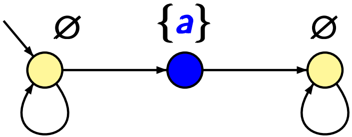

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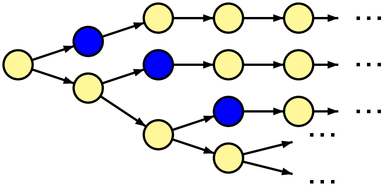

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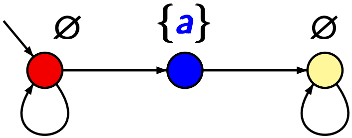

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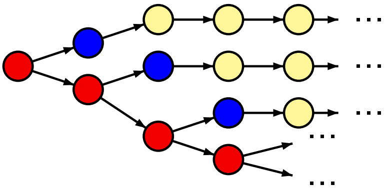

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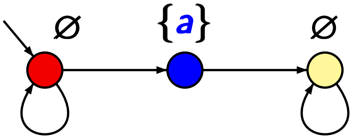

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hence: ●●●... $\models \Box\exists\Diamond a$

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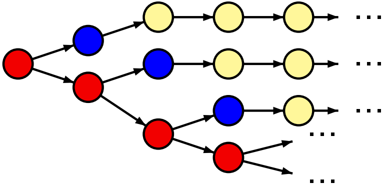

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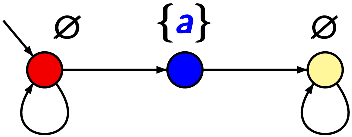


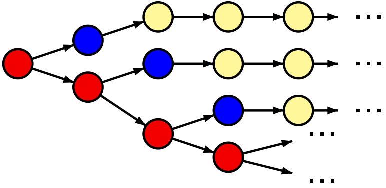
$$\exists\Box\exists\Diamond a \not\equiv \exists\Box\Diamond a$$

$\mathcal{T} \not\models \exists\Box\Diamond a$

$\mathcal{T} \models \exists\Box\exists\Diamond a$ note: $Sat(\exists\Diamond a) = \{$ ● (red), ● (blue) $\}$

hence: ●●●… (red) $\models \Box\exists\Diamond a$

computation tree:



Equivalence of CTL*-formulas

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$\exists\Diamond\Box\varphi \equiv \exists\Diamond\exists\Box\varphi$

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$\exists\Diamond\Box\varphi \equiv \exists\Diamond\exists\Box\varphi$ $\exists\Box\Diamond\varphi \not\equiv \exists\Box\exists\Diamond\varphi$

CTL* model checking

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given: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

CTL* formula Φ

question: does $\mathcal{T} \models \Phi$ hold **?**

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main procedure as for **CTL**:

```
FOR ALL subformulas Ψ of Φ DO
     compute Sat(Ψ) = {s ∈ S : s ⊨ Ψ}
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```

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main procedure as for **CTL**:

```
FOR ALL subformulas Ψ of Φ DO
     compute Sat(Ψ) = {s ∈ S : s ⊨ Ψ}
OD
IF S₀ ⊆ Sat(Φ)
     THEN   return "yes"
     ELSE   return "no"
FI
```

Recursive computation of satisfaction sets

Recursive computation of satisfaction sets

$$
\begin{aligned}
Sat(true) &= S \\
Sat(a) &= \{s \in S : a \in L(s)\} \\
Sat(\Phi_1 \wedge \Phi_2) &= Sat(\Phi_1) \cap Sat(\Phi_2) \\
Sat(\neg\Phi) &= S \setminus Sat(\Phi)
\end{aligned}
$$

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$$Sat(\forall\varphi) = Sat_{LTL}(\varphi) \quad \}$$ using an **LTL** model checker

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$$\left.\begin{array}{lcl} Sat(\forall\varphi) & = & Sat_{LTL}(\varphi) \\ Sat(\exists\varphi) & = & S \setminus Sat_{LTL}(\neg\varphi) \end{array}\right\} \text{ using an } \mathbf{LTL} \text{ model checker}$$

Example: CTL* model checking

$$\Phi = \exists\Diamond\Box a \wedge \exists\Box\big(\bigcirc b \wedge \Diamond \neg\exists(a \,\mathsf{U}\, b)\big)$$

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more precisely: existential **LTL** model checker

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more precisely: existential **LTL** model checker

1. construct an **NBA** for φ
2. check via nested DFS whether $\mathcal{T} \otimes \mathcal{A} \models \exists\Box\Diamond F$

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Fairness in CTL*

Correct or wrong?

Let $fair = \bigwedge_{1 \leq i \leq k} \Box\Diamond c_i$ be an unconditional **LTL** fairness assumption

$$s \models_{fair} \exists\Box a \quad \text{iff} \quad s \models \exists(fair \wedge \Box a)$$

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CTL with fairness → $\models_{fair}$

CTL* semantic → $\models$

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↑ **CTL*** path formula

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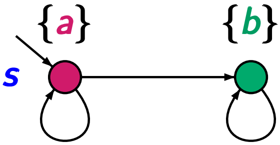

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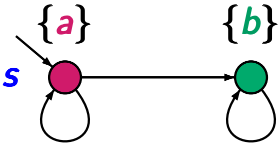

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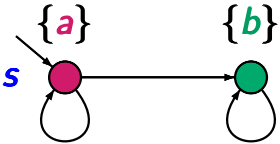

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wrong. But we have:

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall(fair \rightarrow \Box a)$$

FairCTL*

CTL* fairness assumptions are conjunctions of **CTL*** path formulas of the type

$\Box\Diamond\Phi$	unconditional fairness
$\Box\Diamond\Psi \to \Box\Diamond\Phi$	strong fairness
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obvious definition of the satisfaction relation $\models_{fair}$

Satisfaction relation $\models_{fair}$ for CTL*

$s \models_{fair} \exists\varphi$ iff there exists $\pi \in Paths(s)$ with $\pi \models fair$ and $\pi \models_{fair} \varphi$

$\models$ standard **CTL*** satisfaction relation

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iff $s \models \forall(fair \rightarrow \varphi)$ ← if φ is quantifier-free

$\models$ standard **CTL*** satisfaction relation

Complexity of CTL/LTL/CTL* model checking

Complexity of CTL/LTL model checking

	CTL	LTL	
		PSPACE-complete	
$\models$	$size(\mathcal{T}) \cdot \lvert\Phi\rvert$	$size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert)$	

Complexity of CTL/LTL model checking

	CTL	LTL	
	PTIME-complete	*PSPACE*-complete	
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Complexity of CTL/LTL model checking

	CTL	LTL
	PTIME-complete	*PSPACE*-complete
$\models$	$size(\mathcal{T}) \cdot \lvert\Phi\rvert$	$size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert)$
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Complexity of CTL/LTL/CTL* model checking

	CTL	LTL	CTL*
	PTIME-complete	*PSPACE*-complete	?
$\models$	$size(\mathcal{T}) \cdot \lvert\Phi\rvert$	$size(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert)$	?
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Complexity of CTL/LTL/CTL* model checking

	CTL	**LTL** and **CTL***
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$\models$	$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \lvert\Phi\rvert)$	$\mathcal{O}(\mathit{size}(\mathcal{T}) \cdot \exp(\lvert\varphi\rvert))$
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model complexity, i.e., for fixed formula:
$\mathcal{O}(\mathit{size}(\mathcal{T}))$

correct or wrong?

$$\exists(\Diamond a \ \wedge \ \Diamond b) \ \equiv \ \exists\Diamond(a \wedge \exists\Diamond b) \vee \exists\Diamond(b \wedge \exists\Diamond a)$$

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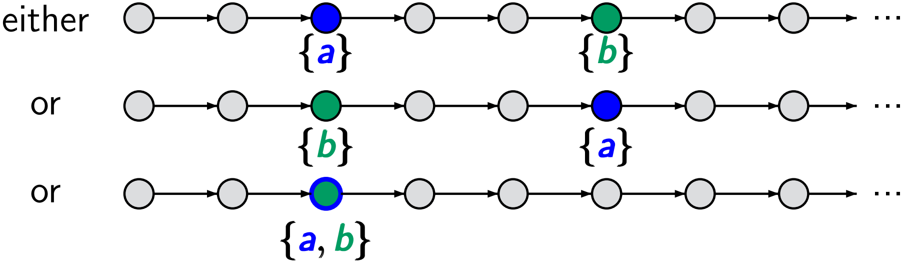
correct.

correct or wrong?

CTLST4.6-17

$$\exists(\Diamond a \wedge \Diamond b) \;\equiv\; \exists\Diamond(\, a \wedge \exists\Diamond b \,) \vee \exists\Diamond(\, b \wedge \exists\Diamond a \,)$$

correct.



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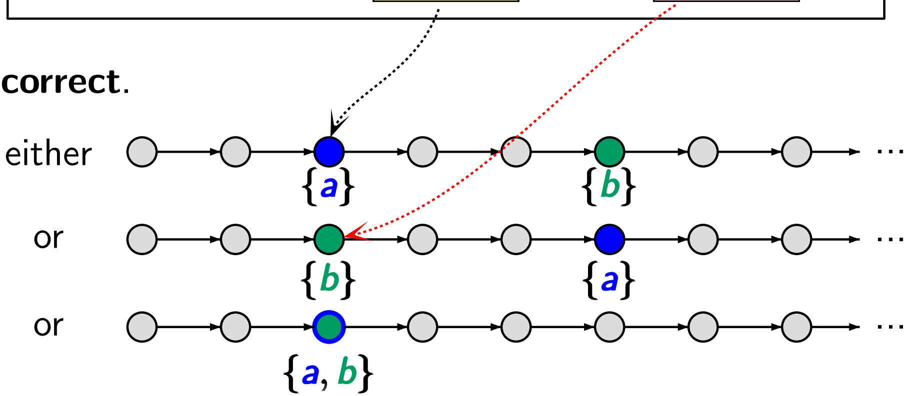

The logic CTL$^+$

The logic CTL$^+$

- **CTL** with Boolean operators for path formulas

The logic CTL$^+$

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- sublogic of **CTL***

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CTL$^+$ state formulas

$$\Phi ::= true \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

CTL$^+$ path formulas

$$\varphi ::= \ldots$$

The logic CTL^+

- **CTL** with Boolean operators for path formulas
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universal quantification can be derived: $\forall\varphi \overset{\text{def}}{=} \neg\exists\neg\varphi$

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e.g., $\exists(\Diamond b \wedge \Box a)$ and $\exists(\bigcirc b \rightarrow (a \, \mathsf{U} \, c))$
are **CTL$^+$** formulas

Expressiveness of CTL^+

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For each **CTL**$^+$-formula there exists an equivalent **CTL** formula.

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$$\exists((\Psi_1 \mathsf{U} \Psi_2) \wedge (\Phi_1 \mathsf{U} \Phi_2)) \rightsquigarrow \ldots$$

$$\exists(\bigcirc\Psi \wedge (\Phi_1 \mathsf{U} \Phi_2)) \rightsquigarrow \ldots$$

$$\exists((a \,\mathsf{U}\, b) \wedge (c \,\mathsf{U}\, d)) \equiv \exists\Big((a \wedge c) \,\mathsf{U}\, \big(b \wedge \exists(c \,\mathsf{U}\, d)\big)\Big) \vee \exists\Big((c \wedge a) \,\mathsf{U}\, \big(d \wedge \exists(a \,\mathsf{U}\, b)\big)\Big)$$

CTL$^+$ formula | **CTL** formula

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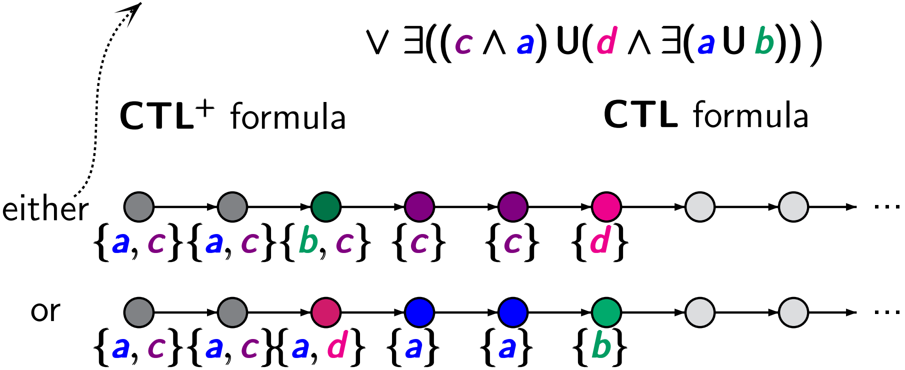
CTL$^+$ formula

CTL formula



From CTL$^+$ to CTL

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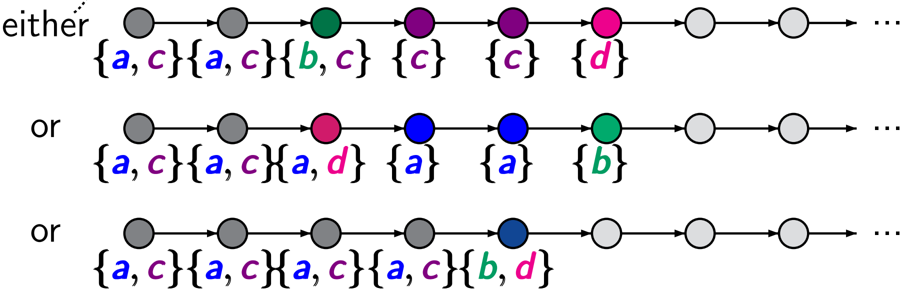
CTL$^+$ formula — **CTL** formula



From CTL$^+$ to CTL

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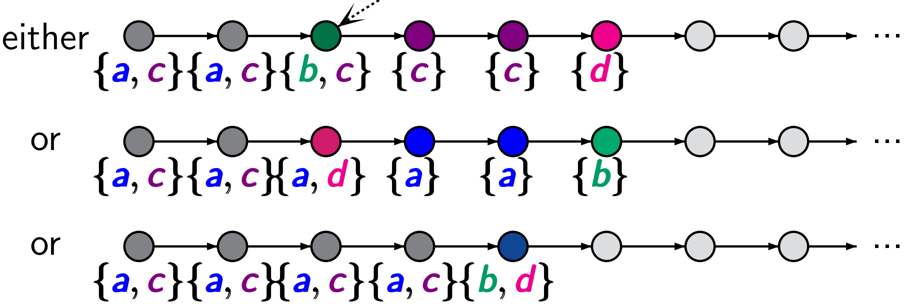
$$\vee\; \exists((c \wedge a)\,\mathsf{U}\,(d \wedge \exists(a\,\mathsf{U}\,b)))$$

CTL$^+$ formula **CTL** formula



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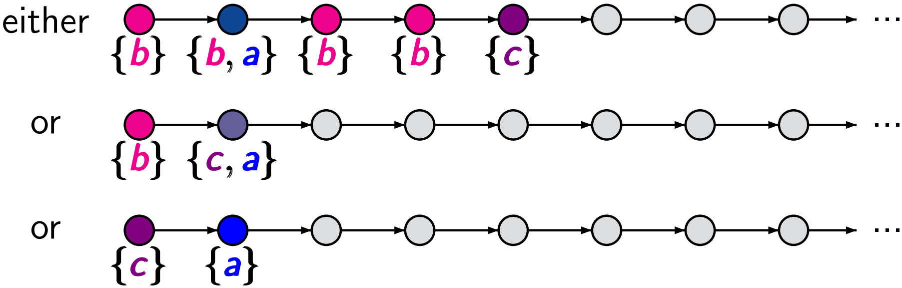

$$\exists(\bigcirc a \ \wedge\ (b \,\mathsf{U}\, c))$$

$$\equiv \quad (c \wedge \exists\bigcirc a) \vee \big(b \wedge \exists\bigcirc\big(\boxed{a \wedge \exists(b \,\mathsf{U}\, c)}\big)\big)$$