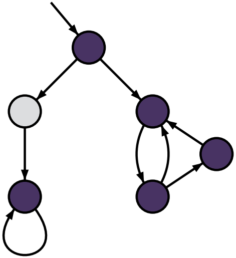
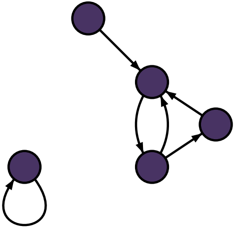
$\exists\Box a$ under strong fairness

does $\mathcal{T} \models_{fair} \exists\Box a$ hold ?



digraph G_a



$\bullet \models a$ $\circ \not\models a$

analyze the digraph G_a that results from $\mathcal{T}$ by removing all states s with $s \not\models a$

$\exists\Box a$ under strong fairness

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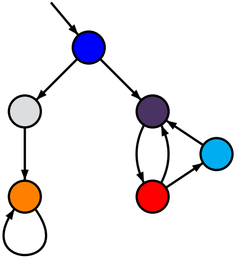

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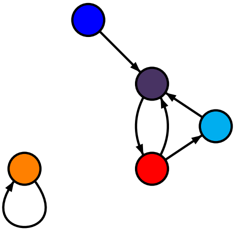

● $\widehat{=} \{b_1\}$ ● $\widehat{=} \{c_1\}$

● $\widehat{=} \{b_2\}$ ● $\widehat{=} \{c_2\}$

$$fair = (\Box\Diamond b_1 \rightarrow \Box\Diamond c_1) \wedge (\Box\Diamond b_2 \rightarrow \Box\Diamond c_2)$$

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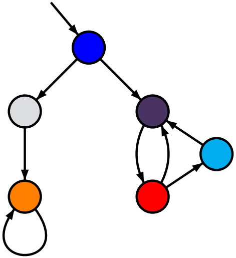

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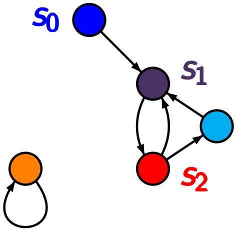

$\hat{=}\{b_1\}$ $\hat{=}\{c_1\}$

$\hat{=}\{b_2\}$ $\hat{=}\{c_2\}$

$s_0\,(s_1\,s_2)^\omega \models \neg\Box\Diamond b_2 \wedge \Box\Diamond c_1$

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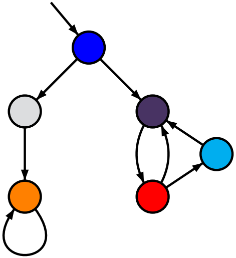

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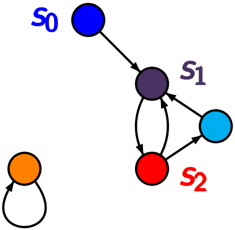

$\widehat{=}\{b_1\}$ $\widehat{=}\{c_1\}$

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$s_0\,(s_1\,s_2)^\omega \models \neg\Box\Diamond b_2 \wedge \Box\Diamond c_1$

$s_0\,(s_1\,s_2)^\omega \models fair$

$$fair = (\Box\Diamond b_1 \rightarrow \Box\Diamond c_1) \wedge (\Box\Diamond b_2 \rightarrow \Box\Diamond c_2)$$

Treatment of $\exists\Box$ under strong fairness

$$fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \to \Box\Diamond c_i)$$

$s \models_{fair} \exists\Box a$ iff there exists a path fragment

$$s_0\, s_1 \ldots s_n \ldots s_{n+r}$$

such that $r \geq 1$, $s = s_0$, $s_n = s_{n+r}$ and

- $s_j \models a$ for all $0 \leq j \leq n+r$
- for all $1 \leq i \leq k$: $\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(b_i) = \varnothing$
  or $\{s_{n+1}, \ldots, s_{n+r}\} \cap Sat(c_i) \neq \varnothing$

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↑

Thus: $D = \{s_{n+1}, \ldots, s_{n+r}\}$ is a strongly connected node-set of the digraph G_a

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↑

Thus: $D = \{s_{n+1}, \ldots, s_{n+r}\}$ is a strongly connected node-set of the digraph G_a (possibly not an SCC)

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G_a: digraph that arises from $\mathcal{T}$ by removing all states s' with $s' \not\models a$

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(2) for all $1 \leq i \leq k$:

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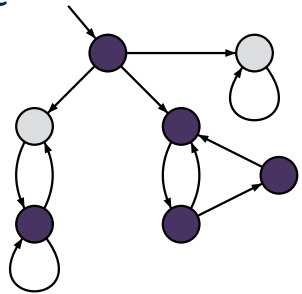
note: if $s \models_{fair} \exists\Box a$ then there might be **no SCC** D where (1) and (2) hold

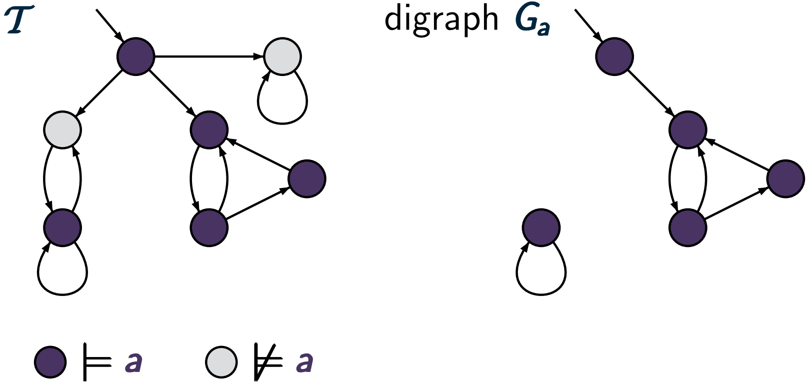
$\mathcal{T}$



$\models a$ $\not\models a$

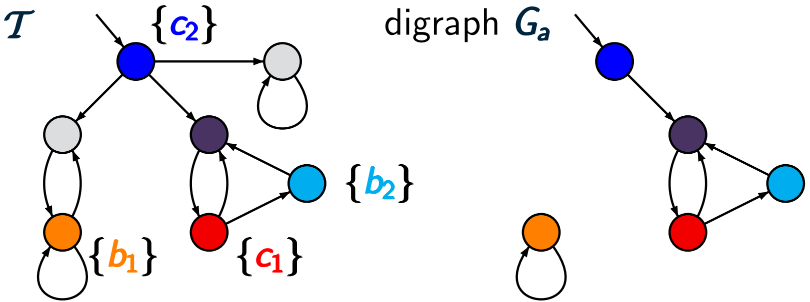
computation of $Sat_{fair}(\exists\Box a)$

Example: computation of $Sat_{fair}(\exists\Box a)$



computation of $Sat_{fair}(\exists\Box a)$
by analyzing the digraph G_a

Example: computation of $Sat_{fair}(\exists\Box a)$



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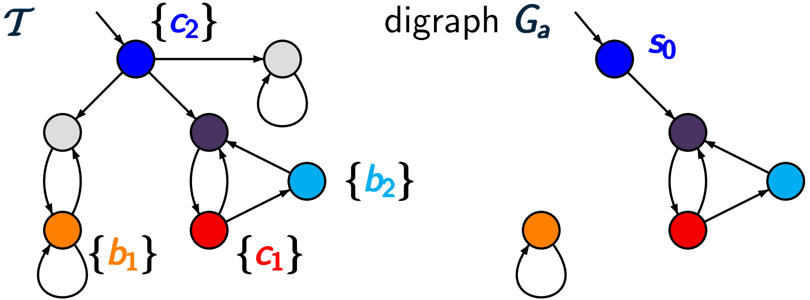

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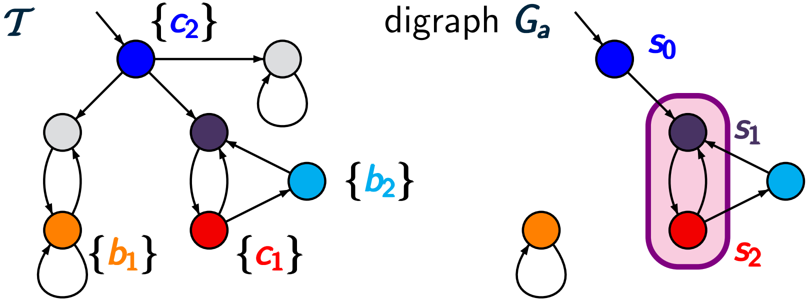
$s_0 \models_{fair} \exists\Box a$

Example: computation of $Sat_{fair}(\exists\Box a)$



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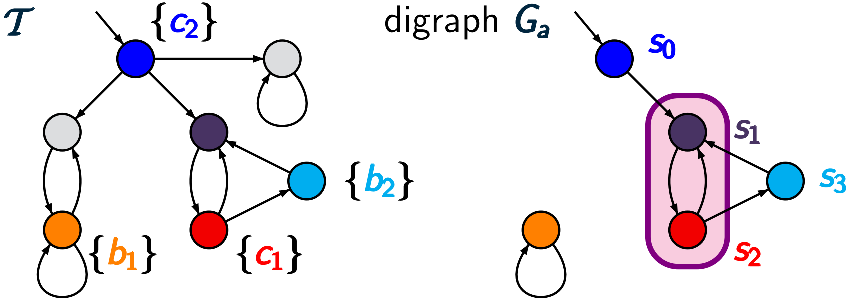
Example: computation of $Sat_{fair}(\exists\Box a)$



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$s_0 \models_{fair} \exists\Box a$ as $s_0\, s_1\, s_2\, s_1\, s_2 \ldots \models_{LTL} fair$

$Sat_{fair}(\exists\Box a) = \{s_0, s_1, s_2, s_3\}$

CTL model checking with fairness

treatment of $\exists\Box$ for **CTL** with fairness treatment of $\exists\Box$ for **CTL** with fairness

here: explanations only for strong fairness

weak fairness and combinations of weak/strong fairness can be treated in an analogous way

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treatment of $\exists\Box$ for **CTL** with fairness

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case 1: unconditional fairness

case 2: $fair = \Box\Diamond b \rightarrow \Box\Diamond c$

case 3: arbitrary strong fairness assumption

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$s \models_{fair} \exists\Box a$ iff **?**

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$s \models_{fair} \exists\Box a$ iff there exists a nontrivial **SCC** C in G_a that is reachable from s and $C \cap Sat(c_i) \neq \varnothing$ for $i = 1, \ldots, k$

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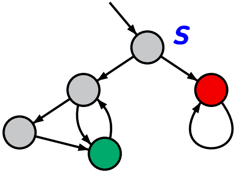
digraph G_a



fairness assumption:
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$\exists\Box a$ under unconditional fairness

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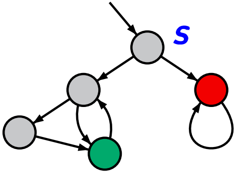

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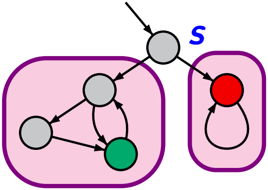

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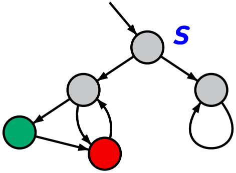

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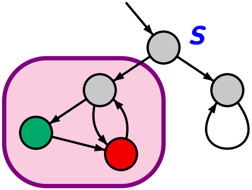

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CTL model checking with fairness

treatment of $\exists\Box$ for CTL with fairness

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case 1: unconditional fairness ✓

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Strong fairness: 1 fairness requirement

$$fair = \Box\Diamond b \rightarrow \Box\Diamond c$$

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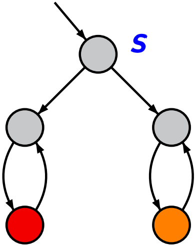
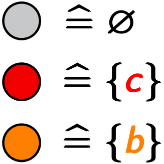
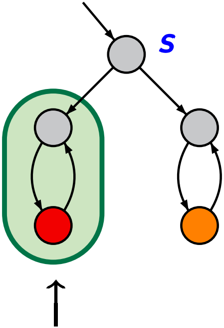


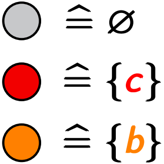


Strong fairness: 1 fairness requirement

$fair = \Box\Diamond b \rightarrow \Box\Diamond c$

digraph G_a





nontrivial **SCC** C of G_a with $C \cap Sat(c) \neq \varnothing$

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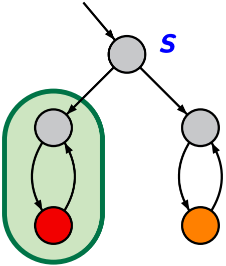
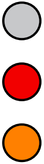
digraph G_a



$\longleftarrow$ $s \models_{fair} \exists\Box a$



$\uparrow$

nontrivial SCC C of G_a with $C \cap Sat(c) \neq \varnothing$

Strong fairness: 1 fairness requirement

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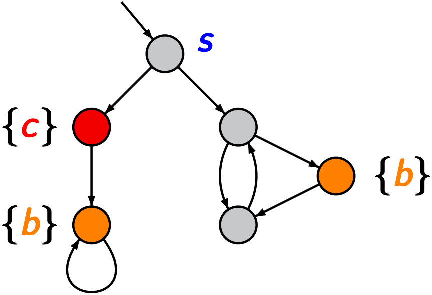
digraph G_a



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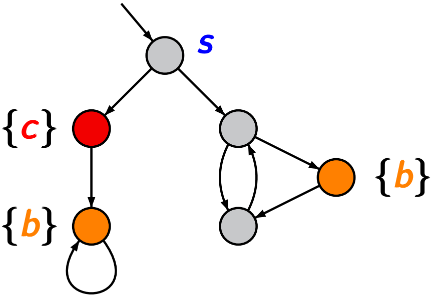

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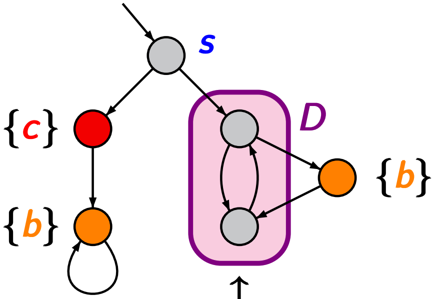

$s \models_{fair} \exists\Box a$

strongly connected node-set D of G_a with $D \cap Sat(b) = \varnothing$

Strong fairness: 1 fairness requirement

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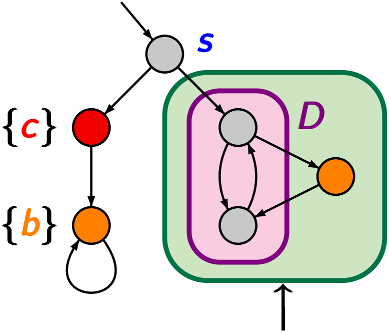
digraph G_a



$s \models_{fair} \exists\Box a$

nontrivial **SCC** C of G_a that contains a nontrivial **SCC** D of $G_a|_C \setminus Sat(b)$

CTL model checking with fairness

treatment of $\exists\Box$ for CTL with fairness

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Example: 2 strong fairness conditions

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$$\mathit{fair} = (\Box\Diamond b_1 \rightarrow \Box\Diamond c_1) \;\wedge\; (\Box\Diamond b_2 \rightarrow \Box\Diamond c_2)$$

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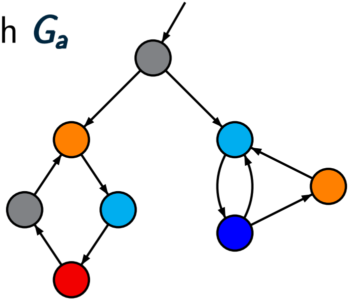

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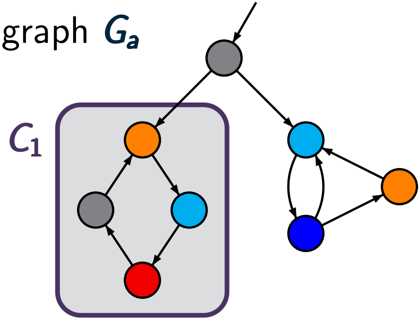

first SCC: $C_1 \cap Sat(c_2) = \varnothing$

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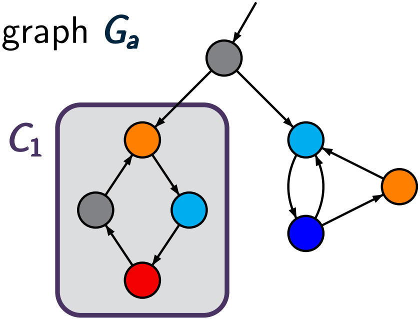

first SCC: $C_1 \cap Sat(c_2) = \varnothing$

analyze $C_1 \setminus Sat(b_2)$ w.r.t. $\Box\Diamond b_1 \rightarrow \Box\Diamond c_1$

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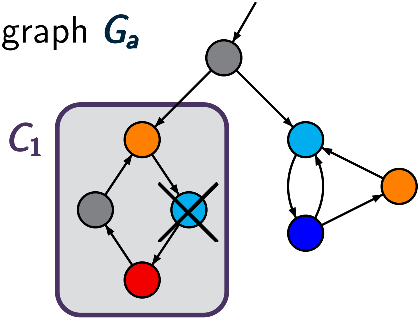

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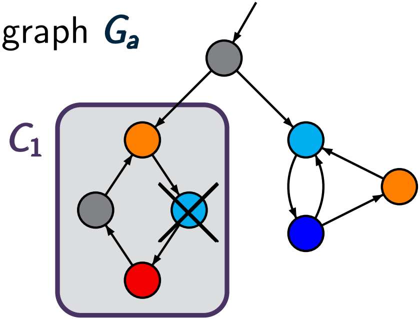
analyze $C_1 \setminus Sat(b_2)$ w.r.t. $\Box\Diamond b_1 \rightarrow \Box\Diamond c_1$

Example: 2 strong fairness conditions

$$fair = (\Box\Diamond b_1 \rightarrow \Box\Diamond c_1) \;\wedge\; (\Box\Diamond b_2 \rightarrow \Box\Diamond c_2)$$

digraph G_a



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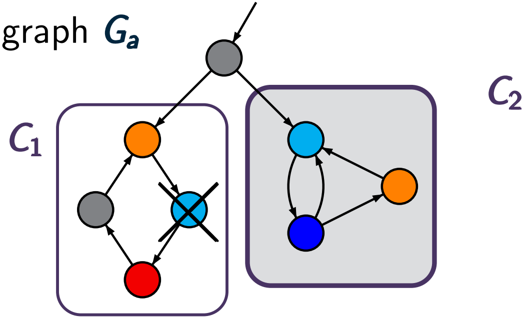

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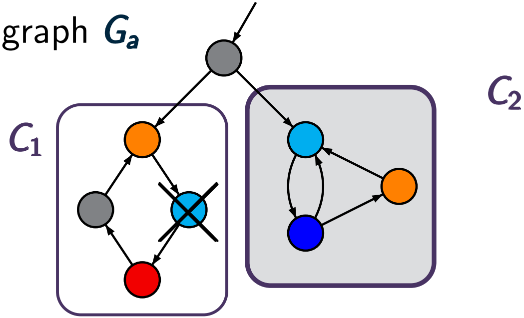

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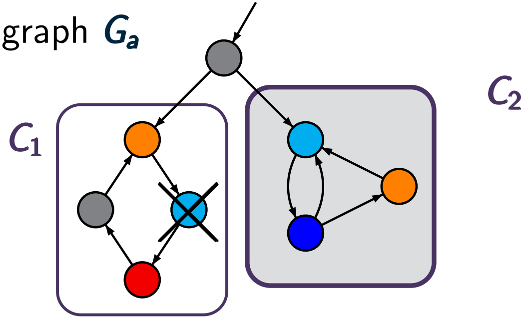

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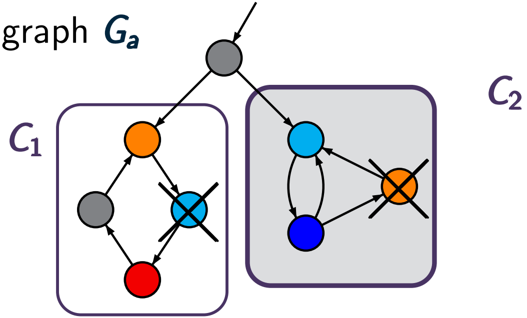

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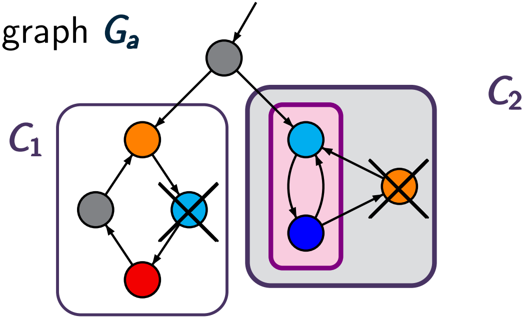

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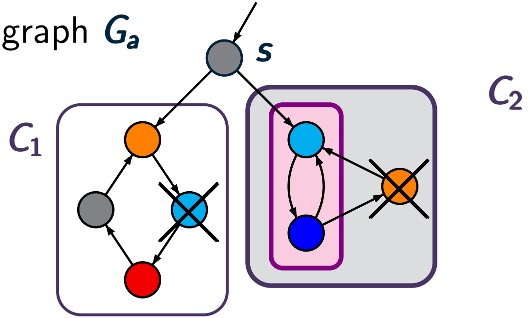

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hence: $s \models_{fair} \exists\Box a$

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↑ backward search from T

```
compute the SCCs of the digraph G_a;
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FOR ALL nontrivial SCCs C of G_a DO
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OD
Sat_fair(∃□a) := {s ∈ S : Reach_{G_a}(s) ∩ T ≠ ∅}
```

(backward search from T)

time complexity: $\mathcal{O}(size(\mathcal{T}) \cdot |fair|)$

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Recursive algorithm *CheckFair*(...)

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Recursive algorithm *CheckFair*(...)

algorithm *CheckFair*$\left(C, k, \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)\right)$ returns

"true" if there exists a cyclic path fragment $s_0\, s_1 \ldots s_n$ in C such that

$$(s_0\, s_1 \ldots s_{n-1})^\omega \models \bigwedge_{1 \leq i \leq k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i)$$

"false" otherwise

Recursive algorithm $\mathit{CheckFair}(\ldots)$

pseudo code for $\mathit{CheckFair}(C, k, \bigwedge_{1 \le i \le k} (\Box\Diamond b_i \rightarrow \Box\Diamond c_i))$

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recurrence for the time complexity:

$T(n, k) = \ \ldots$ where $n = \mathit{size}(C)$

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$\mathcal{O}(\, size(C) \cdot k \,)$

CTL model checking with fairness

input: finite transition system $\mathcal{T}$
CTL fairness assumption *fair*
CTL formula Φ

output: "**yes**", if $\mathcal{T} \models_{fair} \Phi$. "**no**" otherwise.

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transform Φ into an equivalent CTL formula in existential normal form

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↑

i.e., with the basic modalities $\exists\bigcirc$, $\exists \mathsf{U}$ and $\exists\Box$

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calculate $\mathit{Sat}_{\mathit{fair}}(\exists\Box \mathit{true})$;

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CASE Ψ is:

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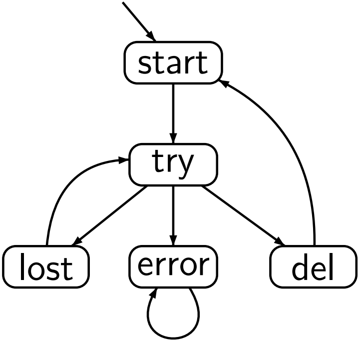
OD

IF $S_0 \subseteq Sat_{fair}(\Phi)$ THEN return "**yes**"

ELSE return "**no**"

FI

Example: CTL model checking with fairness



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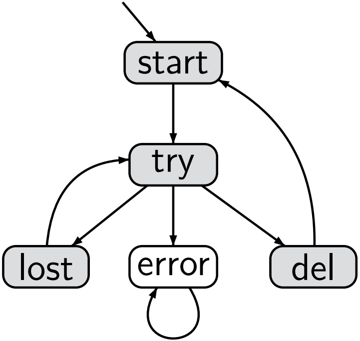

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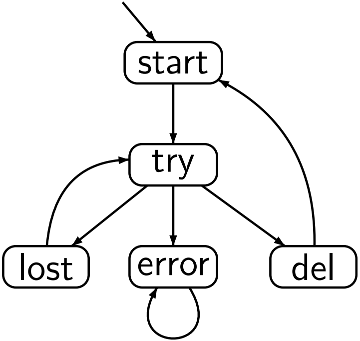

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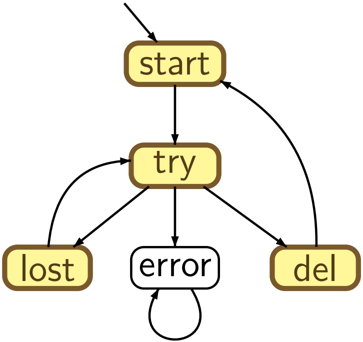

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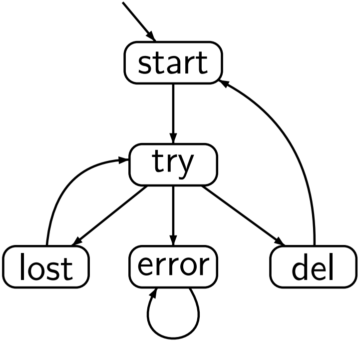

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existential normal form

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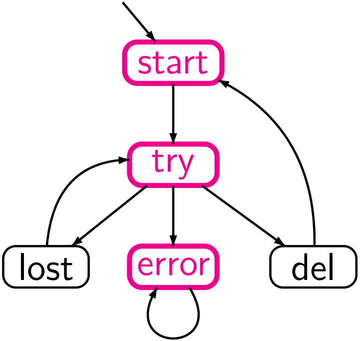

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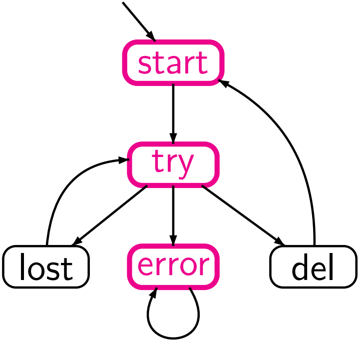

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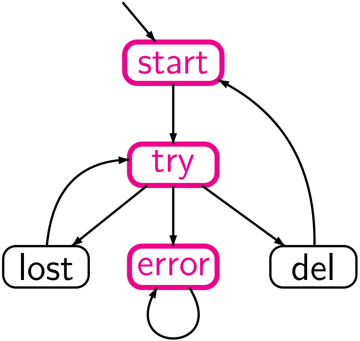

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$Sat_{fair}(\exists\bigcirc a)$

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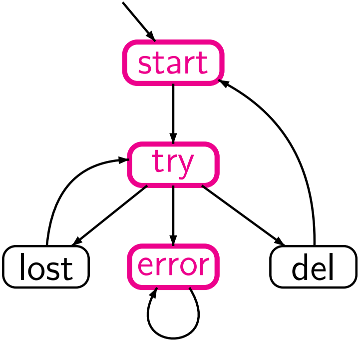

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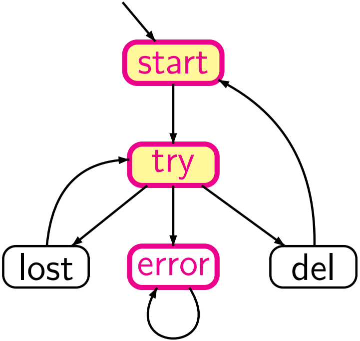

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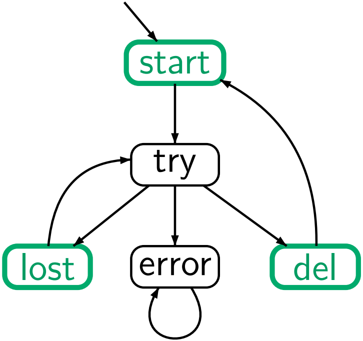

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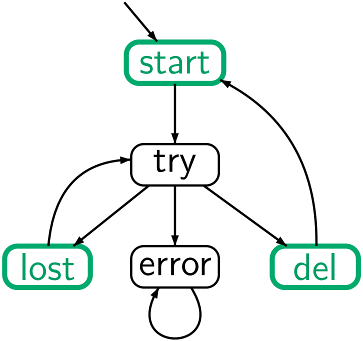
Example: CTL model checking with fairness



$\Phi = \exists\Diamond\ \forall\bigcirc(lost \vee del)$

$\equiv \exists\Diamond\neg\exists\bigcirc\ (\neg lost \wedge \neg del)$

$\rightsquigarrow \exists\Diamond \boxed{\neg\exists\bigcirc a}$

$fair = \Box\Diamond\ \exists\Diamond del \ \rightsquigarrow \Box\Diamond c$ where $Sat(c) = S \setminus \{error\}$

$Sat_{fair}(\exists\Box true) \ = Sat(a_{fair}) = S \setminus \{error\}$

$Sat_{fair}(\exists\bigcirc a) \ = Sat(\exists\bigcirc(\ a \wedge a_{fair}\ )) = \{start, lost, del\}$

$Sat_{fair}(\neg\exists\bigcirc a)$

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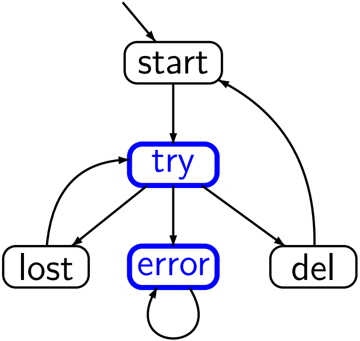

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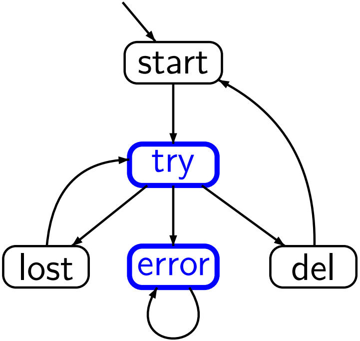

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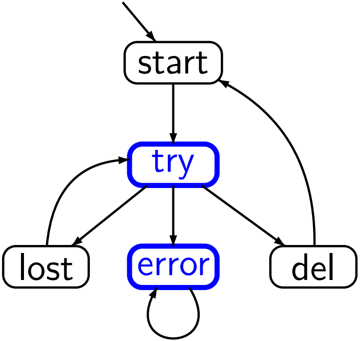

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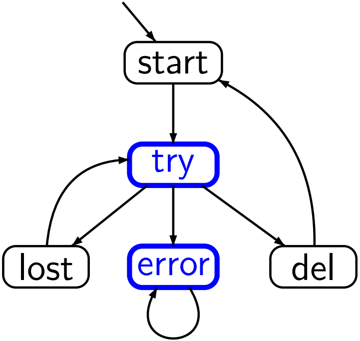

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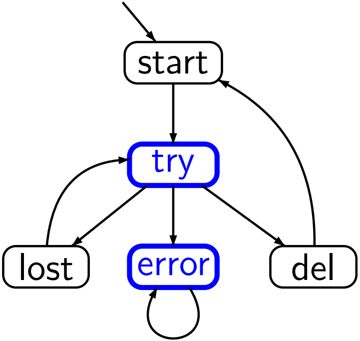

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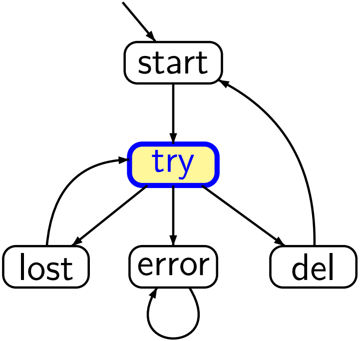

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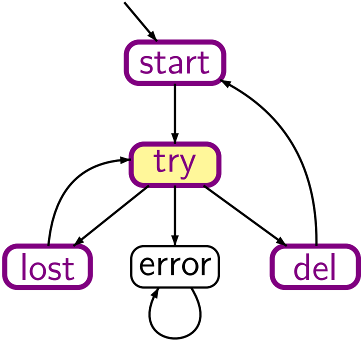

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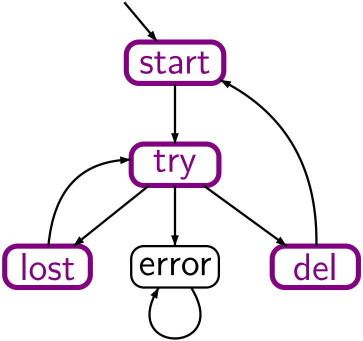

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Correct or wrong?

$$s \models_{fair} \forall\bigcirc a \quad \text{iff} \quad s \models \forall\bigcirc(a \wedge a_{fair})$$

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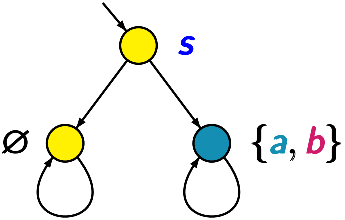

$fair = \Box\Diamond b$

Correct or wrong?

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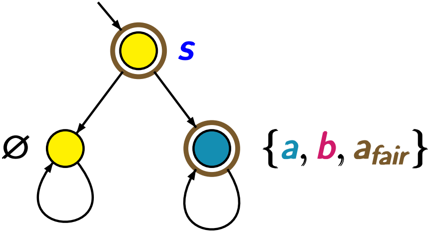

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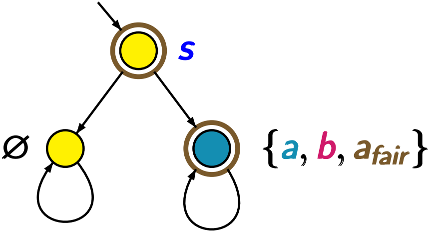

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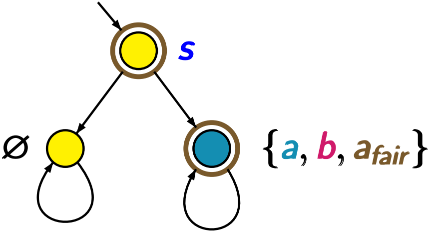

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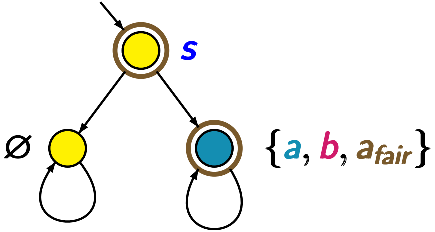

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but correct is:

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad ?$$

Correct or wrong?

$$s \models_{fair} \forall\bigcirc a \quad \text{iff} \quad s \models \forall\bigcirc(a \wedge a_{fair})$$

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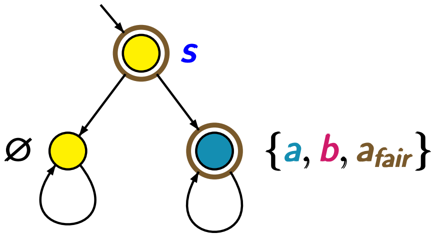

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but correct is:

$$s \models_{fair} \forall\bigcirc a \quad \text{iff} \quad s \models \forall\bigcirc(a_{fair} \rightarrow a)$$

Correct or wrong?

$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$

Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

Correct or wrong?

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correct

$s \models_{fair} \forall\Box a$ iff $s \models_{fair} \neg\exists\Diamond\neg a$

Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

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Correct or wrong?

$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

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$$\text{iff} \quad s \not\models \exists\Diamond(\neg a \wedge a_{fair})$$

Correct or wrong?

$s \models_{fair} \forall\Box a$ iff $s \models \forall\Box(a_{fair} \rightarrow a)$
iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$$s \models_{fair} \forall\Box a \text{ iff } s \models_{fair} \neg\exists\Diamond\neg a$$
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$$s \models_{fair} \forall\Box a \quad \text{iff} \quad s \models \forall\Box(a_{fair} \rightarrow a)$$

iff there is no state s' reachable from s with $s' \models \neg a \wedge a_{fair}$

correct

$$\begin{aligned} s \models_{fair} \forall\Box a \quad &\text{iff} \quad s \models_{fair} \neg\exists\Diamond\neg a \\ &\text{iff} \quad s \not\models_{fair} \exists\Diamond\neg a \\ &\text{iff} \quad s \not\models \exists\Diamond(\neg a \wedge a_{fair}) \\ &\text{iff} \quad s \models \neg\exists\Diamond(\neg a \wedge a_{fair}) \;\equiv\; \forall\Box(a_{fair} \rightarrow a) \end{aligned}$$

Summary

We just saw:

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{fair} \rightarrow a)$$

$$s \models_{fair} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{fair} \rightarrow a)$$

Correct or wrong?

We just saw:

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{fair} \rightarrow a)$$

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Is the following statement correct **?**

$$s \models_{fair} \forall (b \,\mathsf{U}\, a) \quad \text{iff} \quad s \models \forall \big(b \,\mathsf{U} (a_{fair} \rightarrow a)\big)$$

Correct or wrong?

We just saw:

$$s \models_{fair} \forall \bigcirc a \quad \text{iff} \quad s \models \forall \bigcirc (a_{fair} \rightarrow a)$$

$$s \models_{fair} \forall \Box a \quad \text{iff} \quad s \models \forall \Box (a_{fair} \rightarrow a)$$

Is the following statement correct **?**

$$s \models_{fair} \forall (b \, \mathsf{U} \, a) \quad \text{iff} \quad s \models \forall \big(b \, \mathsf{U} (a_{fair} \rightarrow a)\big)$$

wrong.

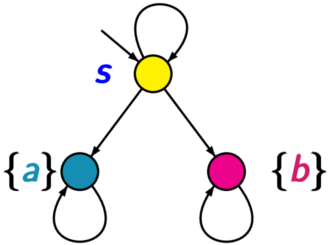
Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

wrong.

$fair = \Box\Diamond b$



$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc((\exists\Diamond a) \wedge a_{fair})$$

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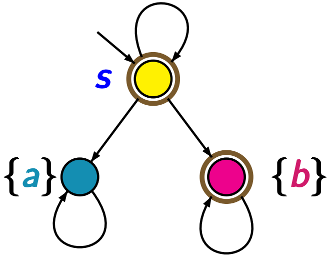

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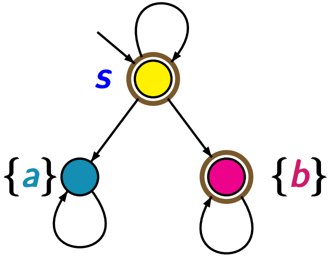

$fair = \Box\Diamond b$

$s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$

Correct or wrong?

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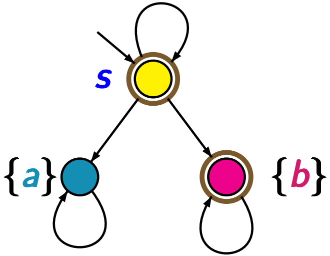

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regard $s \rightarrow s$

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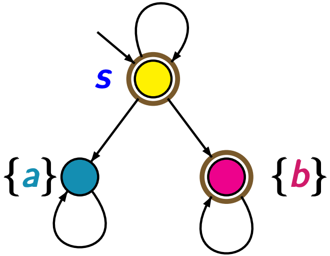

$fair = \Box\Diamond b$

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regard $s \to s$

$s \not\models_{fair} \exists\bigcirc\exists\Diamond a$

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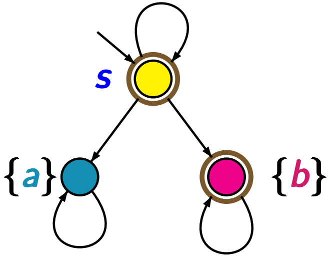

$fair = \Box\Diamond b$

$s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$

regard $s \rightarrow s$

$s \not\models_{fair} \exists\bigcirc\exists\Diamond a$

(note $Sat_{fair}(\exists\Diamond a) = \varnothing$)

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc\big((\exists\Diamond a) \wedge a_{fair}\big)$$

wrong.

$$s \models_{fair} \exists(a\,\mathsf{W}\,c) \quad \text{iff} \quad s \models \exists(a\,\mathsf{W}(c \wedge a_{fair}))$$

remind: W = weak until

Correct or wrong?

$$s \models_{fair} \exists\bigcirc\exists\Diamond a \quad \text{iff} \quad s \models \exists\bigcirc((\exists\Diamond a) \wedge a_{fair})$$

wrong.

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remind: $\mathbf{W}$ = weak until

wrong.

Correct or wrong?

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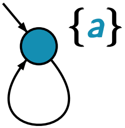

Correct or wrong?

$$s \models_{fair} \exists \bigcirc \exists \Diamond a \quad \text{iff} \quad s \models \exists \bigcirc ((\exists \Diamond a) \wedge a_{fair})$$

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$$s \models_{fair} \exists (a \, \mathsf{W} \, c) \quad \text{iff} \quad s \models \exists (a \, \mathsf{W} (c \wedge a_{fair}))$$

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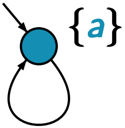

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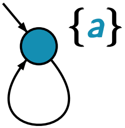

$fair = \Box \Diamond b$

$s \models \exists (a \, \mathsf{W} (c \wedge a_{fair}))$

$s \not\models_{fair} \exists (a \, \mathsf{W} \, c)$

Summary: fairness in CTL

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CTL fairness assumptions: formulas similar to **LTL**

e.g., $fair = \bigwedge_{1 \leq i \leq k} (\Box\Diamond\Psi_i \rightarrow \Box\Diamond\Phi_i)$

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CTL satisfaction relation with fairness:

$s \models_{fair} \exists\varphi$ iff there exists $\pi \in Paths(s)$ with $\pi \models fair$ and $\pi \models_{fair} \varphi$

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model checking for **CTL** with fairness:

CTL fairness assumptions: formulas similar to **LTL**

e.g., $\mathit{fair} = \bigwedge_{1 \le i \le k} (\Box\Diamond\Psi_i \rightarrow \Box\Diamond\Phi_i)$

CTL satisfaction relation with fairness:

$s \models_{\mathit{fair}} \exists\varphi$ iff there exists $\pi \in \mathit{Paths}(s)$ with $\pi \models \mathit{fair}$ and $\pi \models_{\mathit{fair}} \varphi$

model checking for **CTL** with fairness:

- $\exists\bigcirc$, $\exists\mathrm{U}$, $\forall\bigcirc$, $\forall\Box$ via **CTL** model checker

Summary: fairness in CTL

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- complexity: $\mathcal{O}(size(\mathcal{T}) \cdot |\Phi| \cdot |fair|)$