GNBA for LTL-formula φ

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \varnothing$

if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\bigcirc\psi \in B \quad \text{iff} \quad \psi \in B'$$

$$\psi_1 \mathsf{U} \psi_2 \in B \quad \text{iff} \quad (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathsf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathsf{U} \psi_2} : \psi_1 \mathsf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathsf{U} \psi_2} = \{B \in Q : \psi_1 \mathsf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

Induction step: next step

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\, B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0\, A_1\, A_2 \ldots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

Induction step: next step

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iff $\psi' \in B_1$ (definition of δ)

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$\psi \in B_0$

iff $\psi' \in B_1$ (definition of δ)

iff $A_1\, A_2\, A_3 \ldots \models \psi'$ (induction hypothesis)

iff $A_0\, A_1\, A_2\, A_3 \ldots \models \psi$ (semantics of $\bigcirc$)

Induction step: until

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic,
i.e., if ψ, $\psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$\psi \notin B$	iff	$\neg\psi \in B$
$\psi_1 \wedge \psi_2 \in B$	iff	$\psi_1 \in B$ and $\psi_2 \in B$
$\mathit{true} \in cl(\varphi)$	implies	$\mathit{true} \in B$

(ii) B is locally consistent with respect to until **U**,
i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

if $\psi_1 \mathbf{U} \psi_2 \in B$ and $\psi_2 \notin B$ then $\psi_1 \in B$

if $\psi_2 \in B$ then $\psi_1 \mathbf{U} \psi_2 \in B$

Recall: GNBA for LTL-formula φ

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Induction step for $\psi = \psi_1 \, \mathsf{U} \, \psi_2$:

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$$A_j\, A_{j+1}\, A_{j+2} \ldots \models \psi_2$$
$$A_{j-1}\, A_j\, A_{j-1} \ldots \models \psi_1$$
$$A_{j-2}\, A_{j-1}\, A_j \ldots \models \psi_1$$
$$\vdots$$
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$$
\begin{array}{lll}
A_j\, A_{j+1}\, A_{j+2} \ldots \models \psi_2 & \overset{\text{IH}}{\Rightarrow} & \psi_2 \in B_j \\
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B_j is elementary

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$$\left.\begin{array}{l} \quad\ \psi \in B_0 \ \wedge\ \psi_2 \notin B_0 \\ \Rightarrow \psi \in B_1 \ \wedge\ \psi_2 \notin B_1 \\ \Rightarrow \psi \in B_2 \ \wedge\ \psi_2 \notin B_2 \\ \qquad\qquad \vdots \end{array}\right\} \Longrightarrow \forall j \geq 0.\ B_j \notin F_\psi \text{ where}$$

$$F_\psi = \{B : \psi \notin B \text{ or } \psi_2 \in B\}$$

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"$\Longrightarrow$" Suppose $\psi \in B_0$. There exists $j \geq 0$ with $\psi_2 \in B_j$, since otherwise $\forall j \geq 0.\, \psi_2 \notin B_j$ and therefore:

$$\left.\begin{array}{rl} & \psi \in B_0 \ \wedge \ \psi_2 \notin B_0 \\ \Rightarrow & \psi \in B_1 \ \wedge \ \psi_2 \notin B_1 \\ \Rightarrow & \psi \in B_2 \ \wedge \ \psi_2 \notin B_2 \\ & \vdots \end{array}\right\} \Longrightarrow \forall j \geq 0.\ B_j \notin F_\psi \text{ where}$$

$$F_\psi = \{B : \psi \notin B \text{ or } \psi_2 \in B\}$$

Contradiction!

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Complexity: LTL $\rightsquigarrow$ NBA

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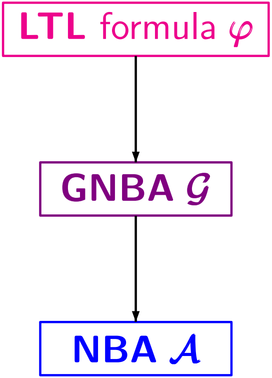

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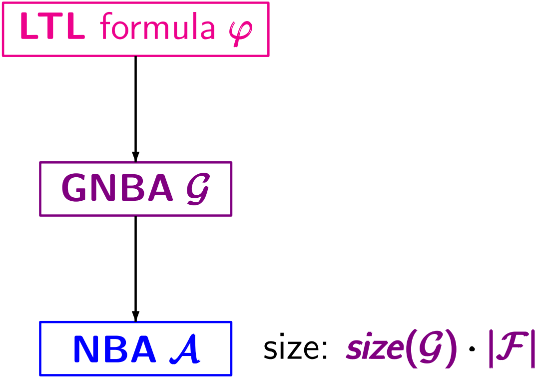

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LTL formula φ

↓

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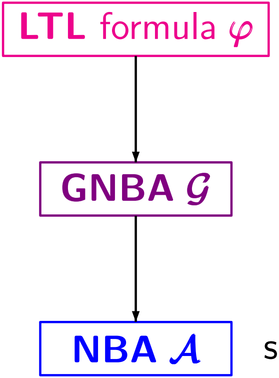

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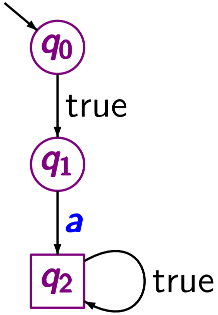
NBA for $\bigcirc a$



constructed GNBA has **4** states and **8** edges

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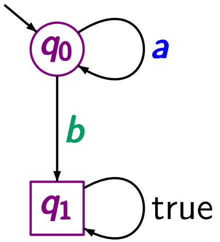
NBA for a **U** b



constructed (G)NBA has **5** states and **20** edges

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... but there exists LTL formulas φ_n such that

- $|\varphi_n| = \mathcal{O}(poly(n))$
- each NBA for φ_n has at least 2^n states

LT-properties that have no "small" NBA

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consider the following family of LT-properties $(E_n)_{n \geq 1}$:

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LT-property E_n for n=1

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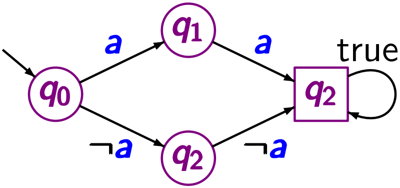

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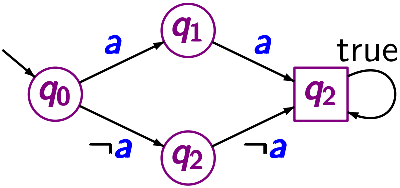

LTL-formula:
$a \leftrightarrow \bigcirc a$

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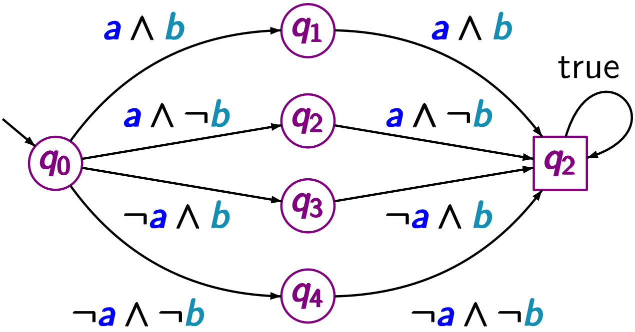
NBA for E_1 if $AP = \{a, b\}$:



$$E_1 = \begin{cases} \text{set of all infinite words over } 2^{AP} \text{ of the form} \\ A\,A\,B_1\,B_2\,B_3\,B_4 \ldots \text{ where } A, B_j \subseteq AP \text{ for } j \geq 0 \end{cases}$$

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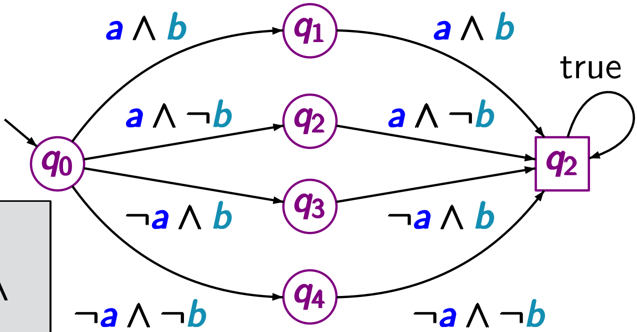
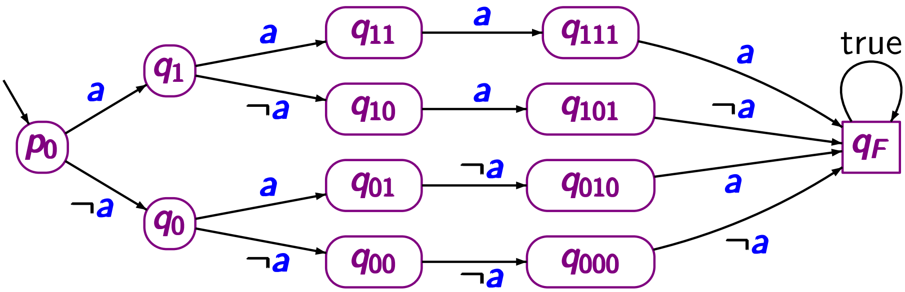


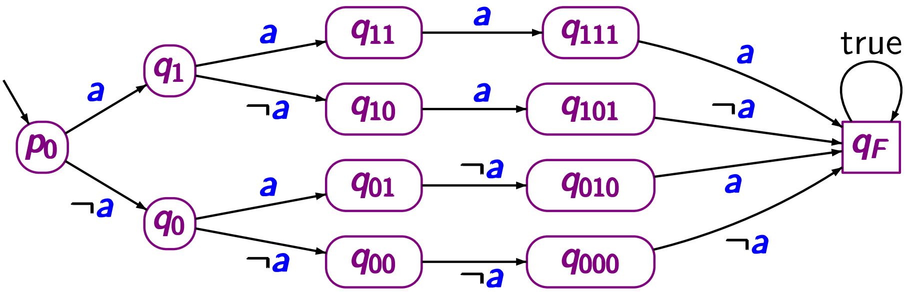
LTL-formula:

$(a \leftrightarrow \bigcirc a) \wedge$
$(b \leftrightarrow \bigcirc b)$

LT property E_n for n=2 and $AP = \{a\}$



$$E_2 = \{ A_1 A_2 A_1 A_2 \sigma : \ A_1, A_2 \subseteq AP, \sigma \in (2^{AP})^\omega \}$$



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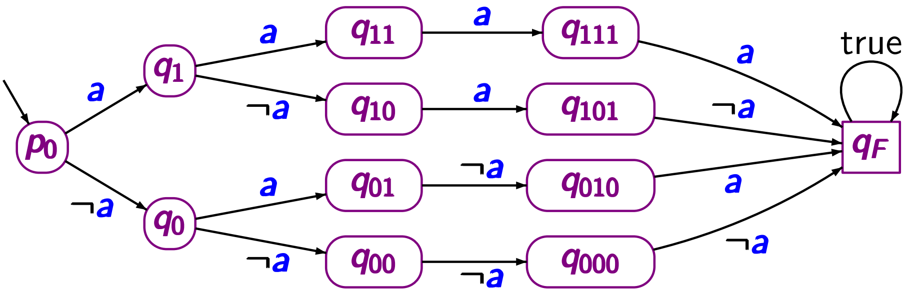

general case: each **NBA** for E_n has $\geq 2^n$ states

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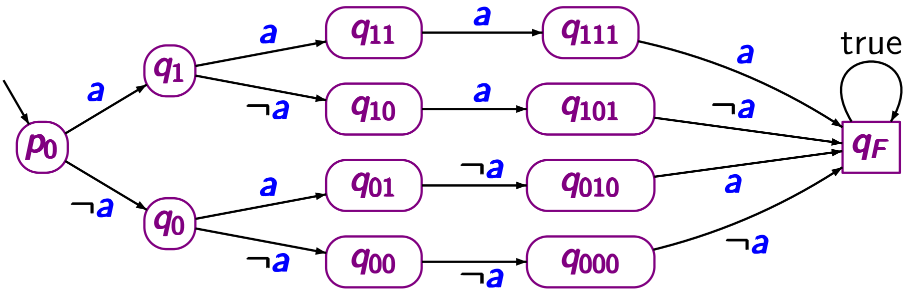
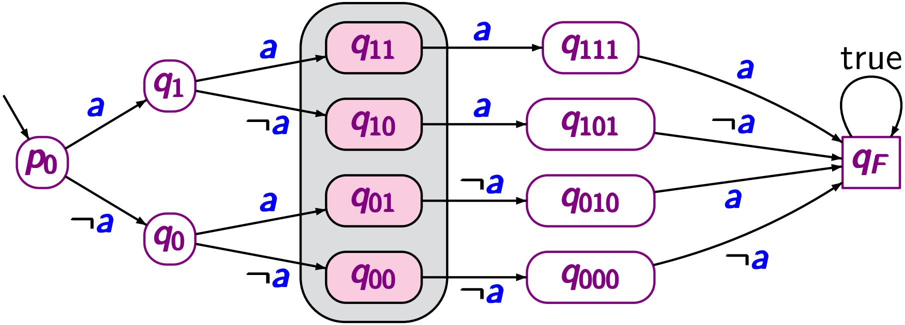


> *general case:* each **NBA** for E_n has $\geq 2^n$ states

$$E_n = Words(\varphi_n) \text{ where } \varphi_n = \bigwedge_{a \in AP} \bigwedge_{0 \leq i < n} (\bigcirc^i a \leftrightarrow \bigcirc^{n+i} a)$$

LT property E_n for $n=2$ and $AP = \{a\}$



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