Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL
- automata-based LTL model checking ←
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

LTL model checking problem

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(without terminal states)
LTL-formula φ over AP

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$$\pi \not\models \varphi, \text{ i.e., } \pi \models \neg\varphi$$

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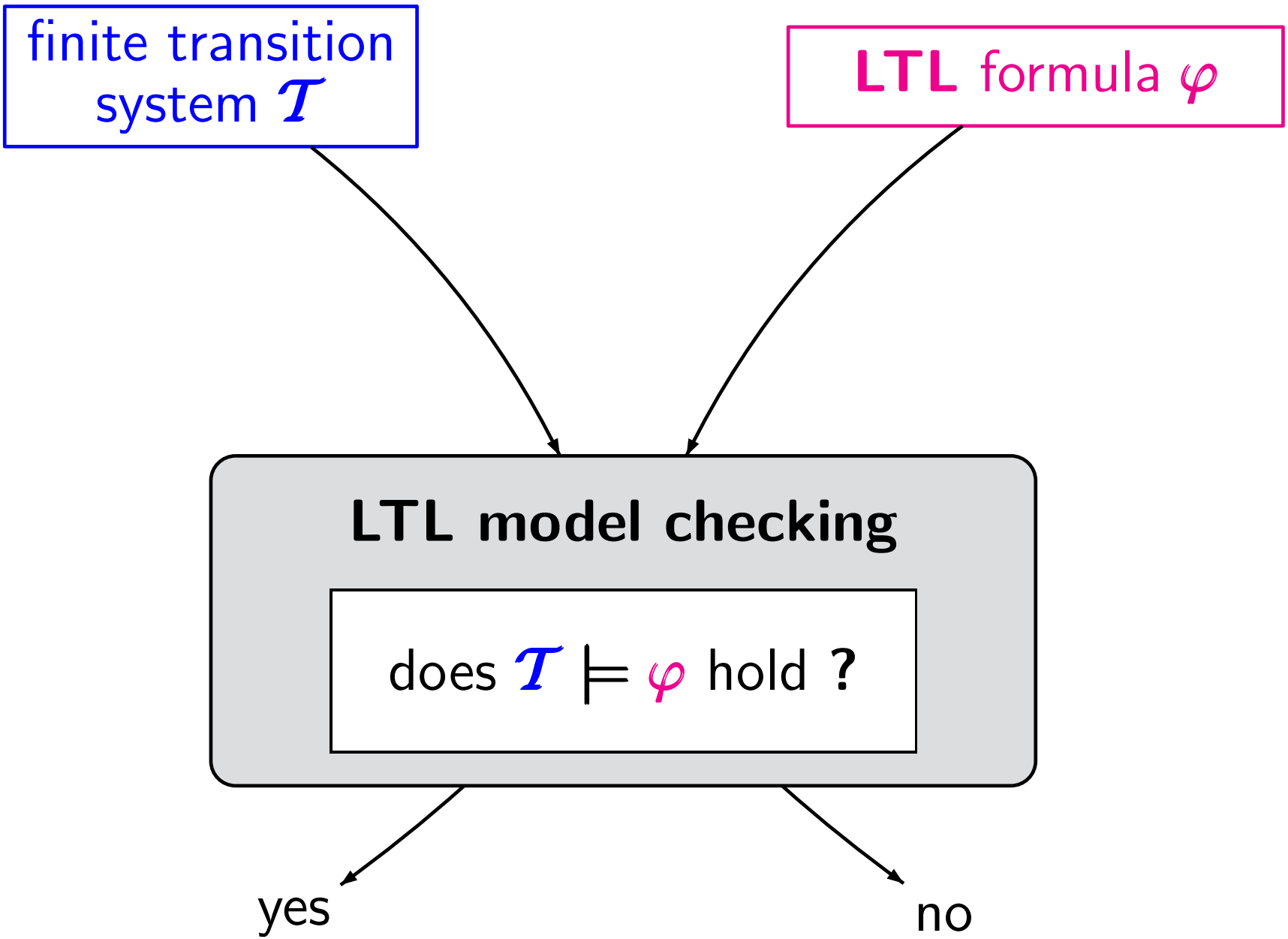
construct the product-TS $\mathcal{T} \otimes \mathcal{A}$
search a path in the product that meets the acceptance condition of $\mathcal{A}$

Automata-based LTL model checking



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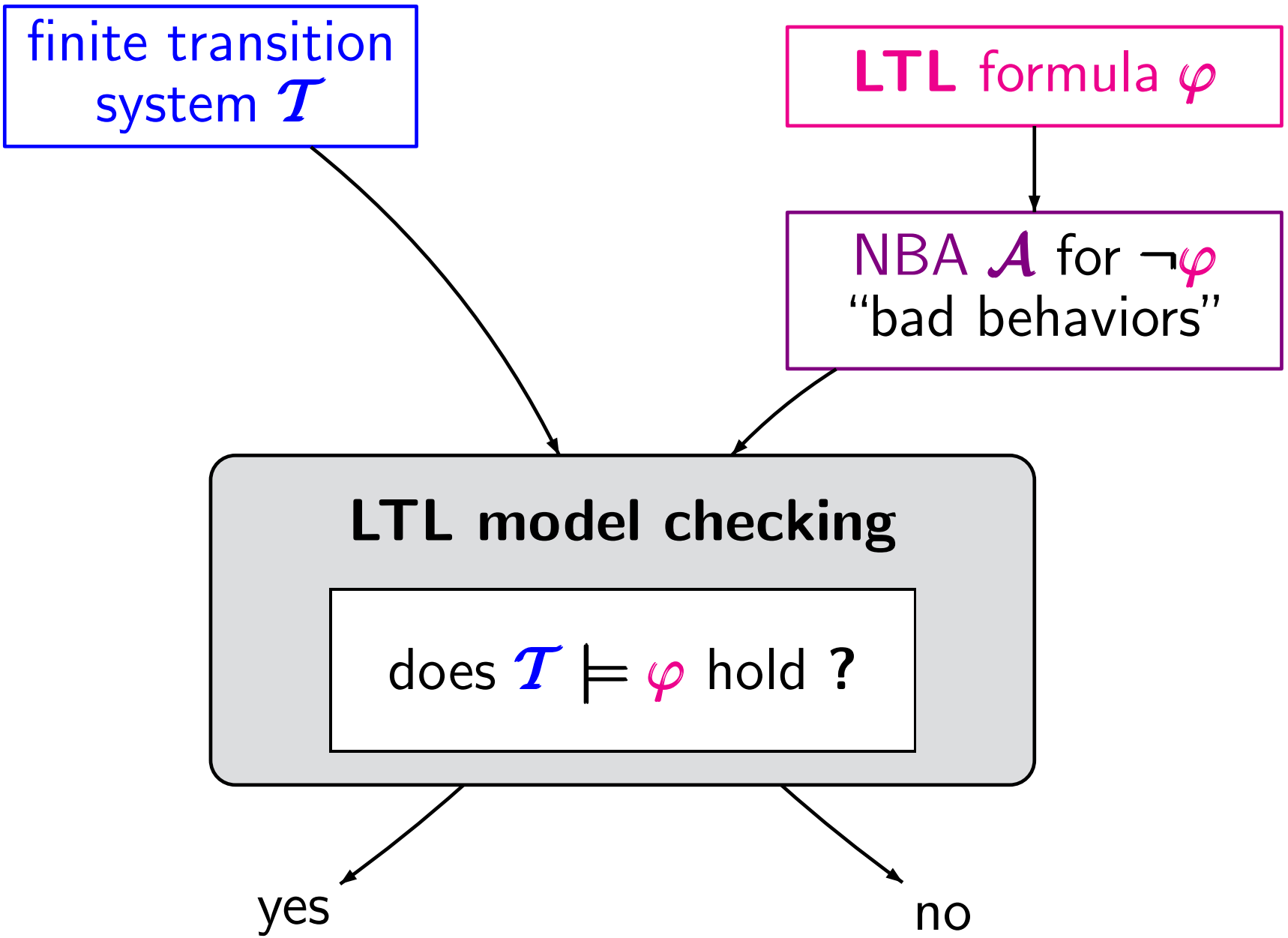

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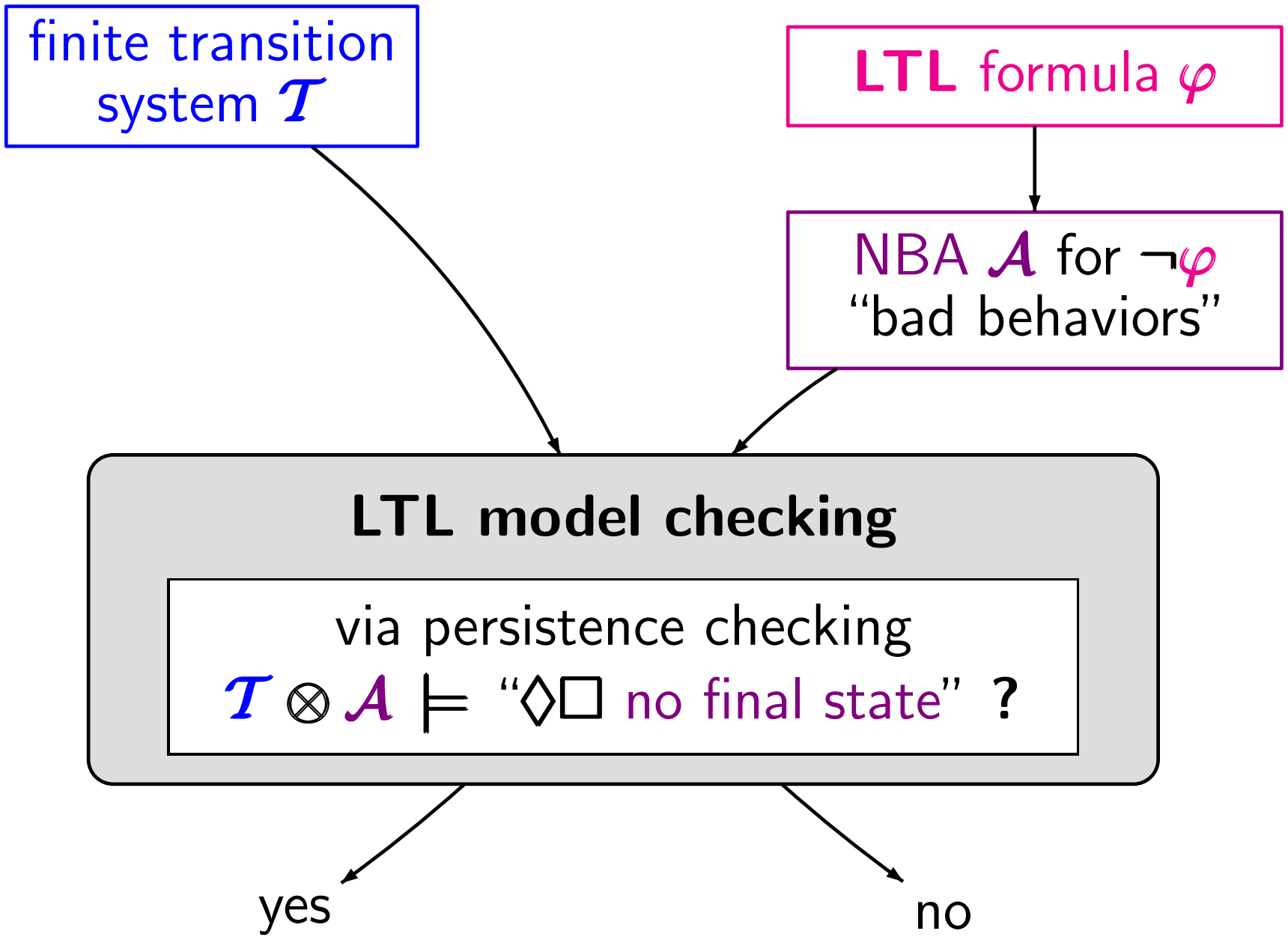

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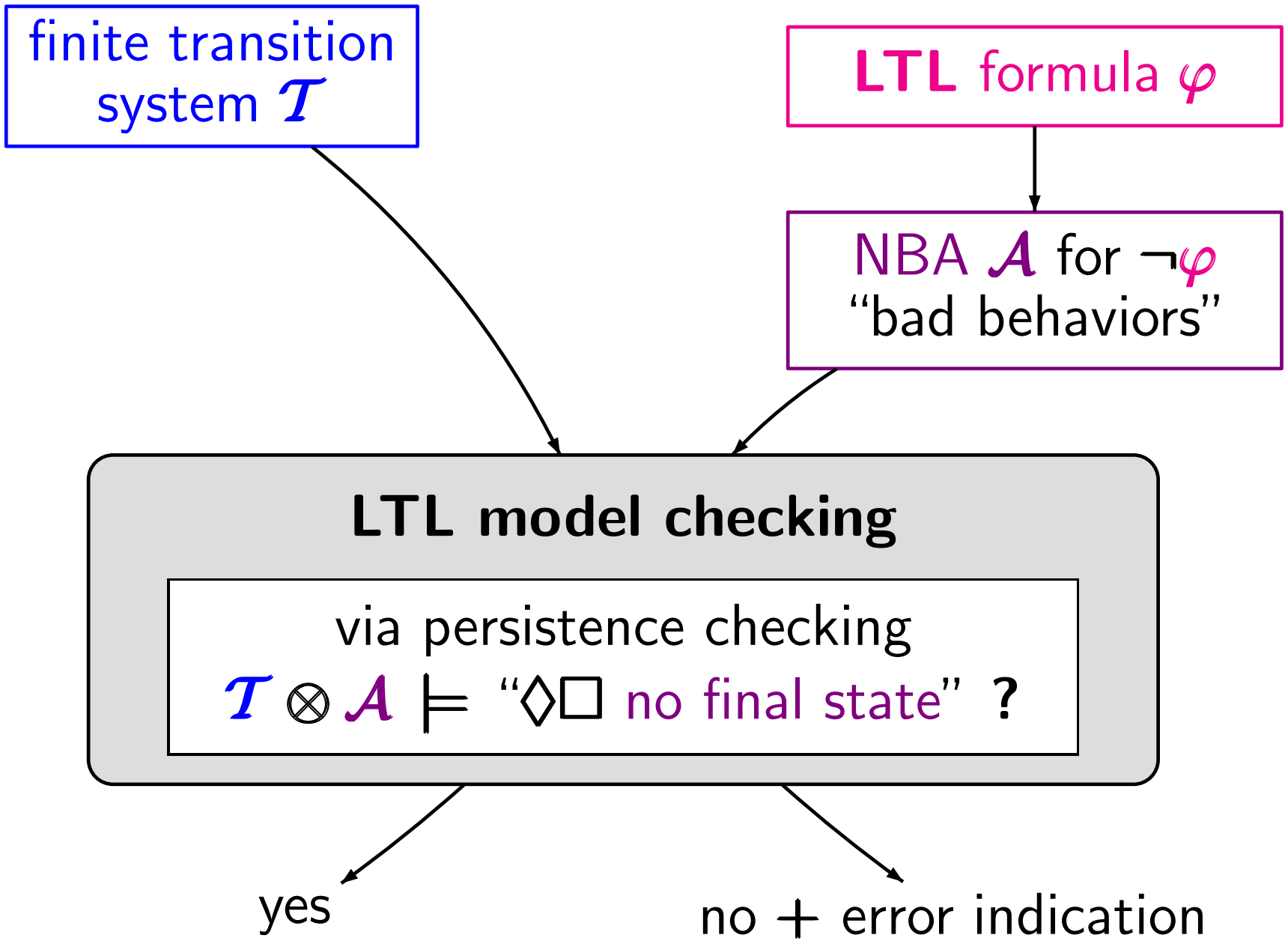

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$\mathcal{T} \models$ safety property E

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where $\mathcal{A}$ is an NFA for the bad prefixes

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iff $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$ $\longleftarrow$ persistence checking

Recall: nondeterministic Büchi automata

NBA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$

- Q finite set of states
- Σ alphabet
- $\delta : Q \times \Sigma \to 2^Q$ transition relation
- $Q_0 \subseteq Q$ set of initial states
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run for a word $A_0 A_1 A_2 \ldots \in \Sigma^\omega$:

state sequence $\pi = q_0 q_1 q_2 \ldots$ where $q_0 \in Q_0$ and $q_{i+1} \in \delta(q_i, A_i)$ for $i \geq 0$

run π is accepting if $\overset{\infty}{\exists}\, i \in \mathbb{N}.\ q_i \in F$

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proof: ... later ...

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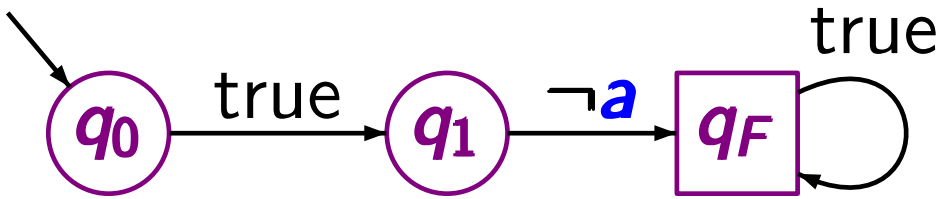

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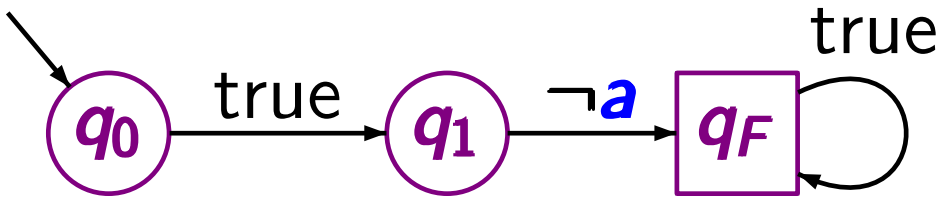

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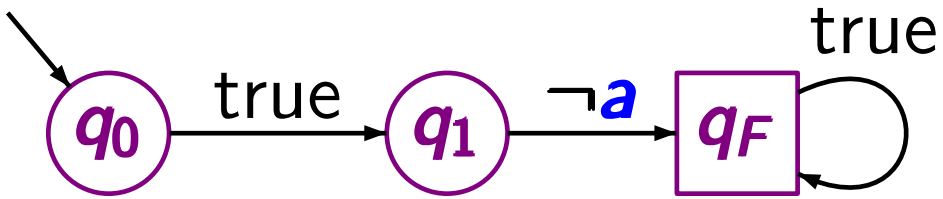

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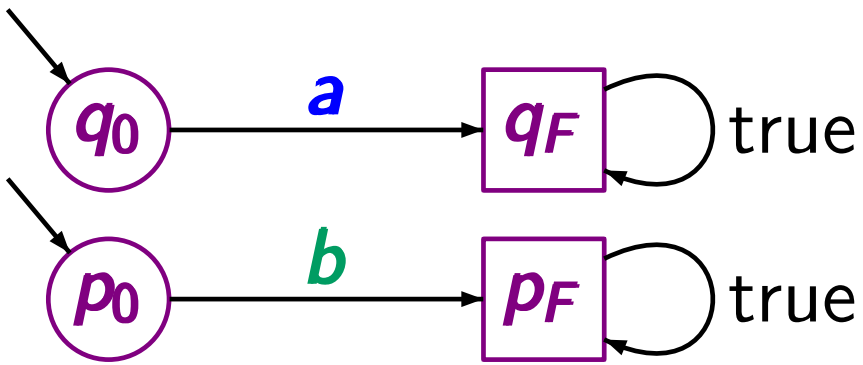

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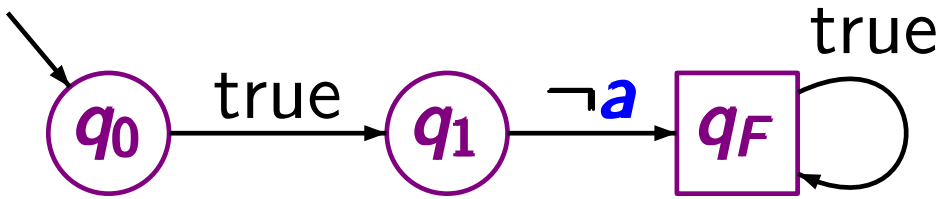

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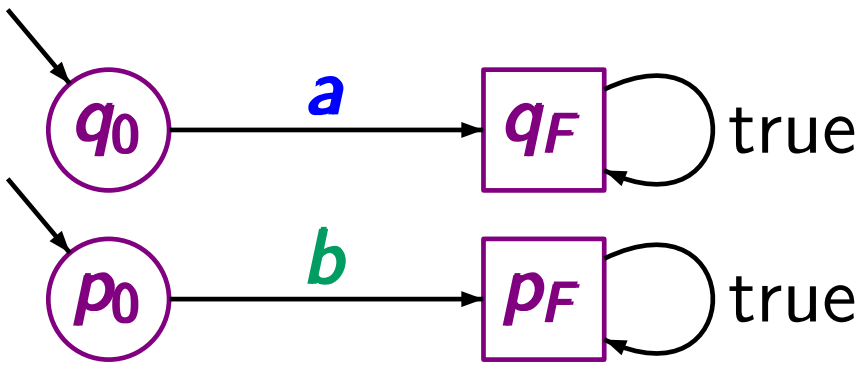

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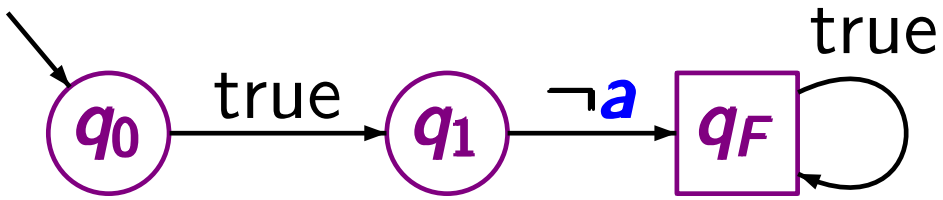

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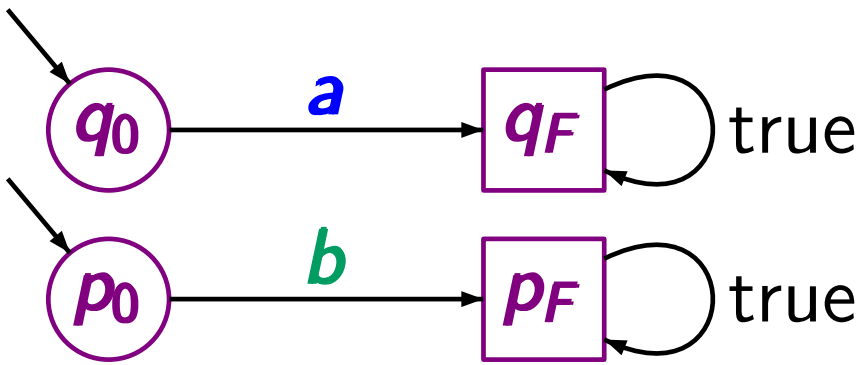

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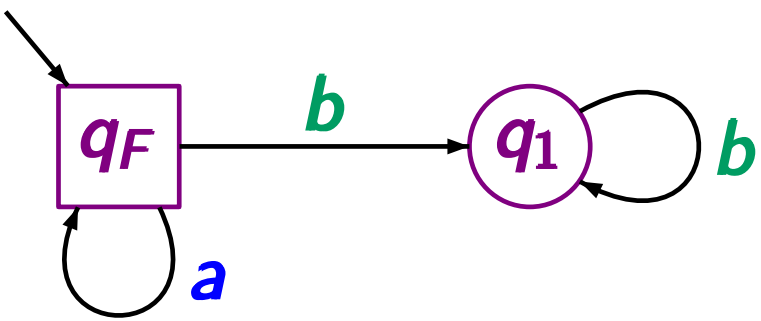

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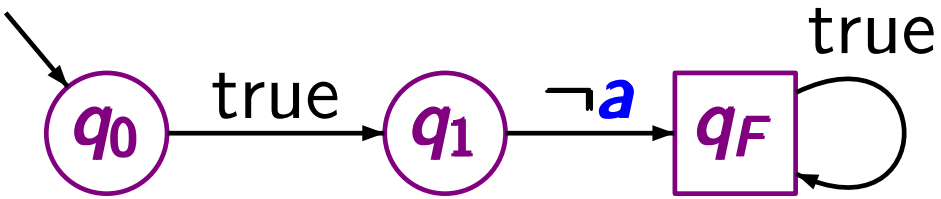

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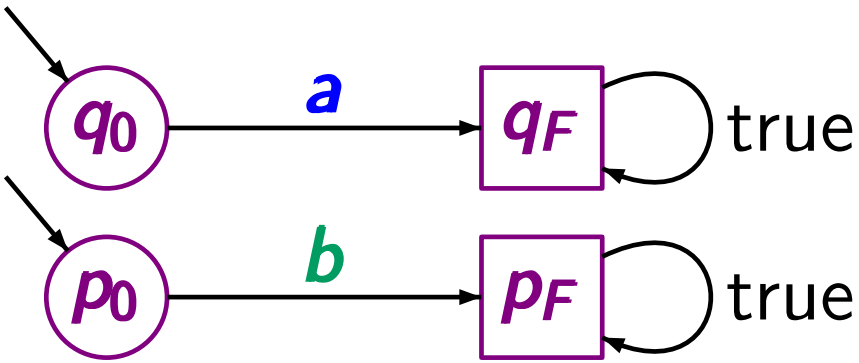

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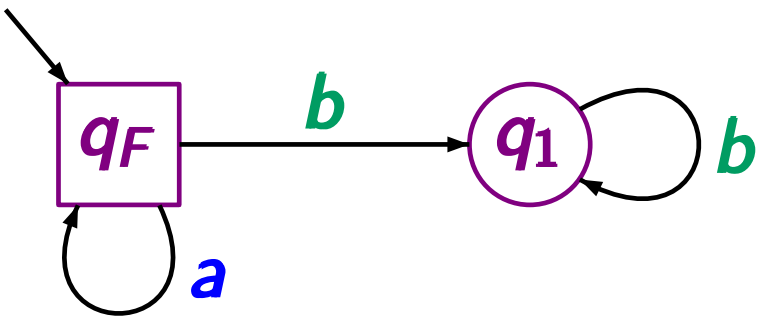

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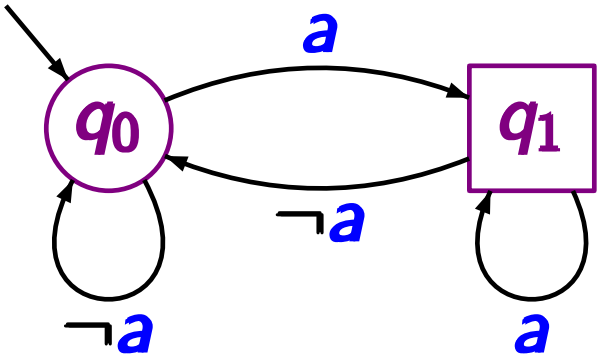

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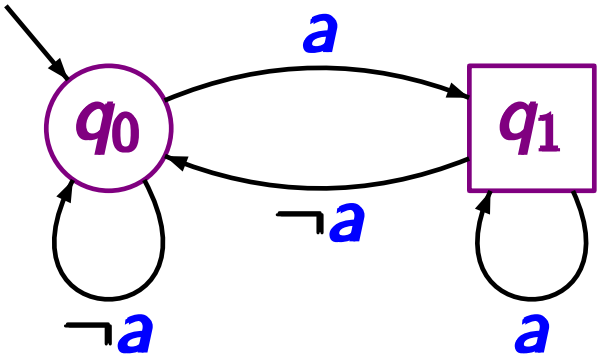

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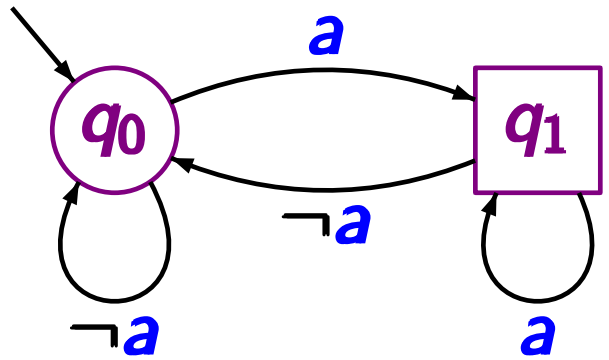

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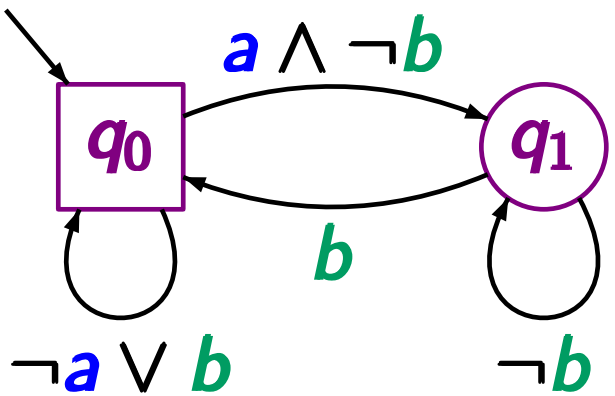

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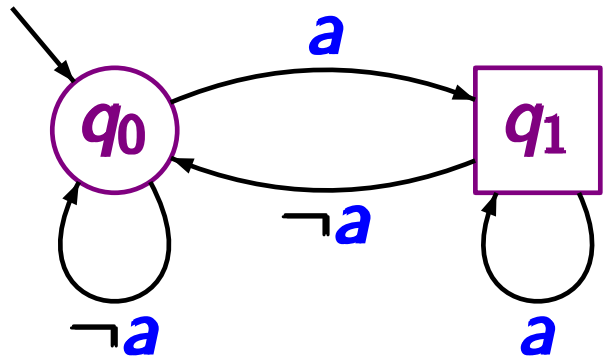

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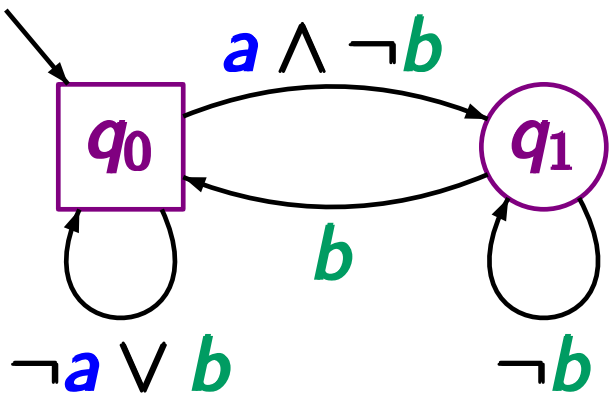

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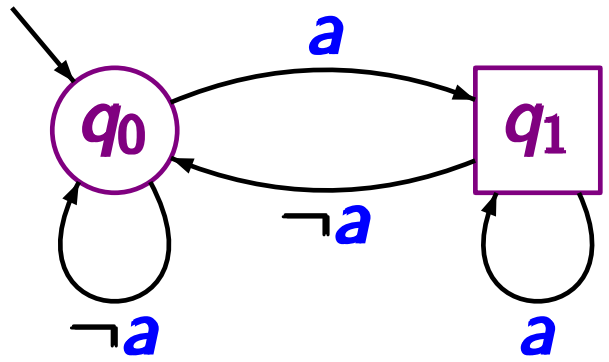

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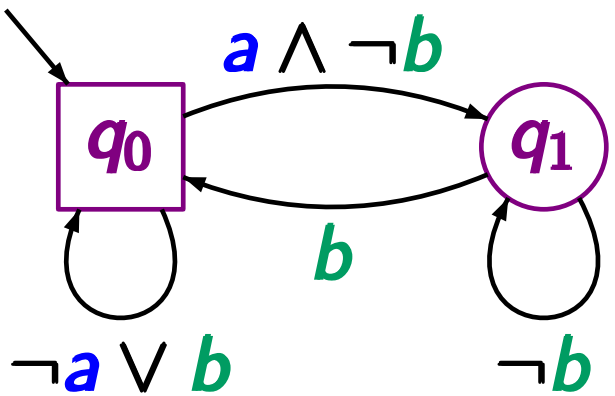

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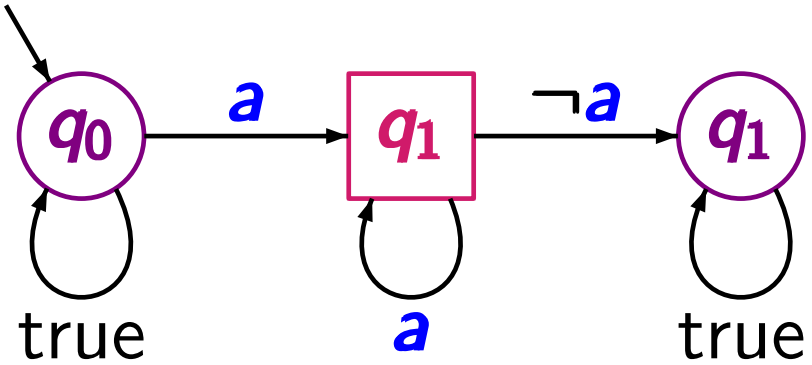

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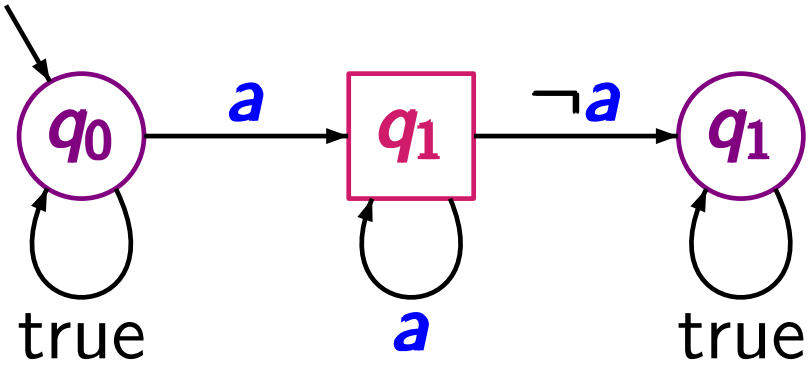
NBA for LTL formula



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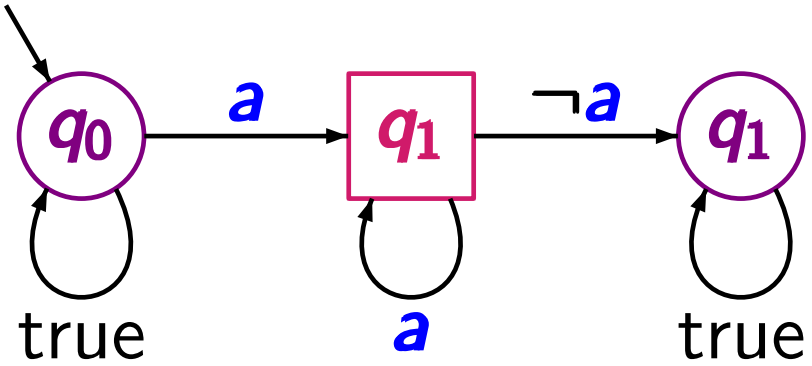

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possible runs for $\{a\}^\omega$

$q_0\ q_0\ q_0\ q_0\ q_0\ q_0\ \ldots$	not accepting
$q_0\ q_1\ q_1\ q_1\ q_1\ q_1\ \ldots$	accepting
$q_0\ q_0\ q_1\ q_1\ q_1\ q_1\ \ldots$	accepting
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$\vdots$	

NFA and NBA for safety properties

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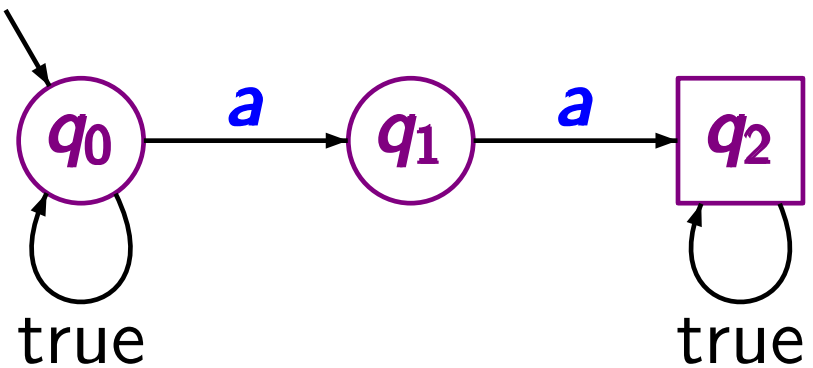

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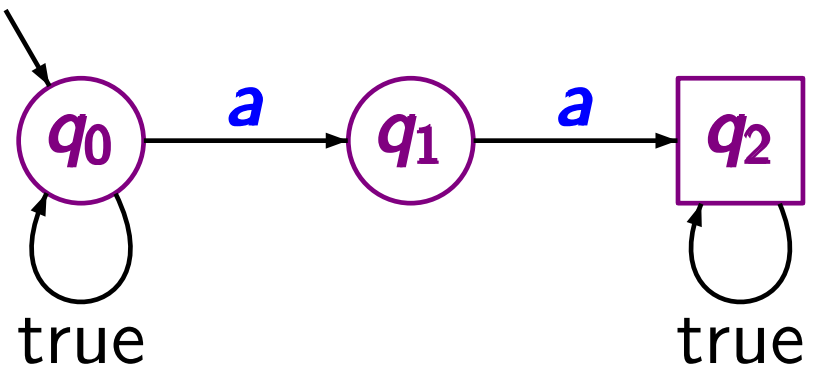

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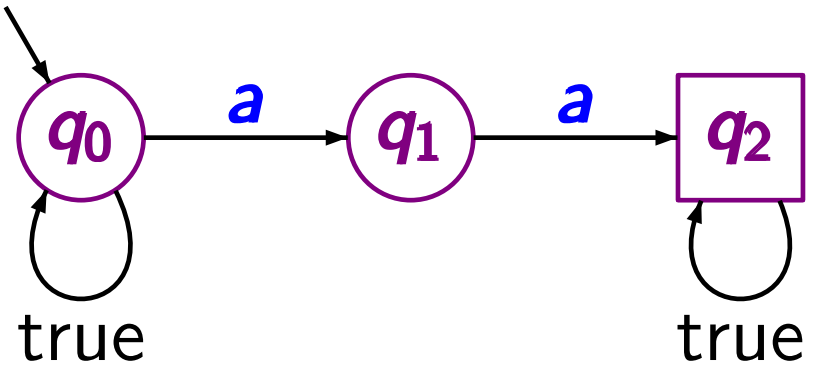

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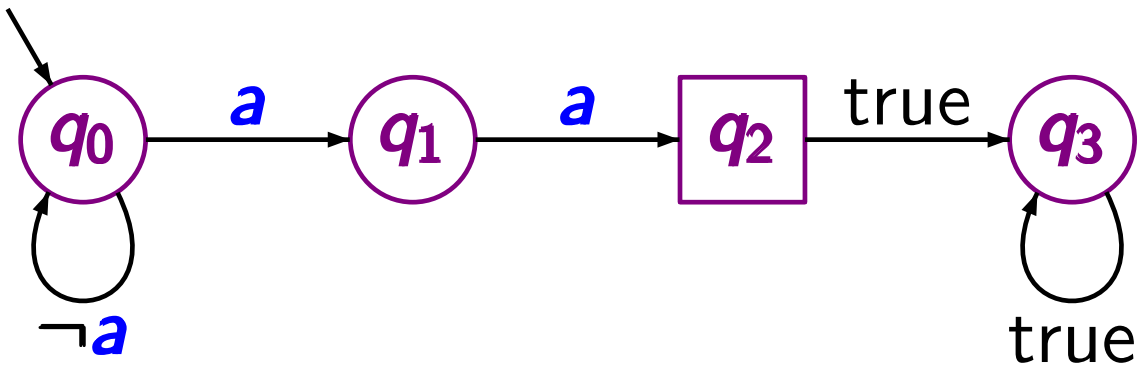

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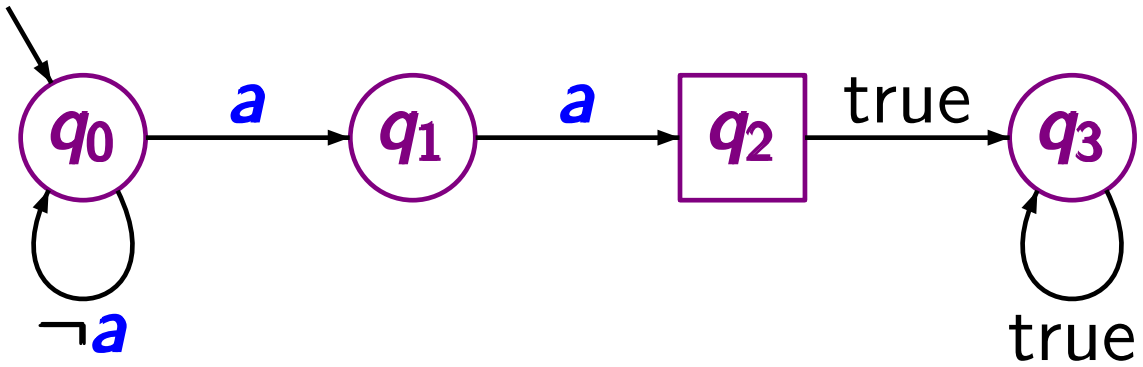

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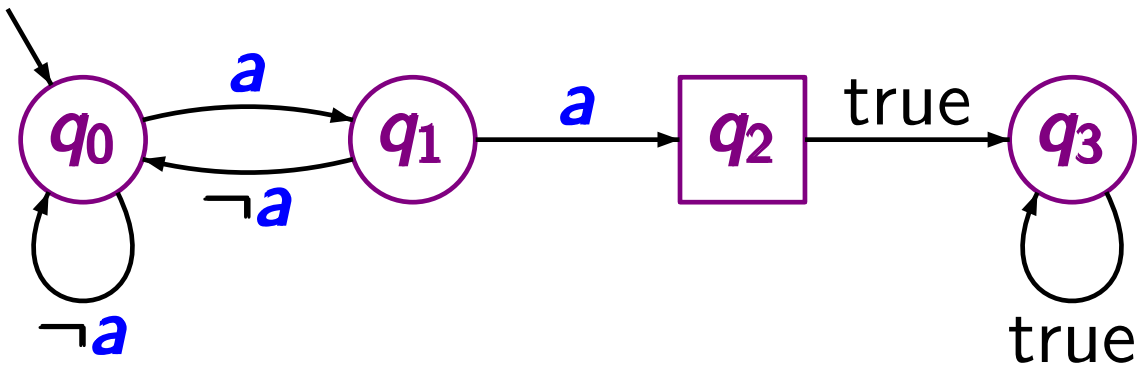

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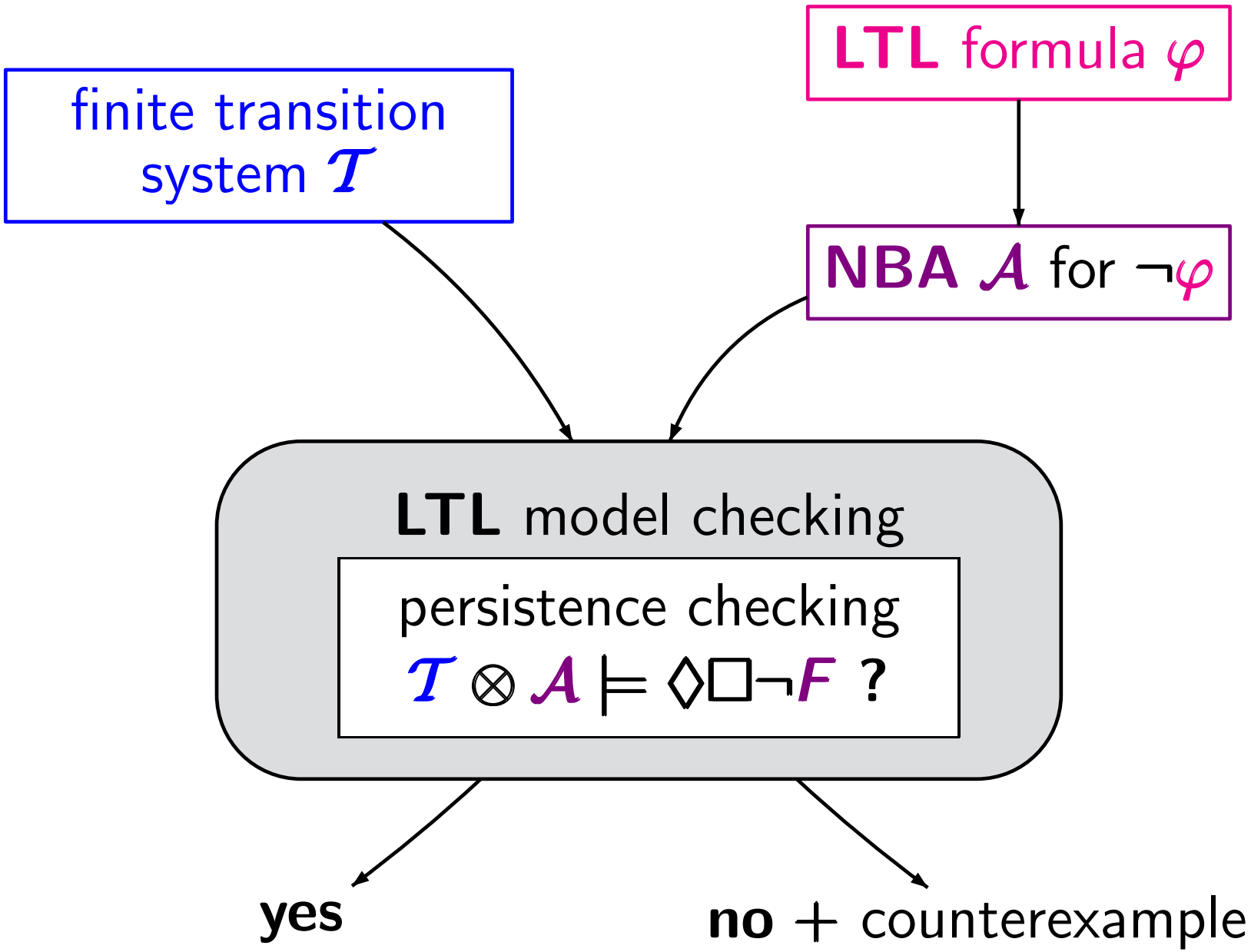
LTL model checking



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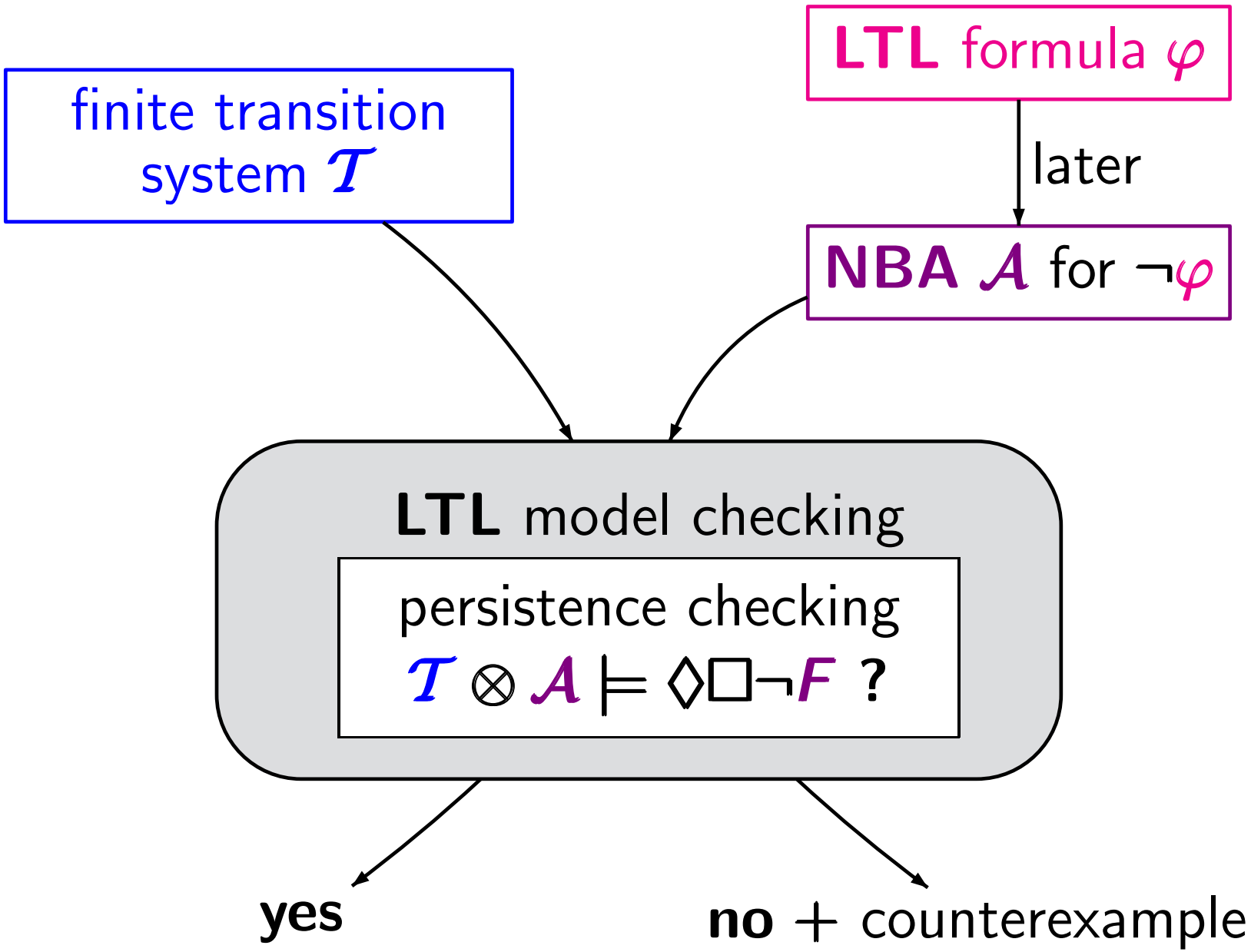

Recall: product transition system

$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ TS without terminal states

$\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$ NBA or NFA

non-blocking, $Q_0 \cap F = \varnothing$

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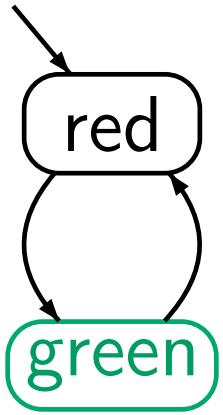
transition relation:

$$\frac{s \xrightarrow{\alpha} s' \ \wedge \ q' \in \delta(q, L(s'))}{\langle s, q \rangle \xrightarrow{\alpha}' \langle s', q' \rangle}$$

Example: LTL model checking

TS $\mathcal{T}$



LTL formula $\varphi = \Box\Diamond$green

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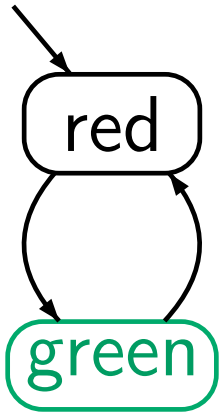

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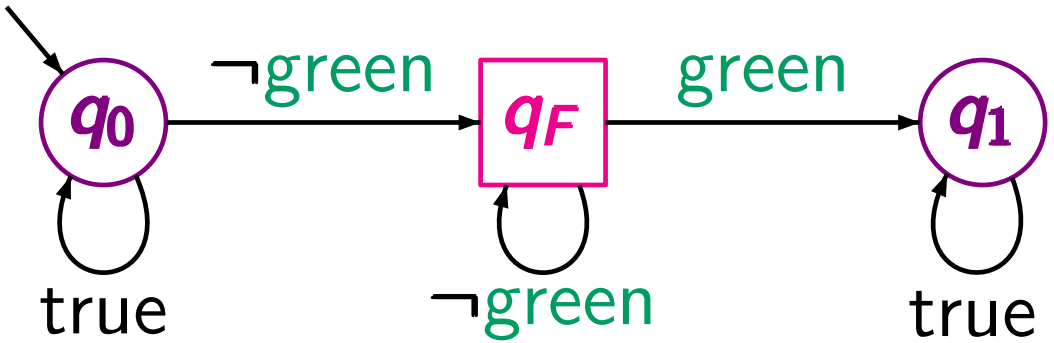

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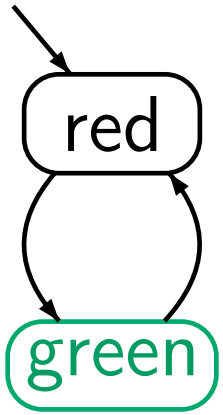

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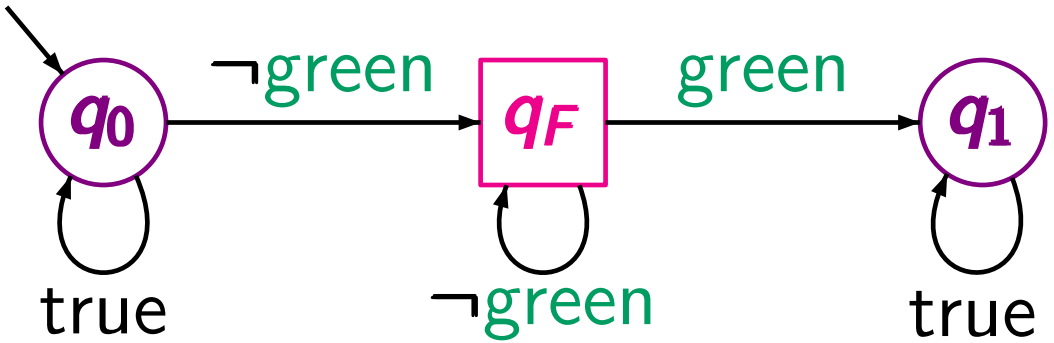

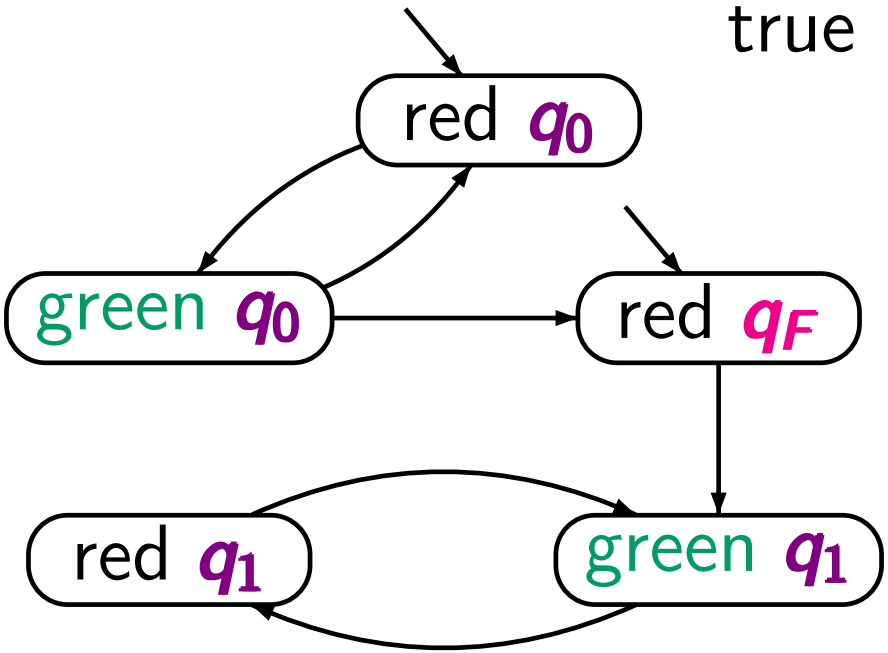

reachable fragment of the product TS $\mathcal{T} \otimes \mathcal{A}$

Example: LTL model checking

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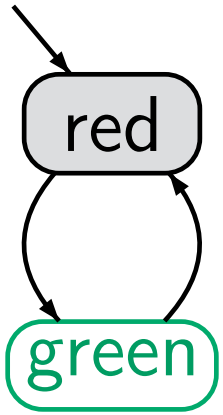

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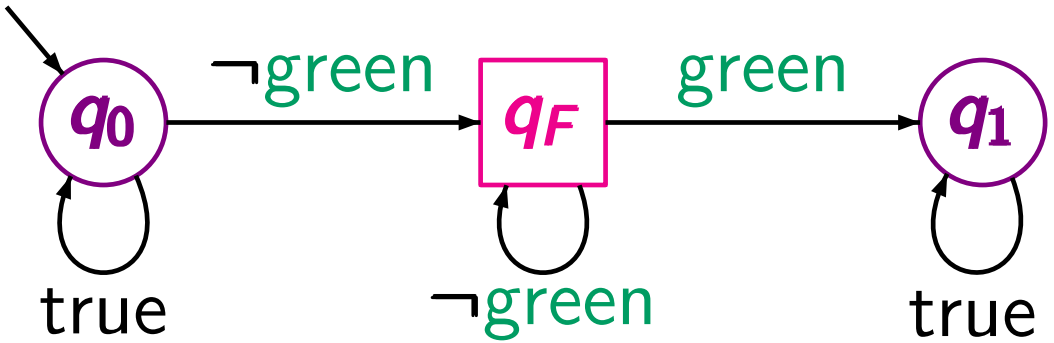

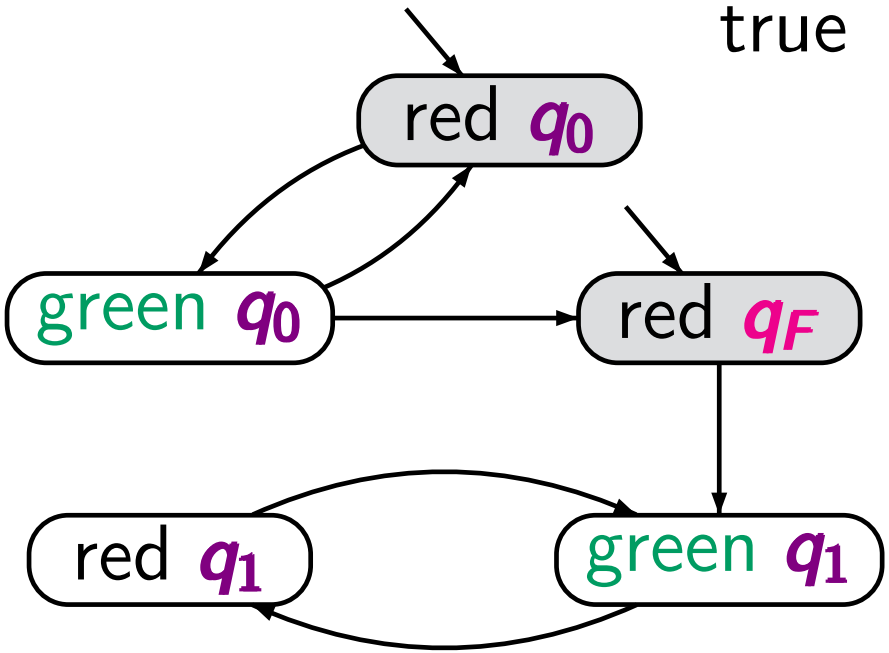

initial states:

$\langle \text{red}, q \rangle$ where

$$q \in \delta(q_0, L(\text{red})) = \delta(q_0, \varnothing) = \{q_0, q_F\}$$

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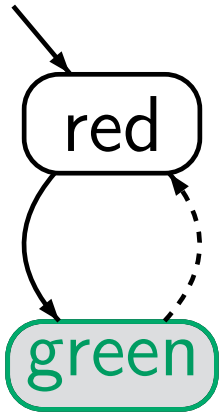

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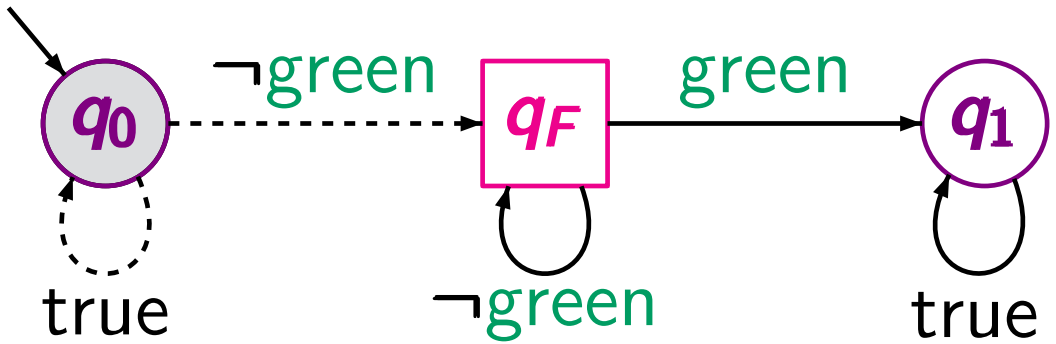
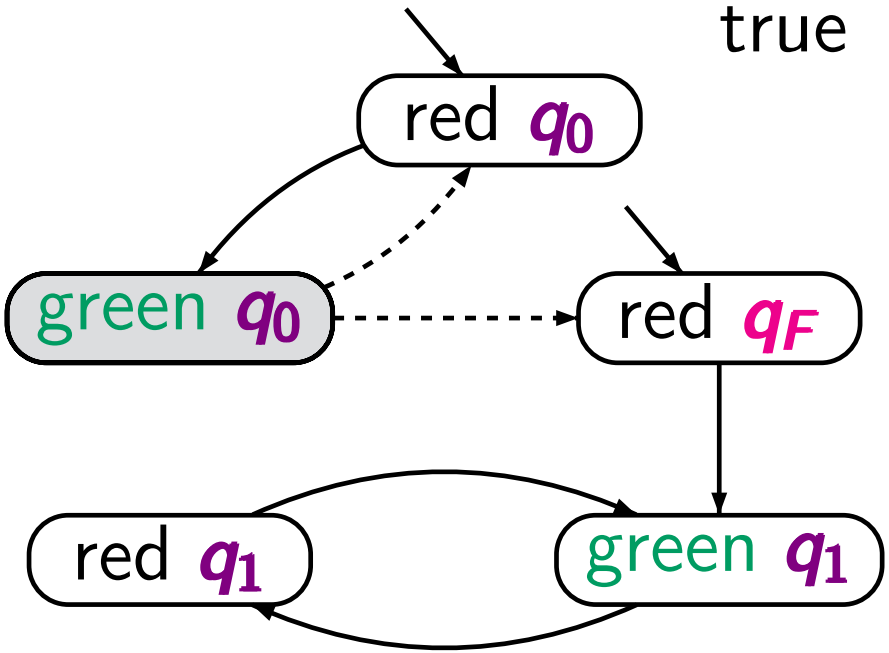




transition

$\langle \text{green}, q_0 \rangle \rightarrow \langle \text{red}, q \rangle$

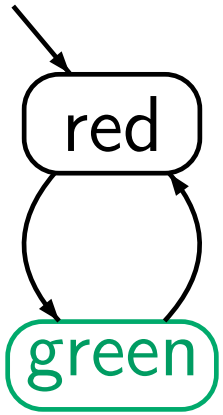
$q \in \delta(q_0, L(\text{red}))$

$= \delta(q_0, \varnothing)$

$= \{q_0, q_F\}$

Example: LTL model checking

TS $\mathcal{T}$



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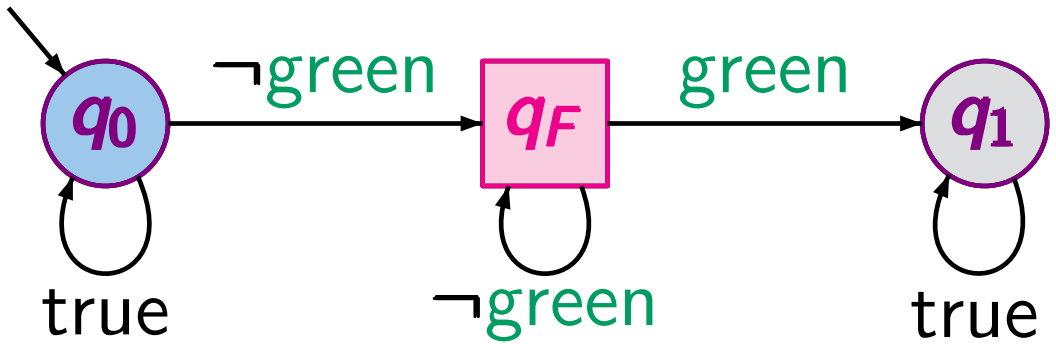
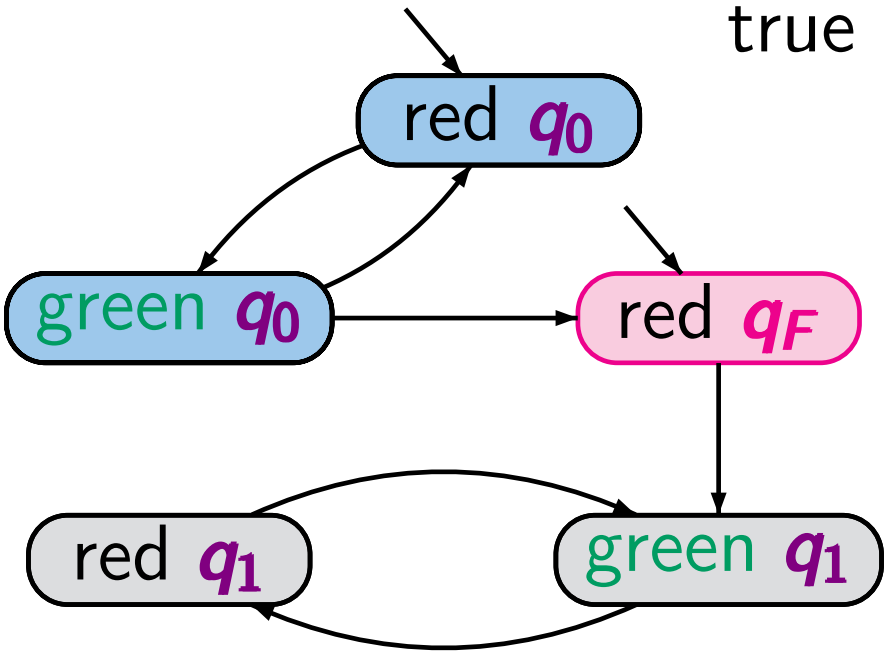




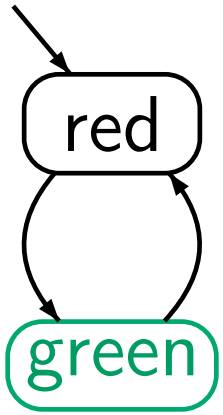
atomic propositions

$AP' = \{q_0, q_F, q_1\}$

obvious labeling function

Example: LTL model checking

TS $\mathcal{T}$



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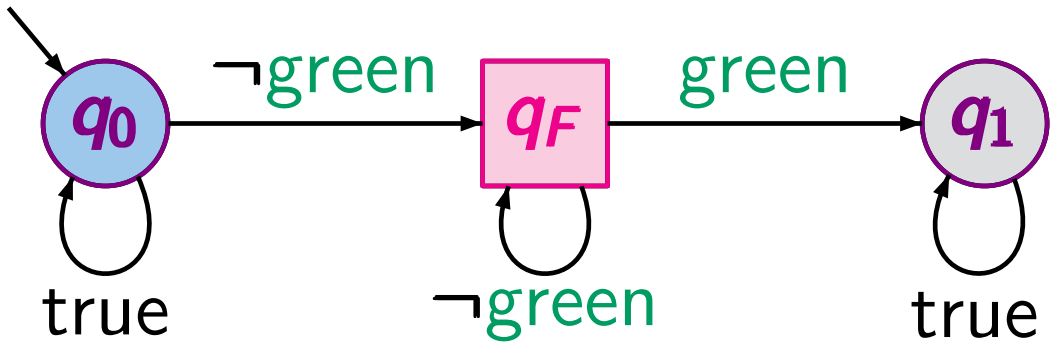

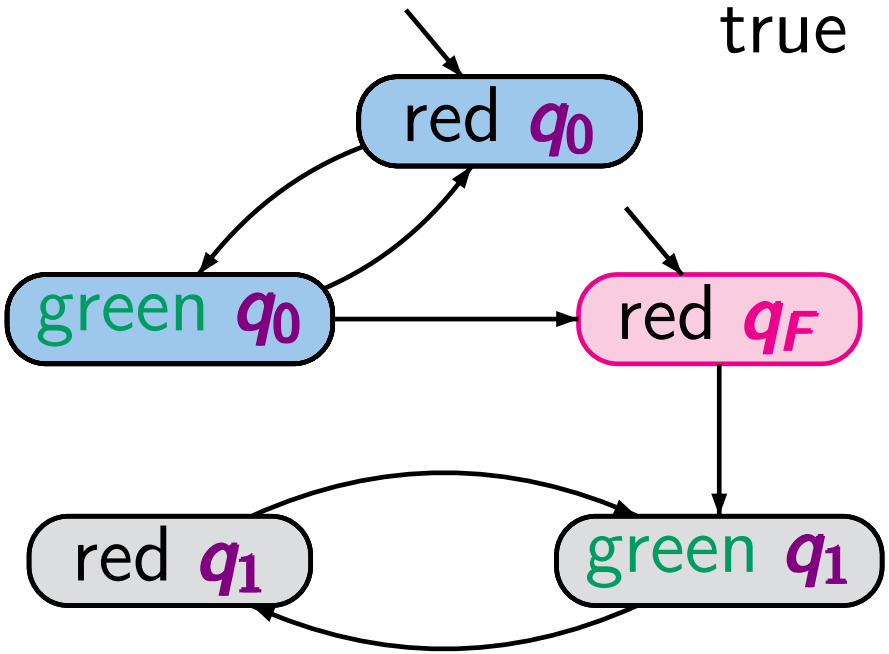

$\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

Example: LTL model checking

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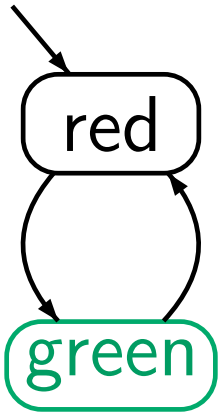

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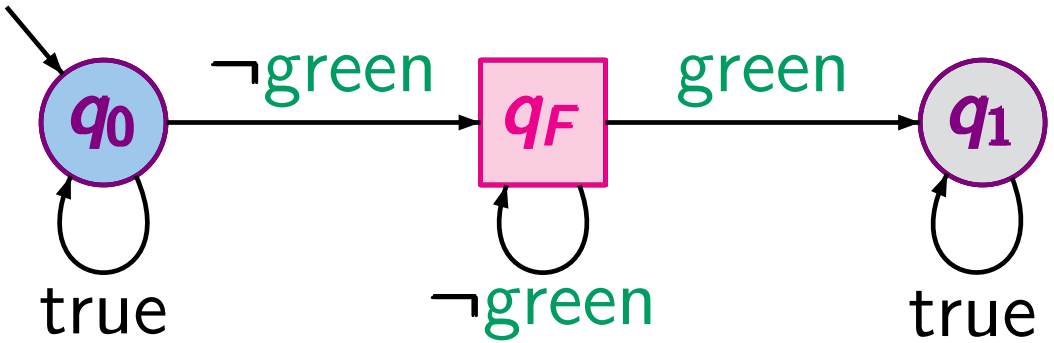
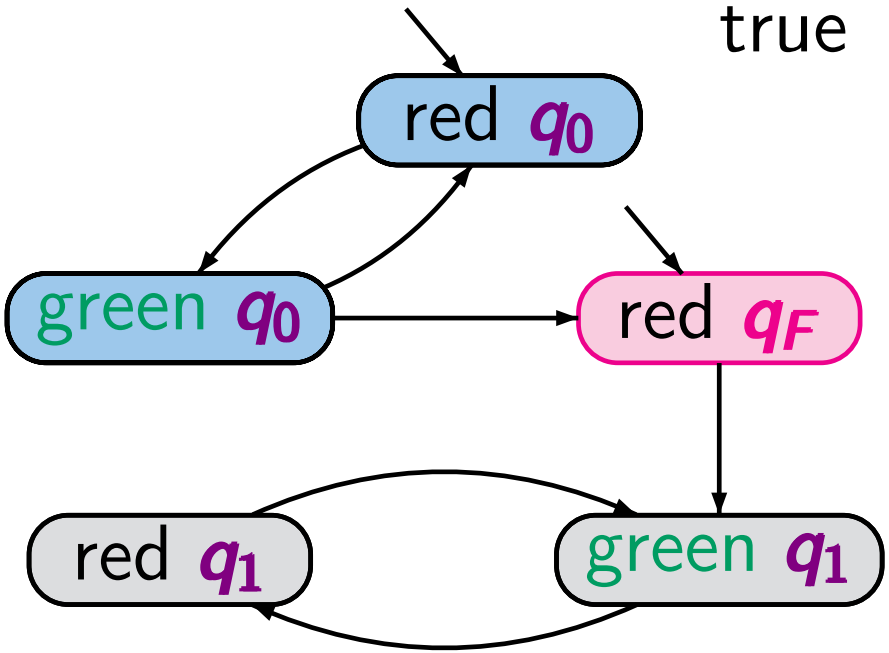
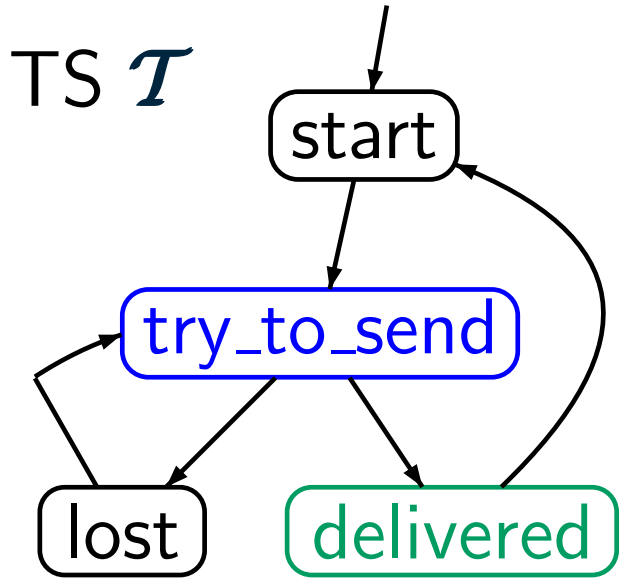




$\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

hence: $\mathcal{T} \models \varphi$

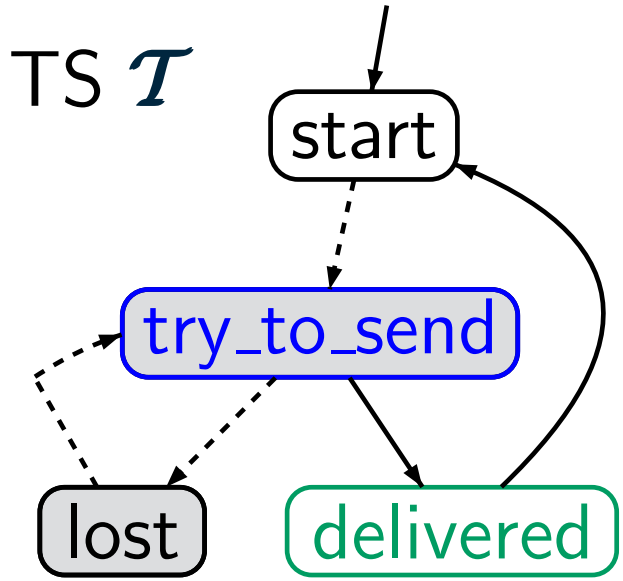
Example: LTL model checking



LTL formula $\varphi = \Box(\textit{try} \rightarrow \Diamond \textit{del})$

"each (repeatedly) sent message will eventually be delivered"

Example: LTL model checking



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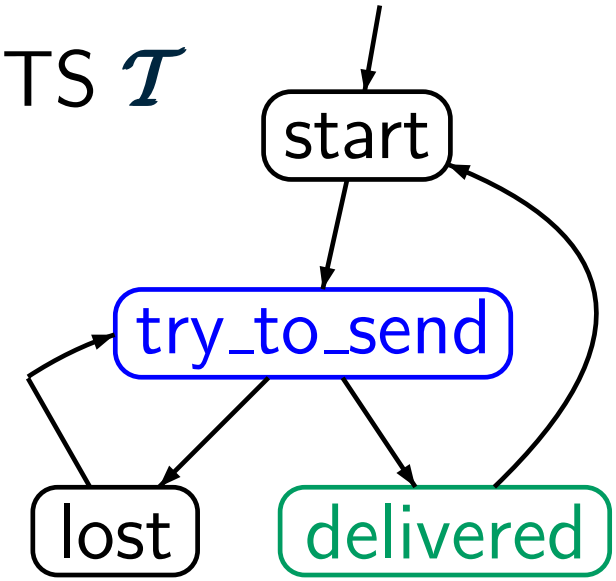
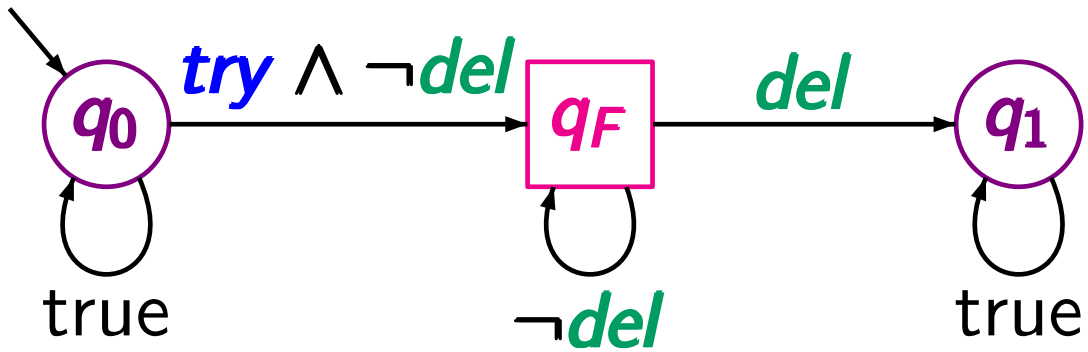


NBA $\mathcal{A}$ for $\neg\varphi \equiv \Diamond(\mathit{try} \wedge \Box\neg\mathit{del})$



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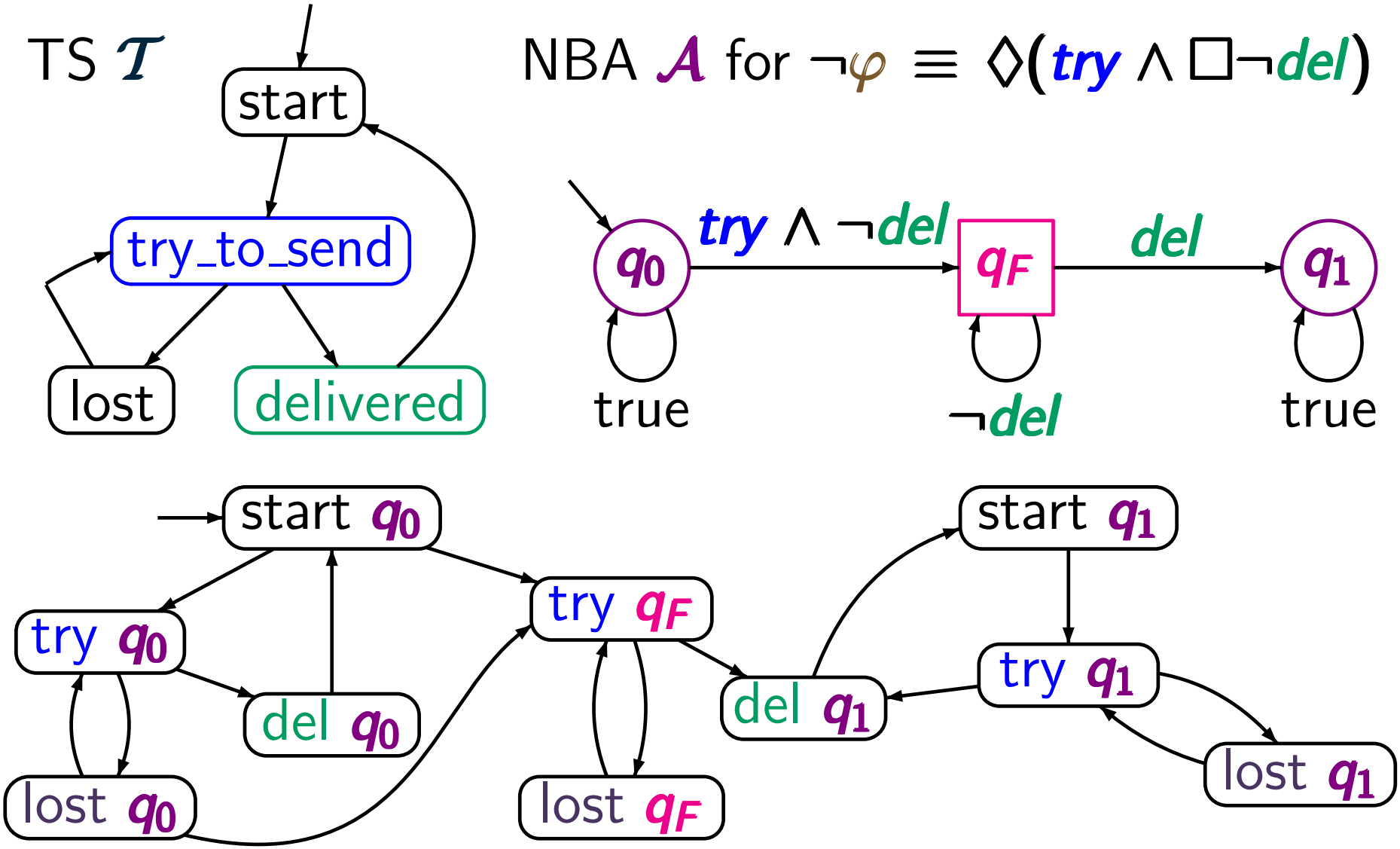
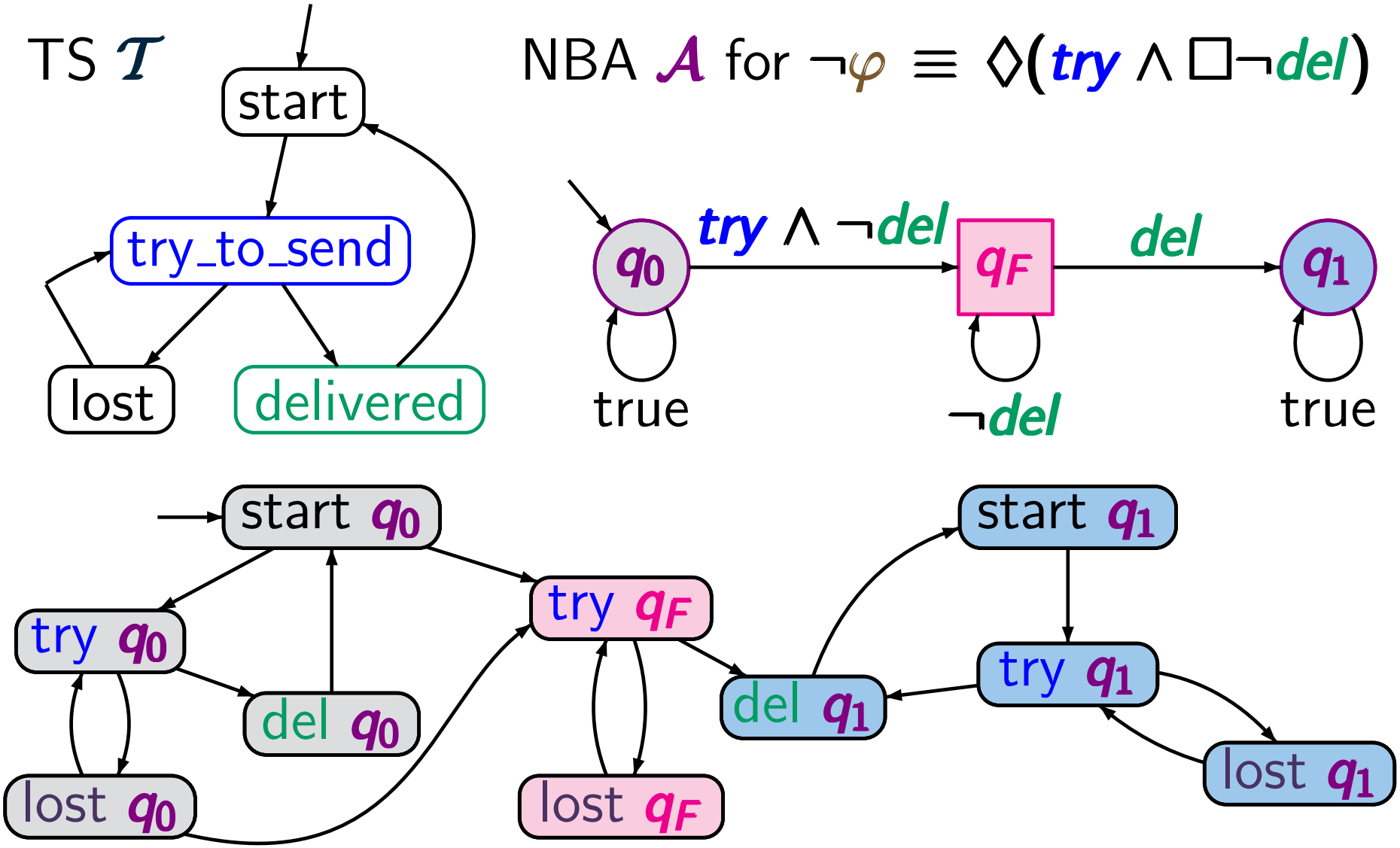


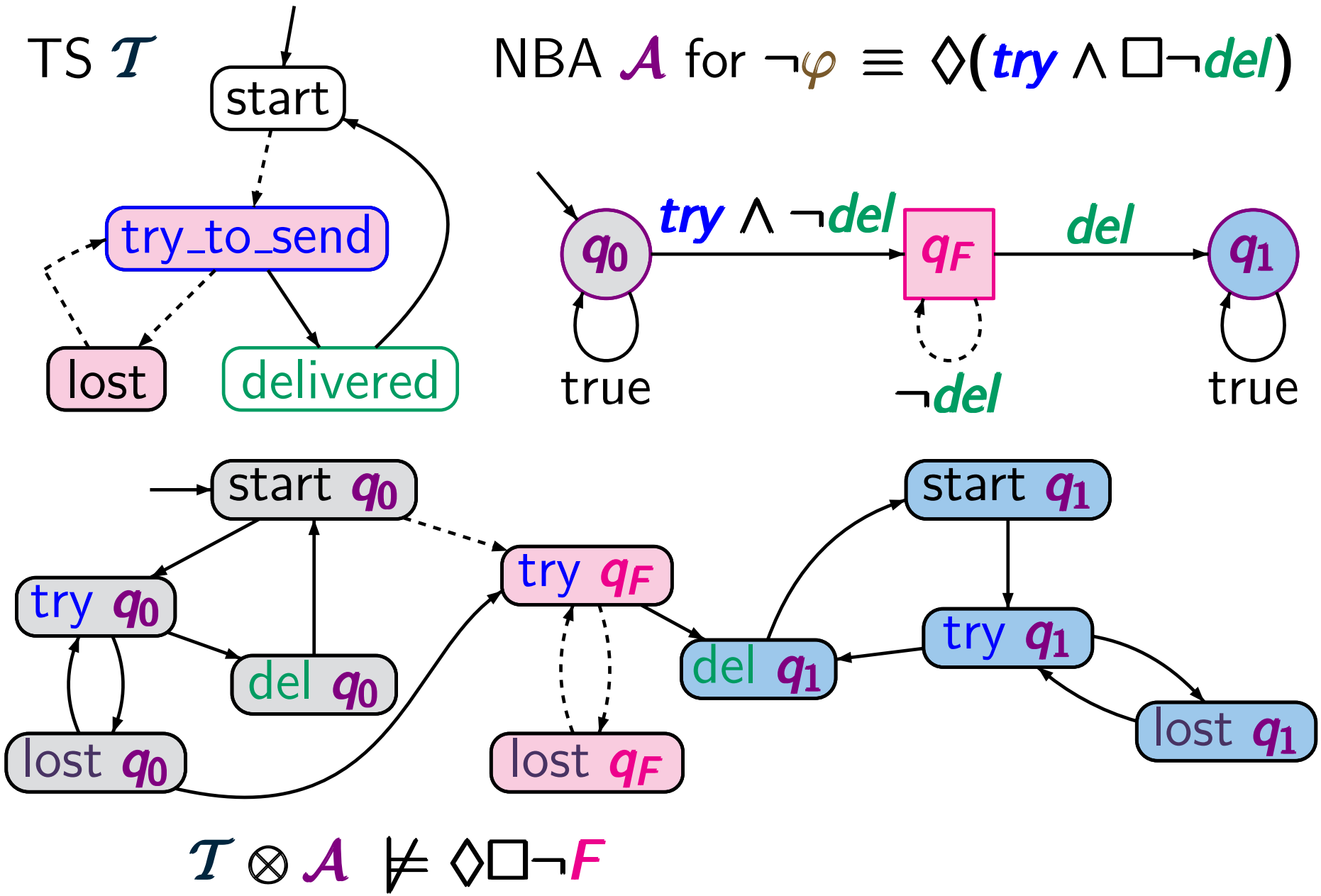
reachable fragment of the product-TS

Example: LTL model checking



set of atomic propositions $AP' = \{q_0, q_1, q_F\}$

Example: LTL model checking



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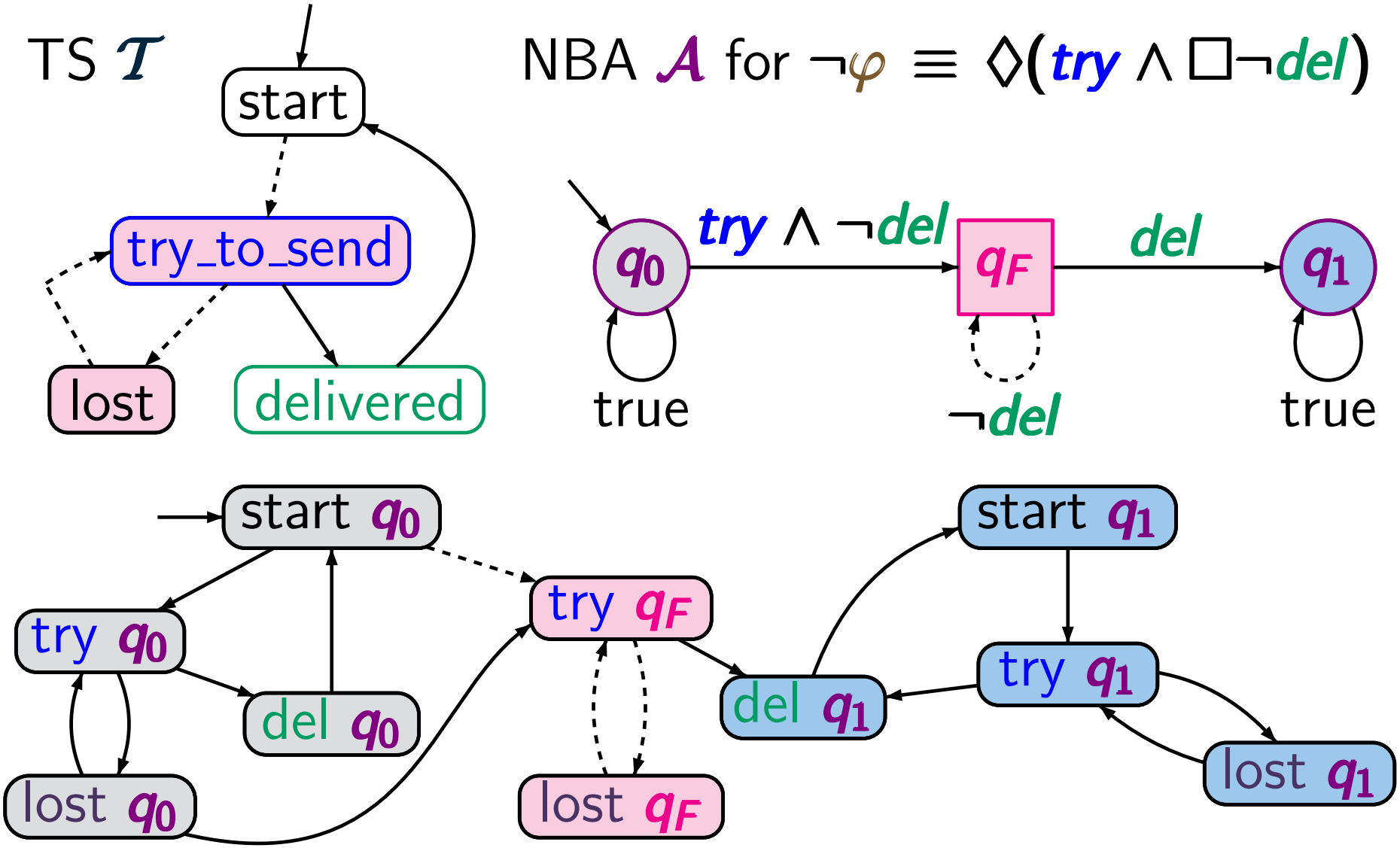

$\mathcal{T} \otimes \mathcal{A} \not\models \Diamond\Box\neg F$ hence: $\mathcal{T} \not\models \varphi$

LTL model checking

given: finite TS $\mathcal{T}$, LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold **?**

LTL model checking

given: finite TS $\mathcal{T}$, LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold **?**

construct an NBA $\mathcal{A}$ for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

LTL model checking

given: finite TS $\mathcal{T}$, LTL-formula φ

question: does $\mathcal{T} \models \varphi$ hold **?**

construct an NBA $\mathcal{A}$ for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$ $\longleftarrow$ persistence checking nested **DFS**

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IF $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

THEN return "**yes**"

ELSE compute a counterexample

$\langle s_0, p_0\rangle \ldots \langle s_n, p_n\rangle \ldots \langle s_n, p_n\rangle$

for $\mathcal{T} \otimes \mathcal{A}$ and $\Diamond\Box\neg F$

return "**no**" and $s_0 \ldots s_n \ldots s_n$

Complexity of LTL model checking

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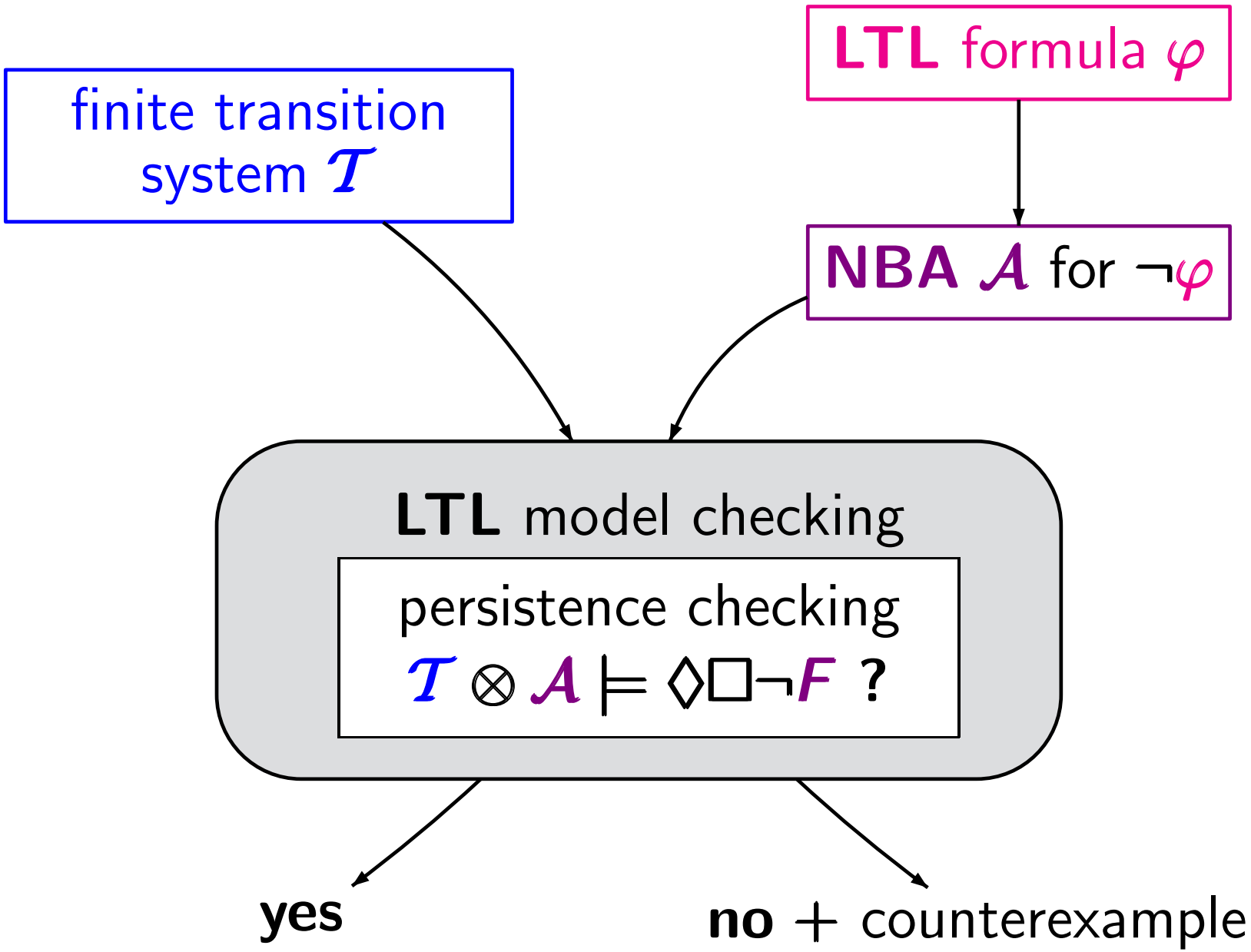
time complexity: $\mathcal{O}(size(\mathcal{T}) \cdot size(\mathcal{A}))$

LTL model checking



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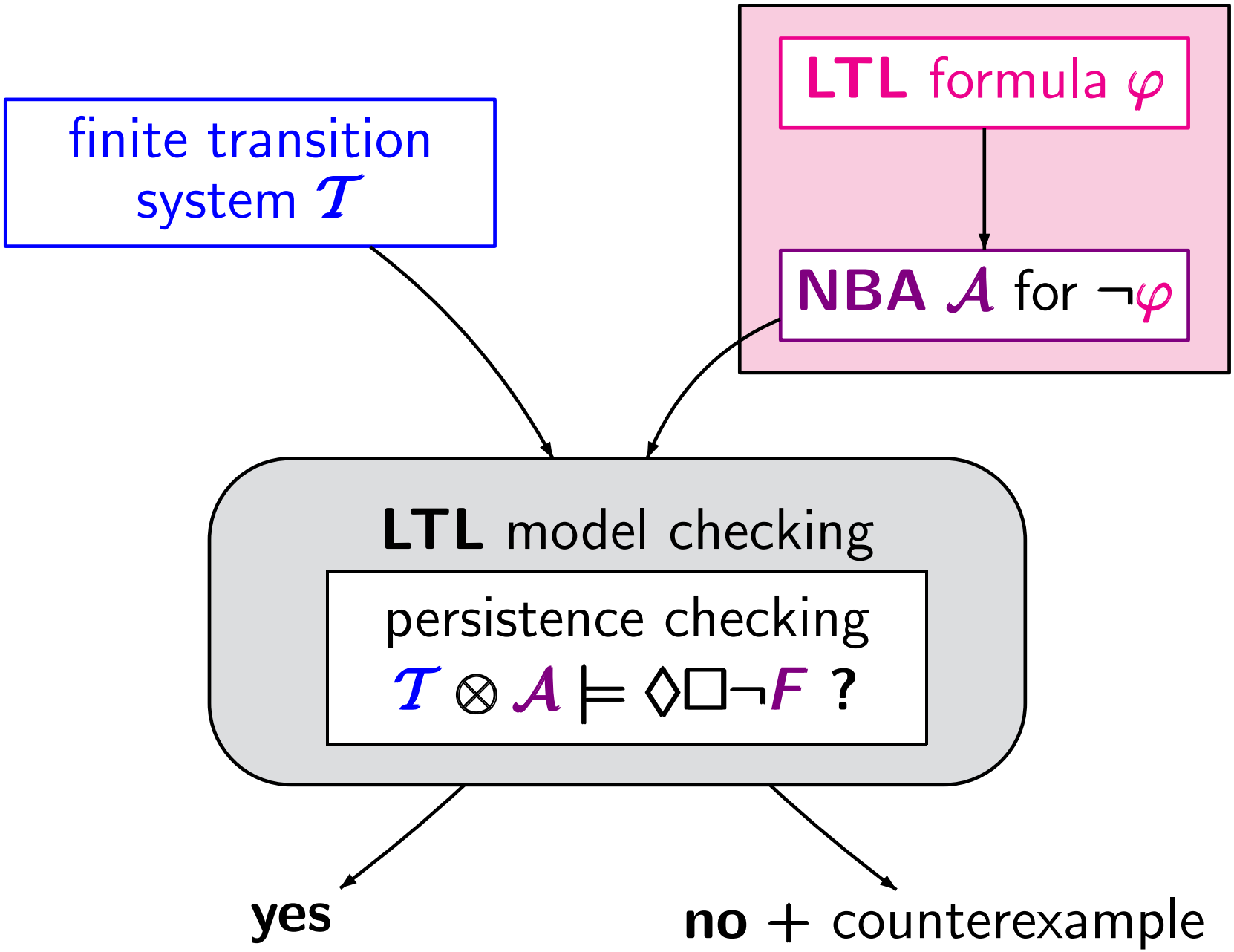

From LTL to NBA

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For each **LTL** formula φ there is an **NBA** $\mathcal{A}$ s.t.

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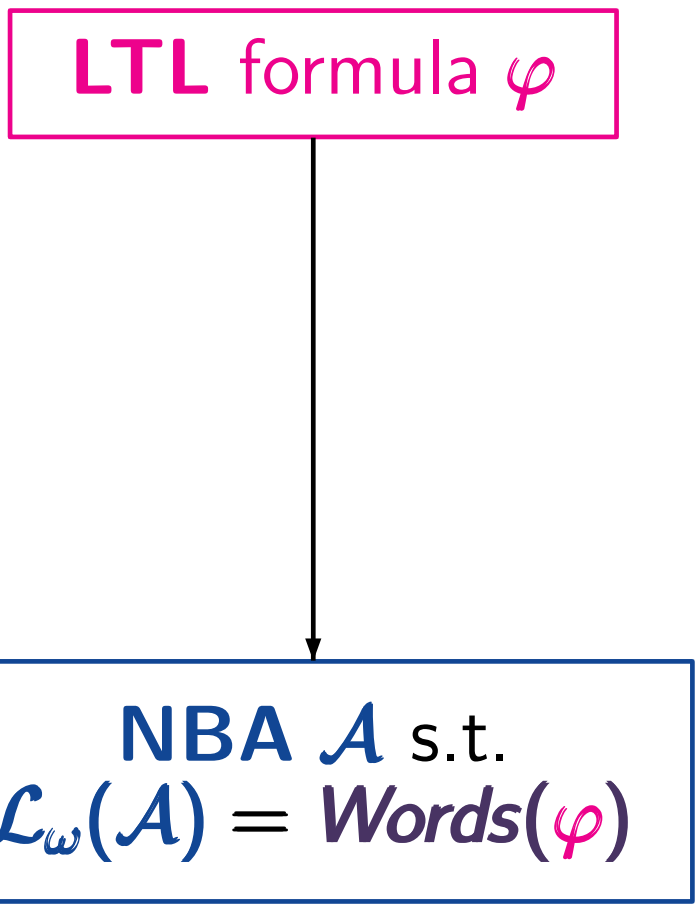

nondeterministic
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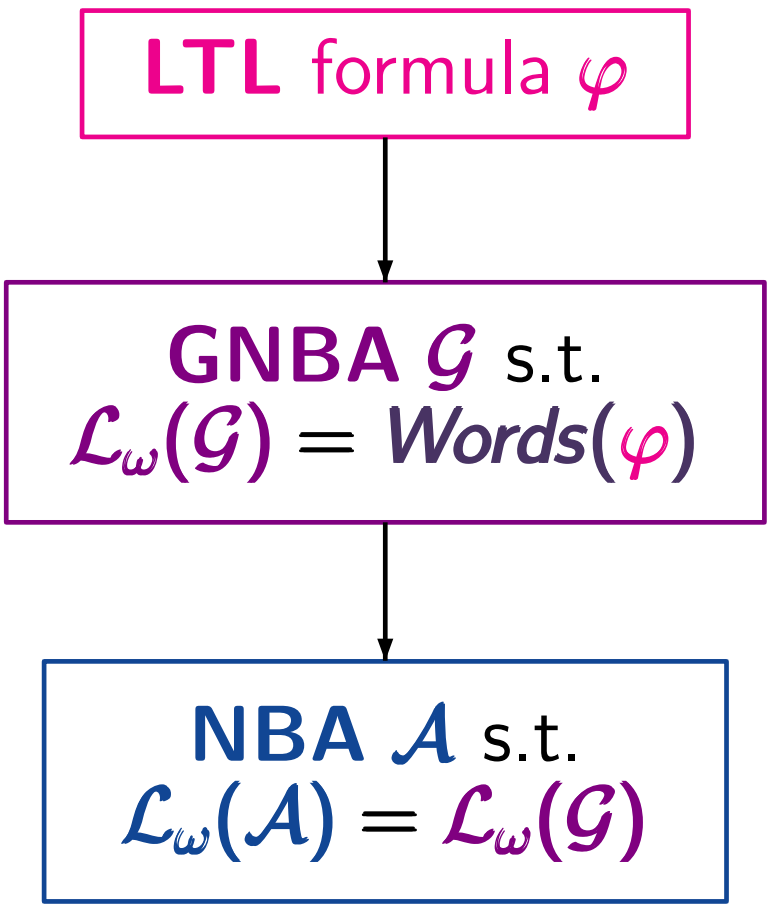

generalized NBA
several acceptance sets

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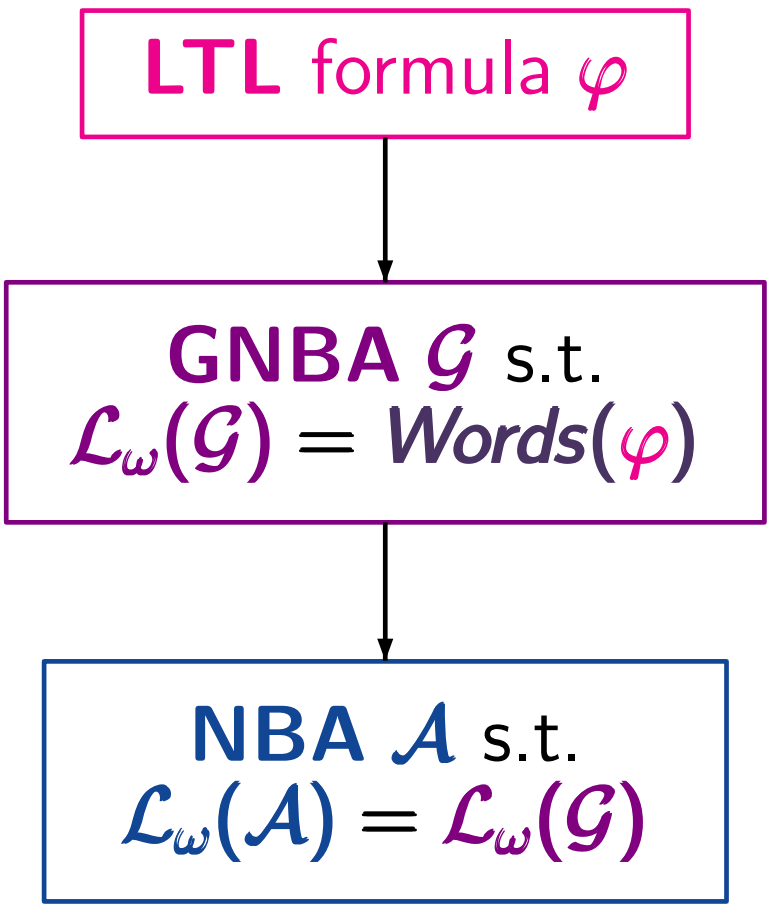

generalized NBA
k acceptance sets

k copies of $\mathcal{G}$

nondeterministic
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Encoding of LTL semantics in a GNBA

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idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

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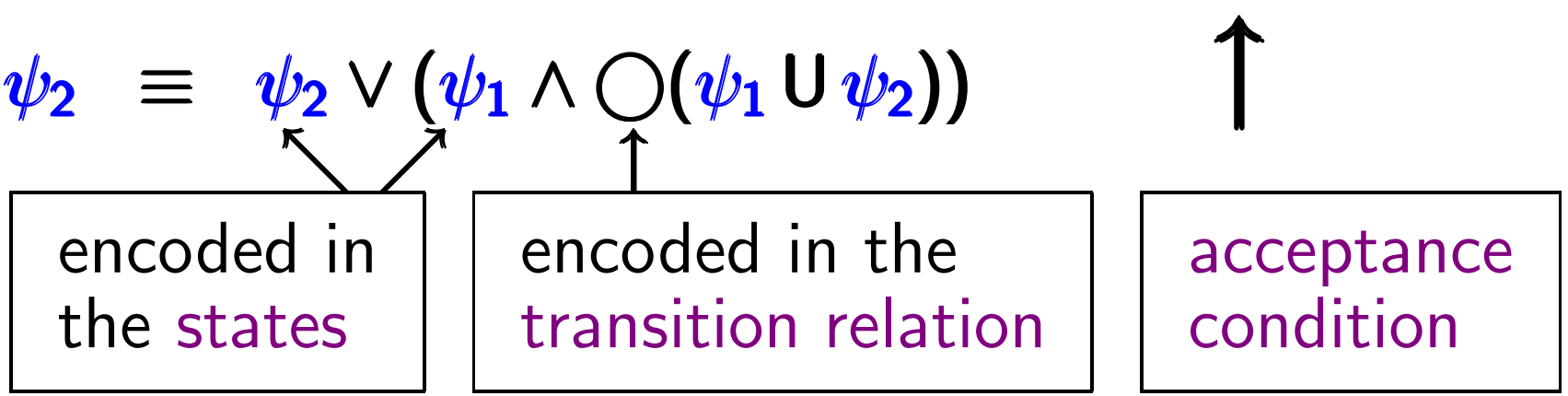

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

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where $B_i = \{\psi \in \mathit{cl}(\varphi) : A_i A_{i+1} A_{i+2} \ldots \models \psi\}$

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set of subformulas of φ and their negations

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where the B_i's are subsets of
$\{a, \neg a, b, \neg b, \psi, \neg\psi, \varphi, \neg\varphi\}$

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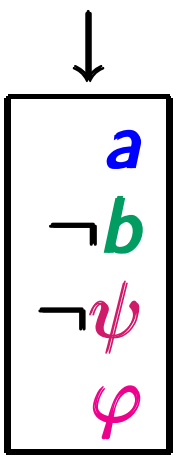

just for better readability:
tuple rather than set notation

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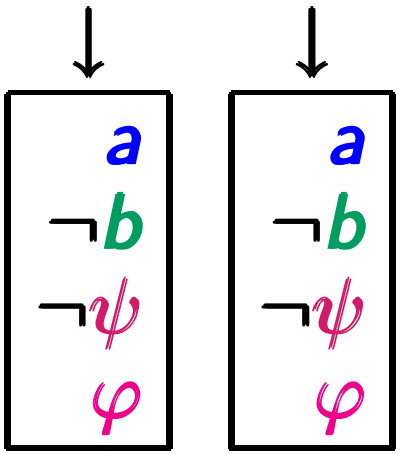

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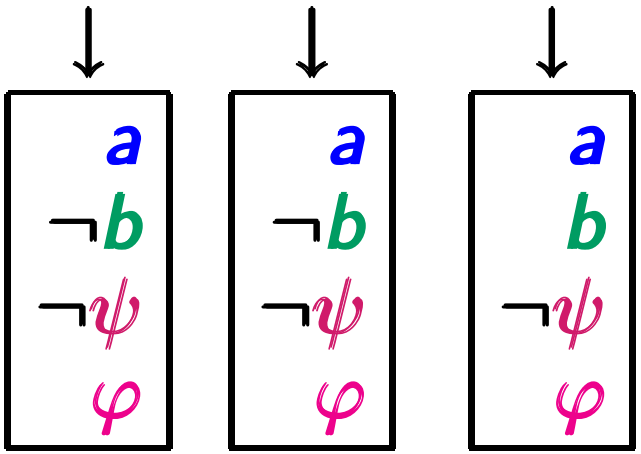

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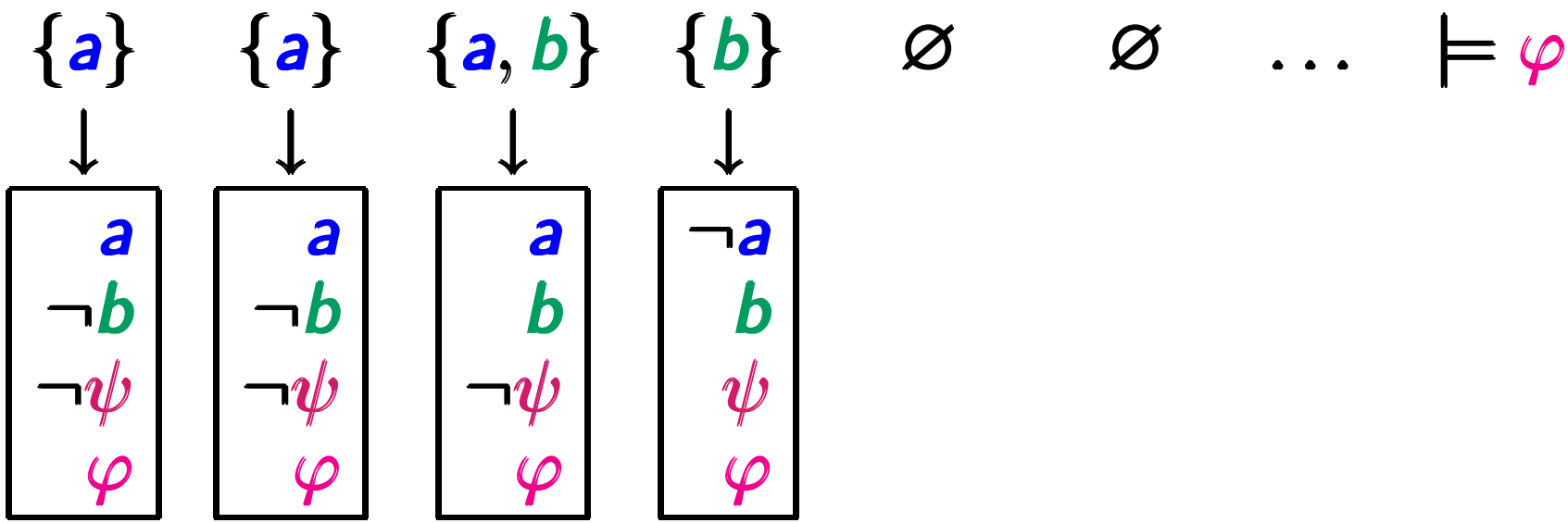

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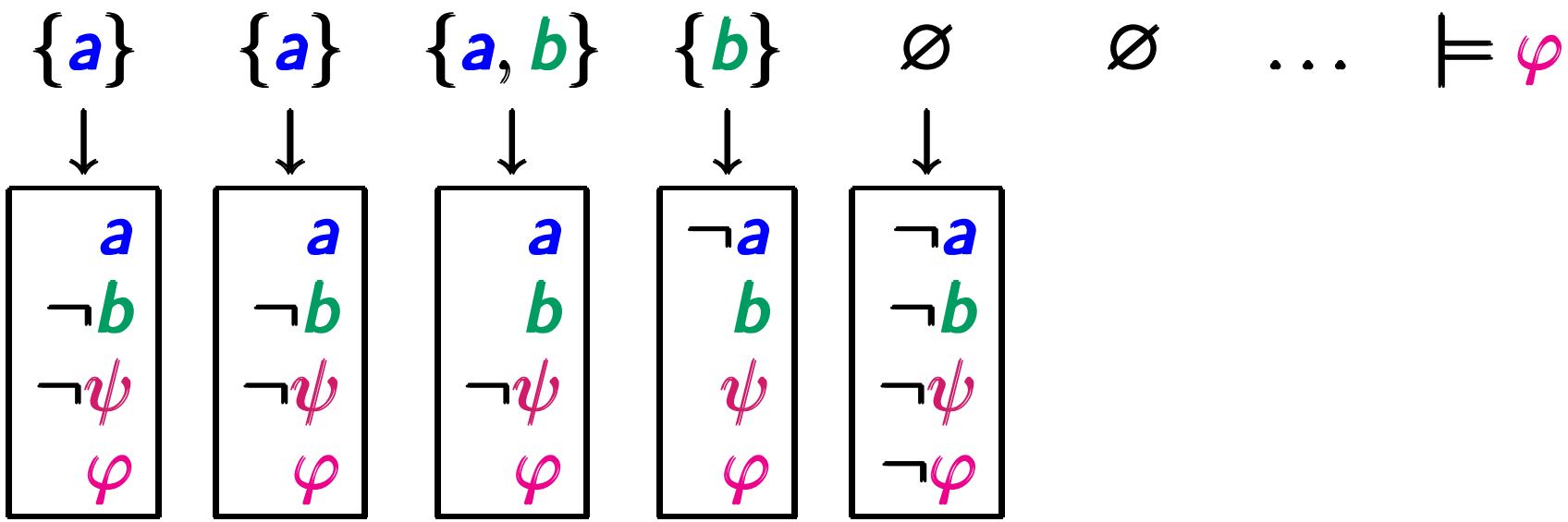

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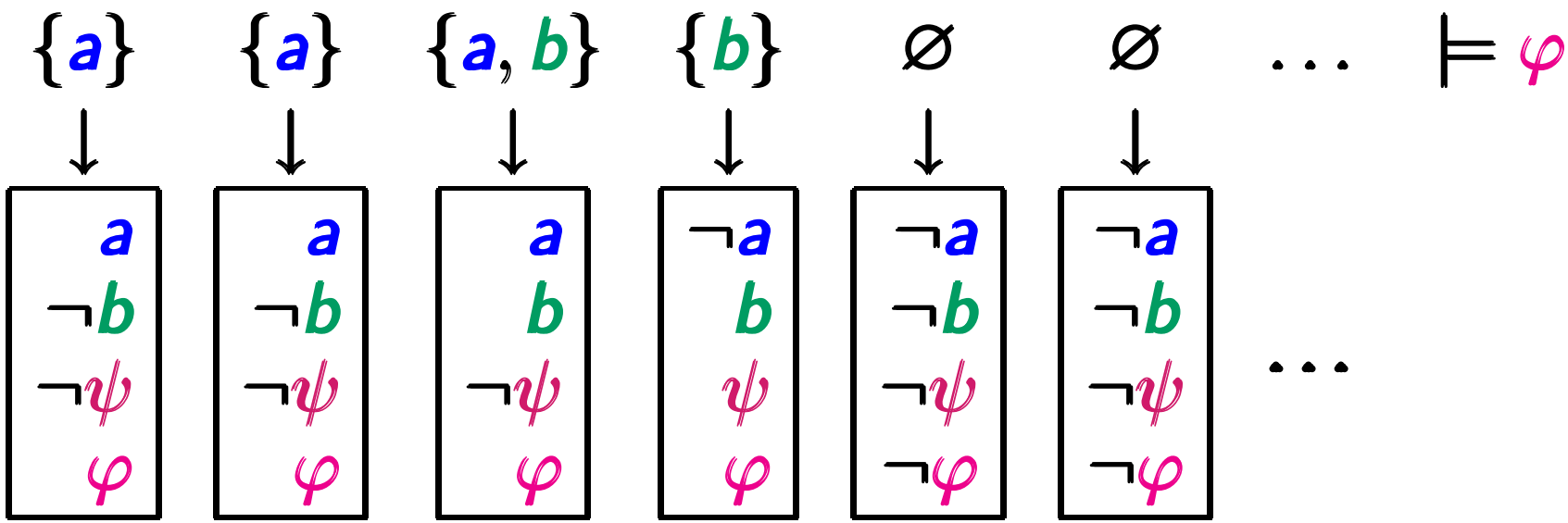

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- if $\mathit{false} \in cl(\varphi)$ then $\mathit{false} \notin B$

(2) B is maximal consistent

- if $\psi \in cl(\varphi) \setminus B$ then $\neg\psi \in B$

(3) B is locally consistent with respect to until U:

- if $\psi_1 \,\mathsf{U}\, \psi_2 \in B$ and $\neg\psi_2 \in B$ then $\neg\psi_1 \notin B$

Elementary formula-sets

Let $B \subseteq cl(\varphi)$. B is called elementary if:

(1) B is consistent w.r.t. propositional logic

- if $\psi \in B$ then $\neg\psi \notin B$
- if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$
- if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$
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(3) B is locally consistent with respect to until U:

- if $\psi_1 \,\mathsf{U}\, \psi_2 \in B$ and $\neg\psi_2 \in B$ then $\neg\psi_1 \notin B$
- if $\psi_2 \in B$ and $\psi_1 \,\mathsf{U}\, \psi_2 \in cl(\varphi)$ then $\neg(\psi_1 \,\mathsf{U}\, \psi_2) \notin B$

Elementary formula-sets

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic,
i.e., if ψ, $\psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$\psi \notin B$	iff	$\neg\psi \in B$
$\psi_1 \wedge \psi_2 \in B$	iff	$\psi_1 \in B$ and $\psi_2 \in B$
$\mathit{true} \in cl(\varphi)$	implies	$\mathit{true} \in B$

(ii) B is locally consistent with respect to until U,
i.e., if $\psi_1 \,\mathsf{U}\, \psi_2 \in cl(\varphi)$ then:

if $\psi_1 \,\mathsf{U}\, \psi_2 \in B$ and $\psi_2 \notin B$ then $\psi_1 \in B$

if $\psi_2 \in B$ then $\psi_1 \,\mathsf{U}\, \psi_2 \in B$

Elementary or not?

Let $\varphi = a \mathsf{U} (\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$

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$B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$ elementary

Example: elementary formula-sets

closure $cl(\varphi)$:

- set of all subformulas of φ and their negations
- ψ and $\neg\neg\psi$ are identified

elementary formula-sets: subsets B of $cl(\varphi)$

- maximal consistent w.r.t. propositional logic
- locally consistent w.r.t. U

For $\varphi = a\,\mathsf{U}(\neg a \wedge b)$, the elementary sets are:

$$\{\ a,\ \ b, \neg(\neg a \wedge b), \varphi\} \qquad \{\ a,\ \ b, \neg(\neg a \wedge b), \neg\varphi\}$$
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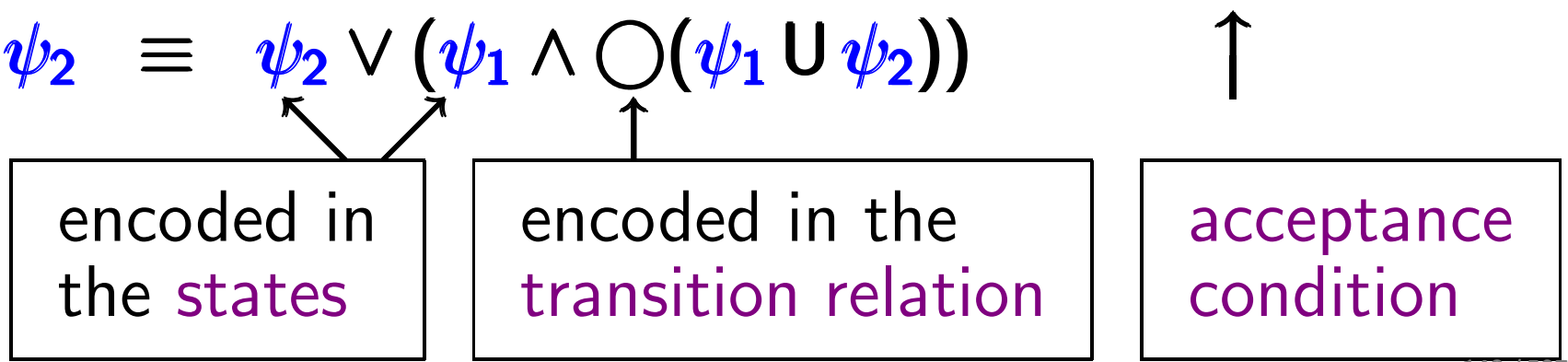
Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$:

semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
until U	expansion law, least fixed point

$$\psi_1 \,\mathsf{U}\, \psi_2 \quad \equiv \quad \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2))$$



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↑

elementary formula sets

encoded in the transition relation

acceptance condition

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Example: GNBA for $\varphi = \bigcirc a$

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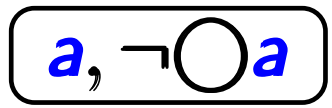

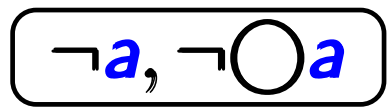

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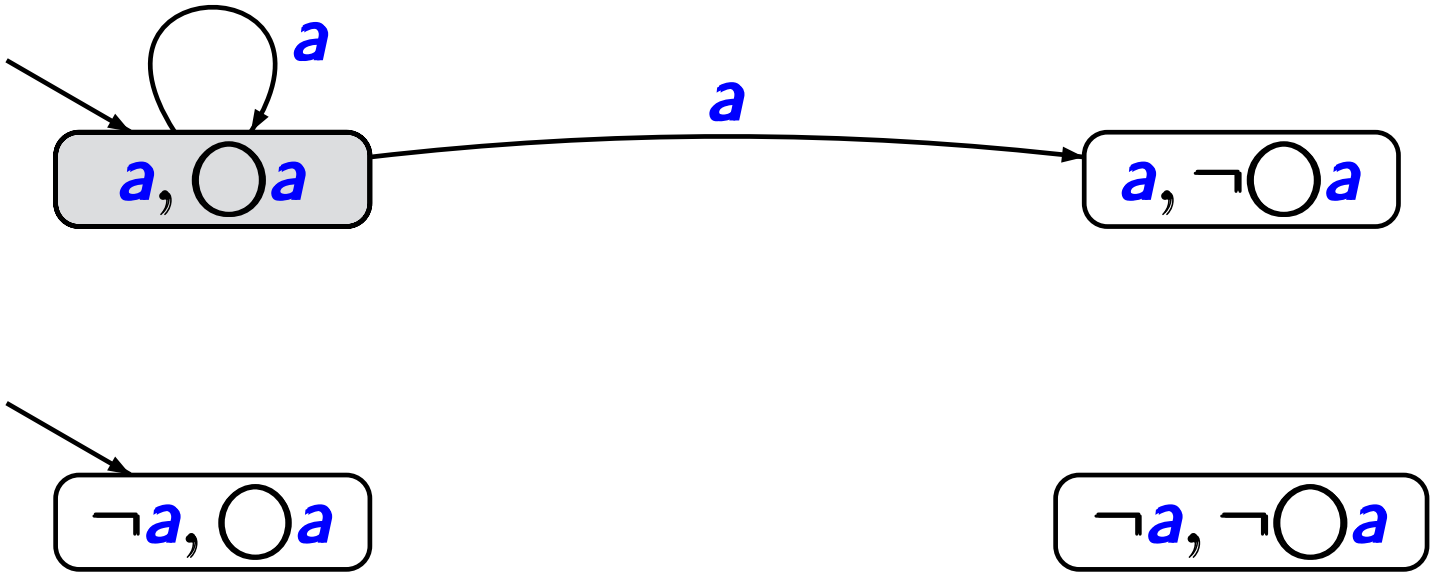

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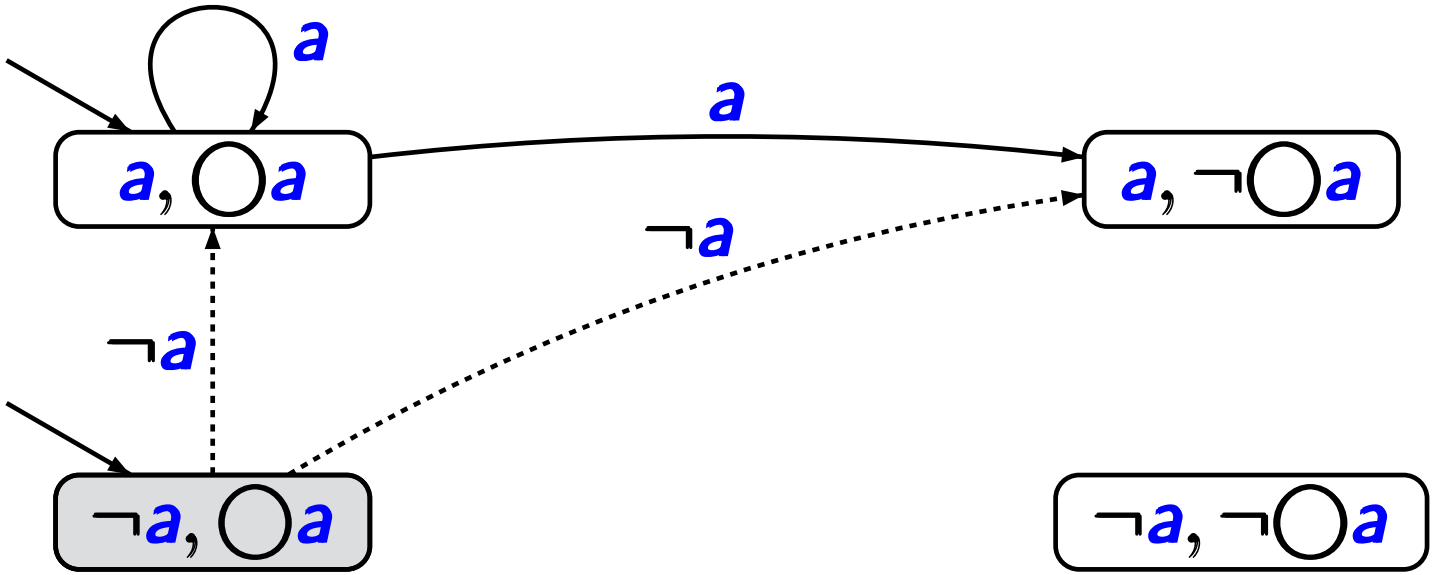

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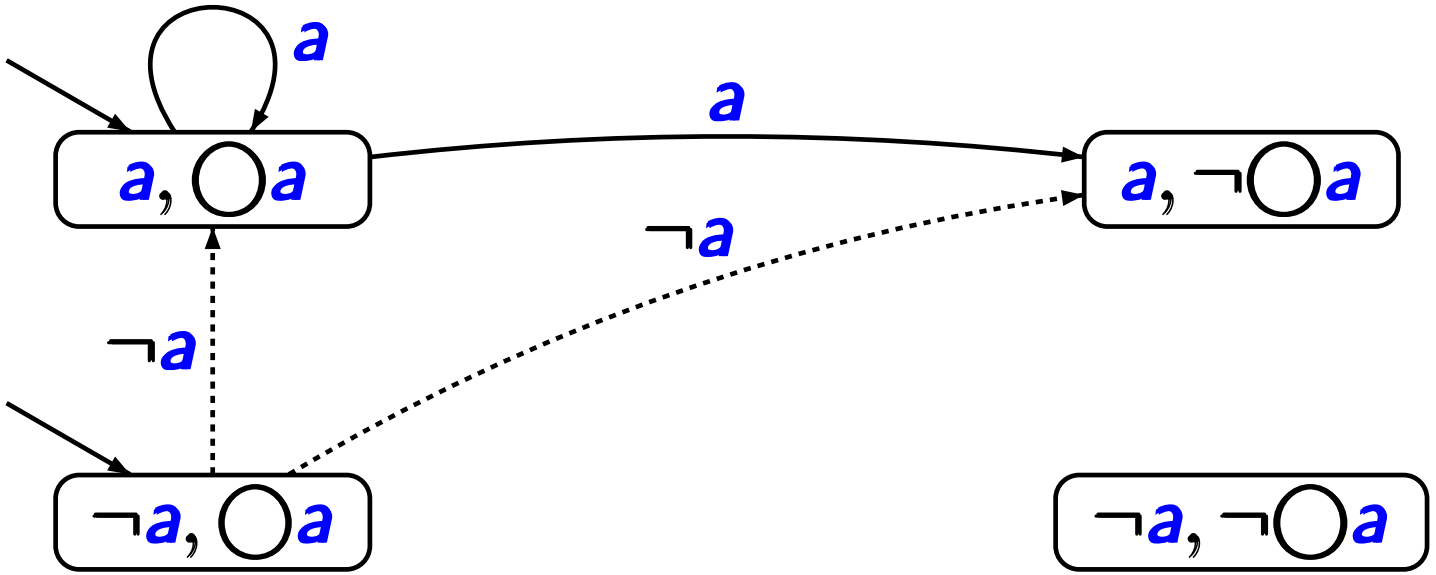

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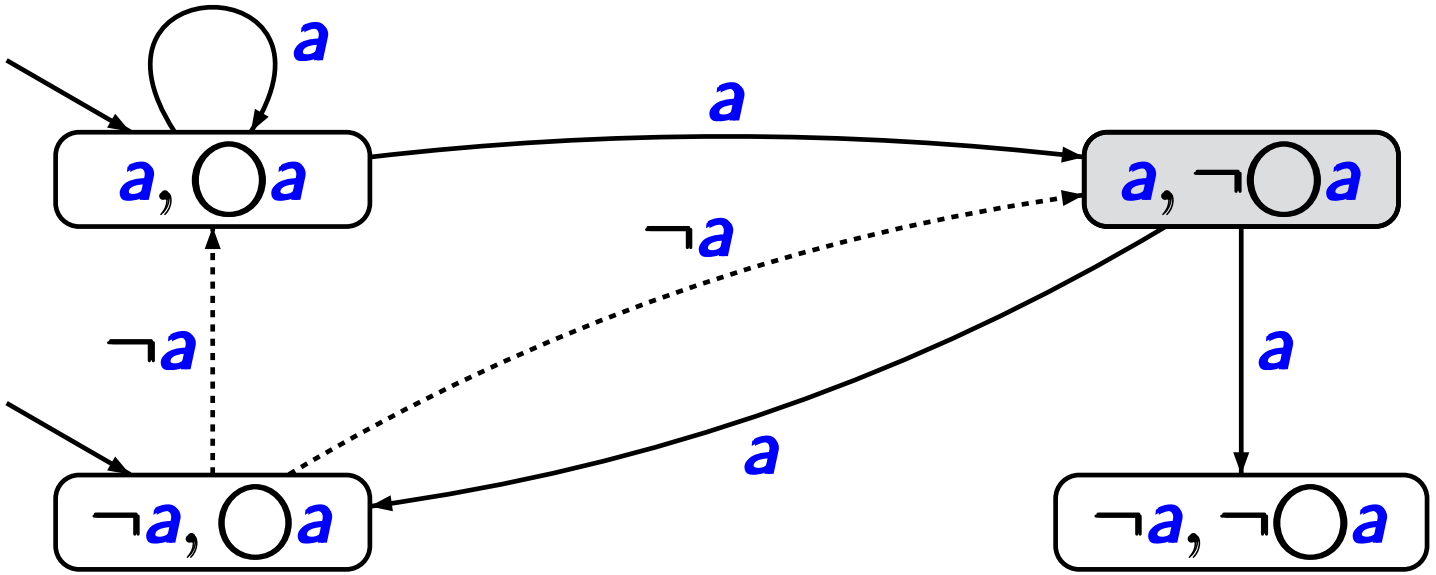

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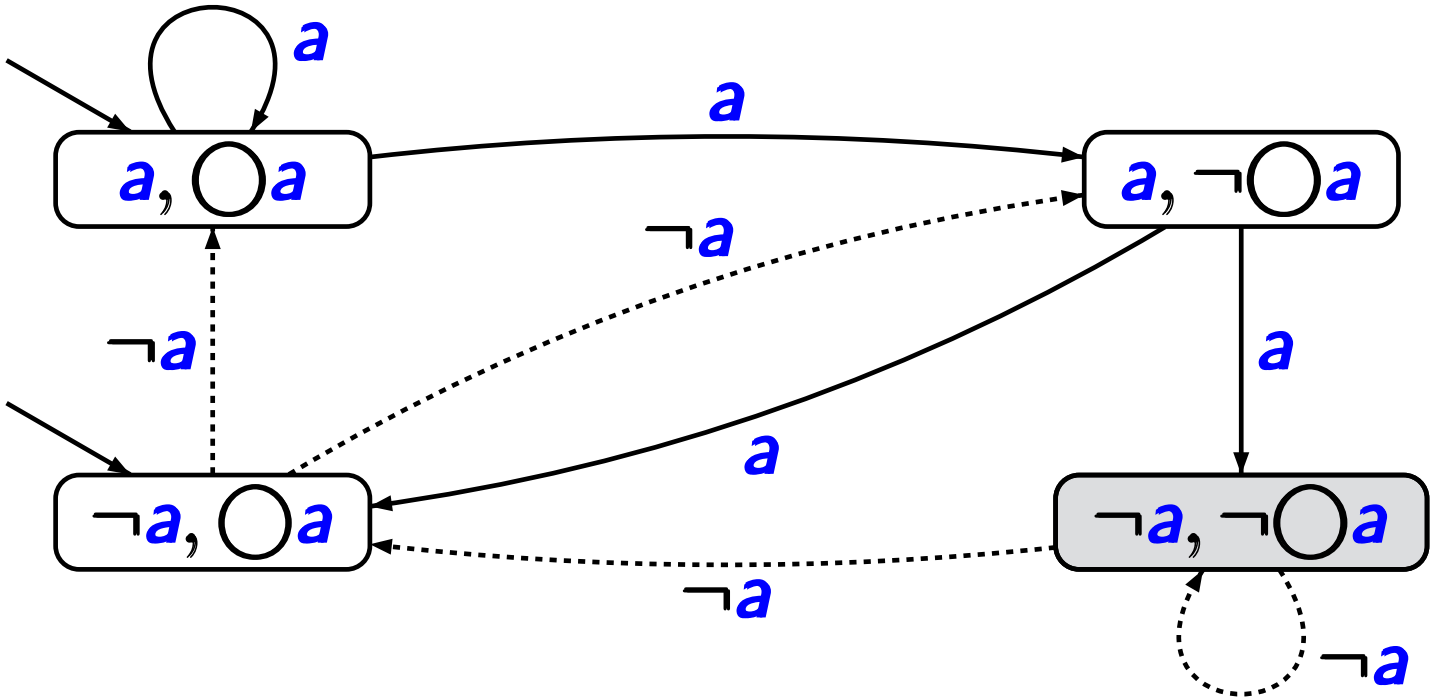

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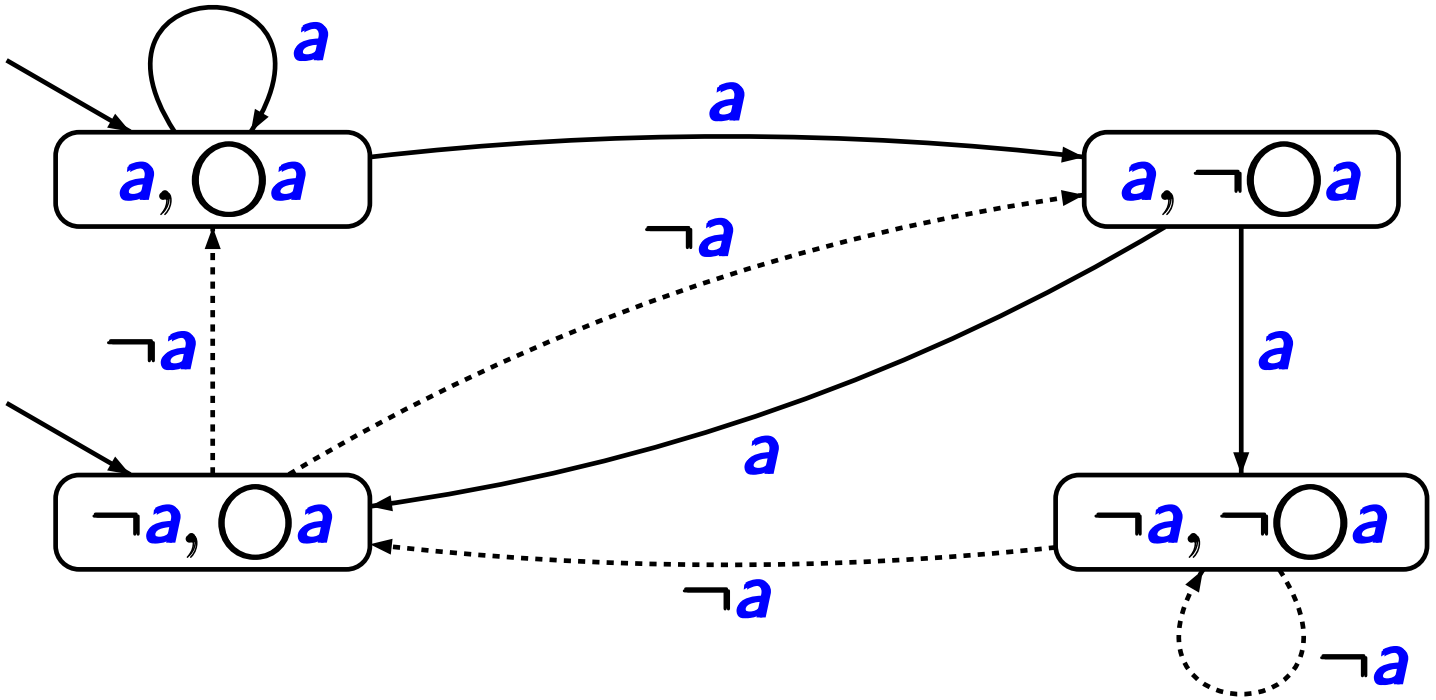
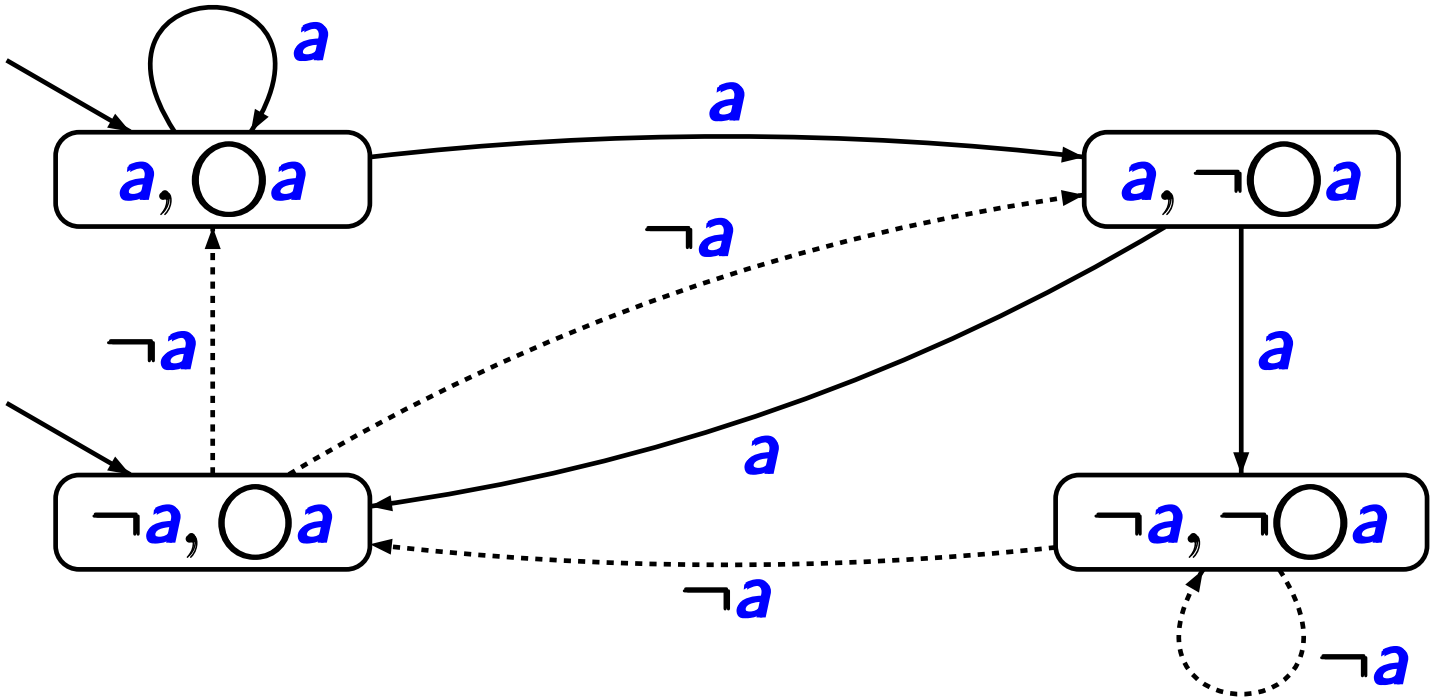


set of acceptance sets:

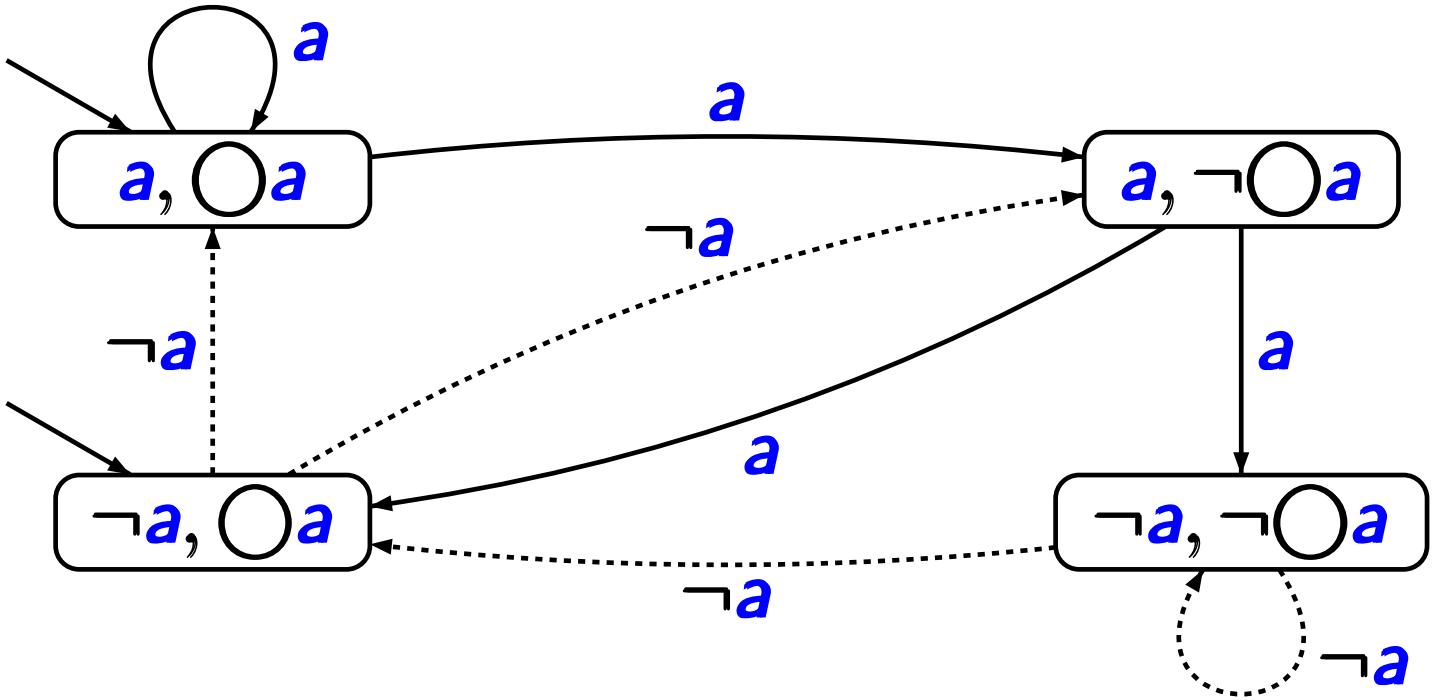
Example: GNBA for $\varphi = \bigcirc a$



set of acceptance sets: $\mathcal{F} = \varnothing$

hence: all words having an infinite run are accepted

Example: GNBA for $\varphi = \bigcirc a$



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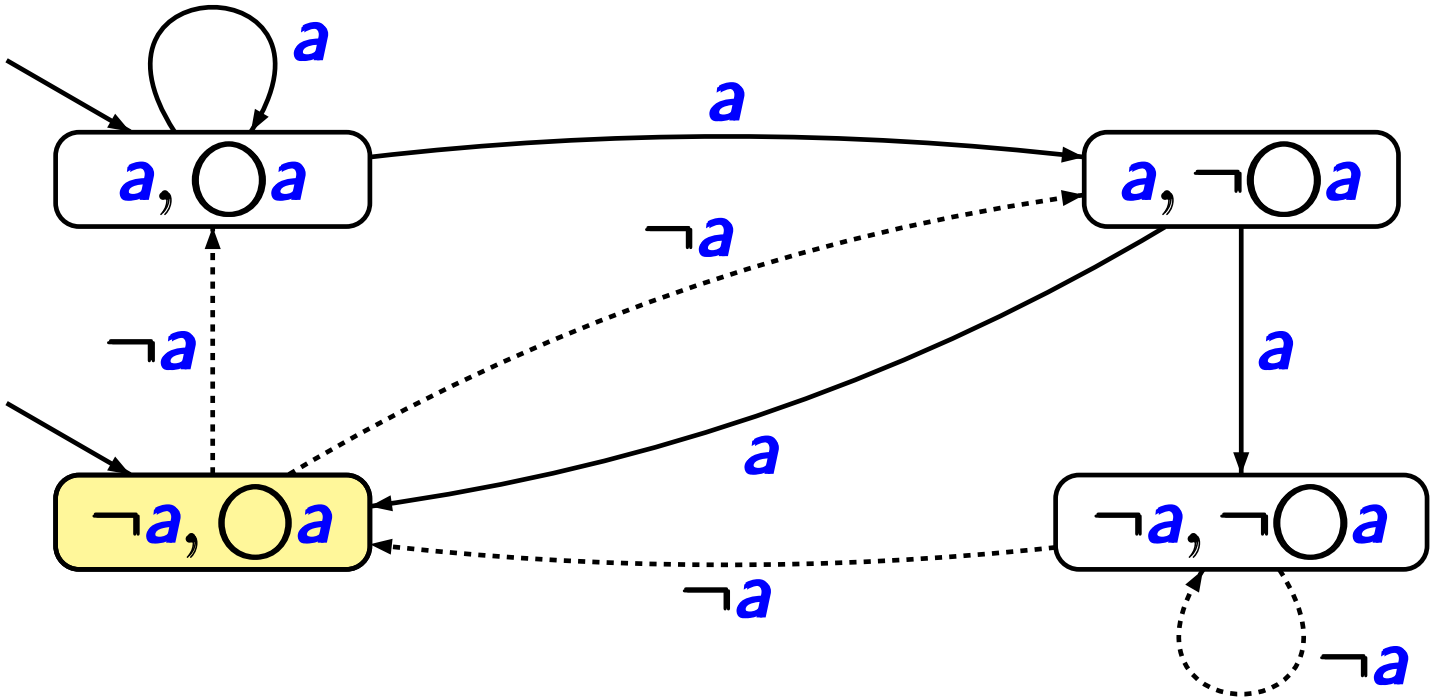

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