Overview

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here: two propositional temporal logics:

LTL: linear temporal logic

CTL: computation tree logic

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL ⟵
- automata-based LTL model checking
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

Linear Temporal Logic (LTL)

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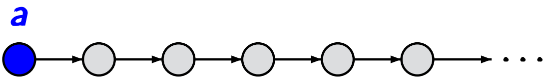

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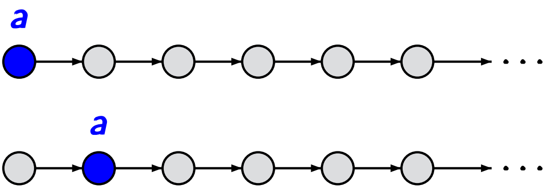

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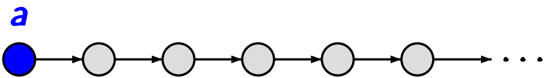

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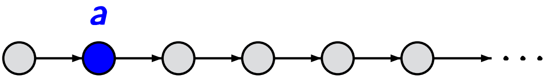
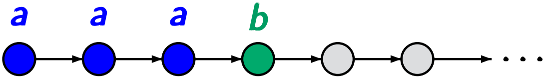


until operator
$a \,\mathsf{U}\, b$



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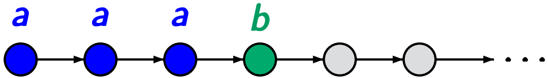
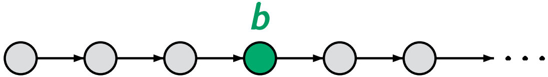


eventually
$\Diamond b$



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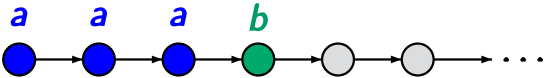

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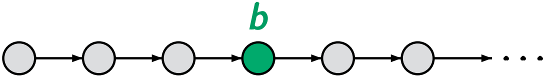
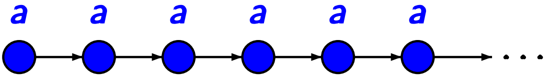
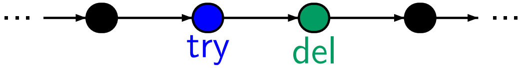


always
$\Box a$



$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\, \text{delivered})$



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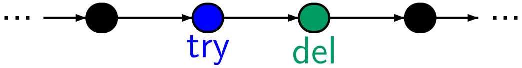
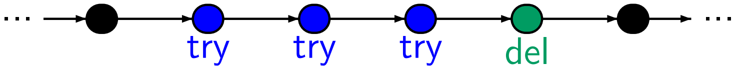


$\Box$ (try_to_send $\longrightarrow$ try_to_send **U** delivered)



Next $\bigcirc$, until U and eventually $\Diamond$

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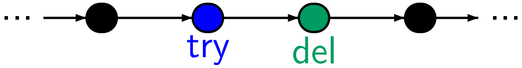

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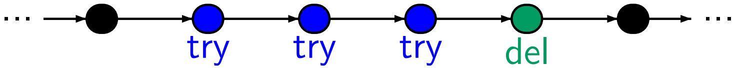

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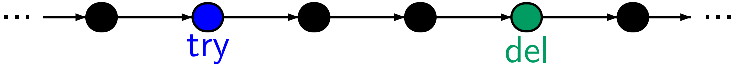

Examples for LTL formulas

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traffic light: $\Box(\mathit{yellow} \vee \bigcirc\neg \mathit{red})$

Infinitely often and eventually forever

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interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

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LT property of formula φ:

$$Words(\varphi) \stackrel{\text{def}}{=} \left\{\sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi\right\}$$

LTL-semantics of derived operators $\Diamond$ and $\Box$

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& & A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1 \quad \text{for } 0 \leq i < j \\
\sigma \models \Diamond\varphi & \text{iff} & \text{there exists } j \geq 0 \text{ such that} \\
& & A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi \\
\sigma \models \Box\varphi & \text{iff} & \text{for all } j \geq 0 \text{ we have:} \\
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\end{array}$$

LTL semantics over TS

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given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

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assumption: $\mathcal{T}$ has no terminal states, i.e.,
all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
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LTL formula φ over AP

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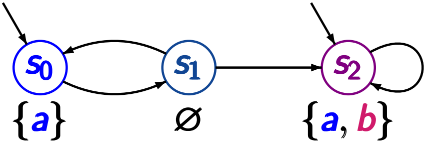
remind: LT property of an LTL formula:

$$Words(\varphi) = \left\{ \sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi \right\}$$

Example: LTL-semantics over paths



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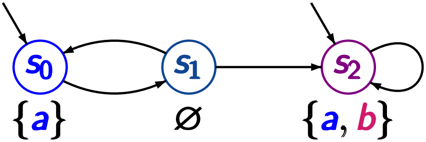

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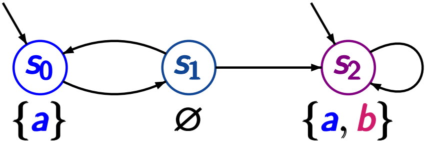
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$

Example: LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$

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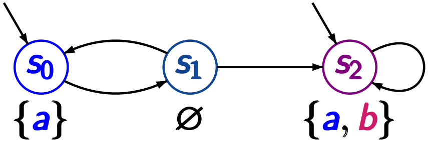

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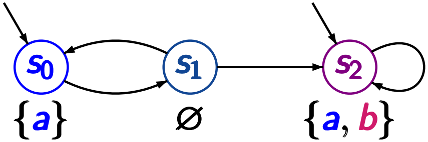

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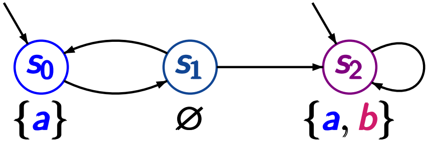

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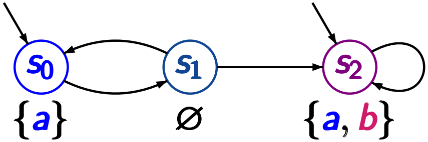

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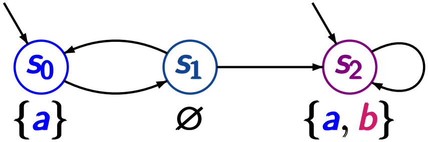

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$\pi \models a$, but $\pi \not\models b$	as $L(s_0) = \{a\}$
$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
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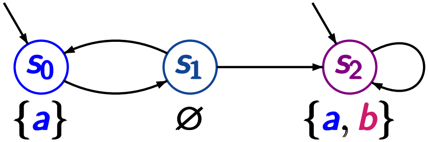

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$\pi \models \bigcirc \bigcirc (a \wedge b)$ $\qquad$ as $L(s_2) = \{a, b\}$

$\pi \models (\neg b) \,\mathsf{U}\, (a \wedge b)$

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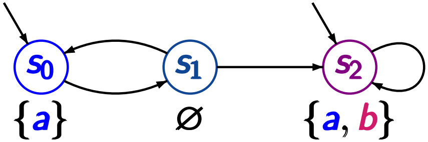

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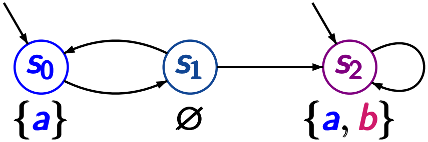

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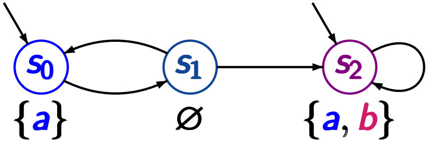
$\pi \models \bigcirc \bigcirc (a \wedge b)$	as $L(s_2) = \{a, b\}$
$\pi \models (\neg b) \, \mathsf{U} \, (a \wedge b)$	as $s_0, s_1 \models \neg b$
$\pi \models (\neg b) \, \mathsf{U} \, \Box(a \wedge b)$	and $s_2 \models a \wedge b$

Correct or wrong ?



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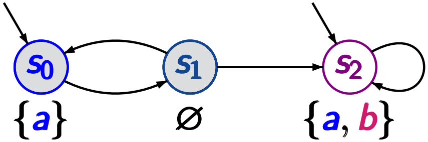

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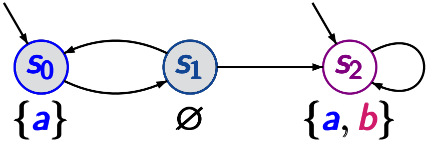

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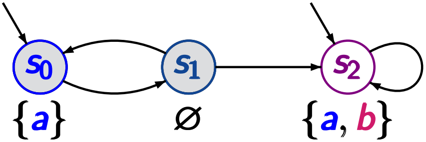

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$\pi \not\models a \,\mathsf{U}\, b$ $\quad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

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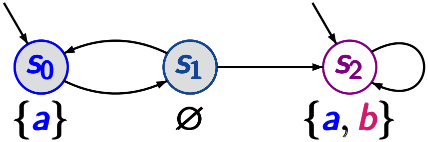

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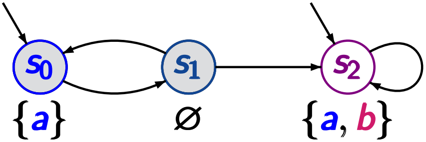
$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$?

Correct or wrong ?



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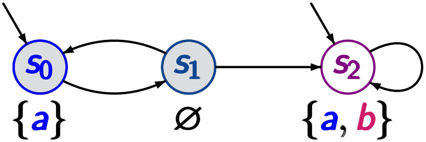

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$\pi \models \bigcirc \bigcirc \neg b$?

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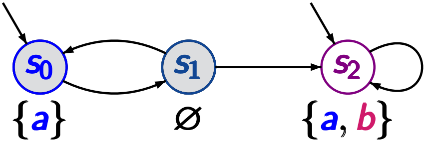

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$\pi \not\models a \, \mathsf{U} \, b$ $\quad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \, \mathsf{U} \, b)$ $\quad$ as $\pi \not\models \Diamond b$

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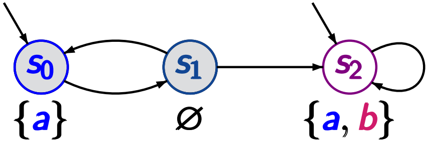

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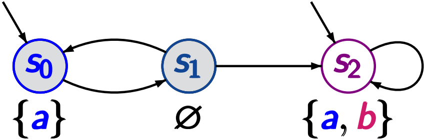
$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \models \Box a$?

Correct or wrong ?



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$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

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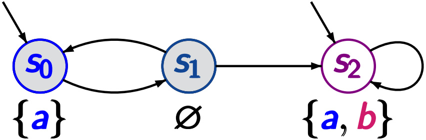

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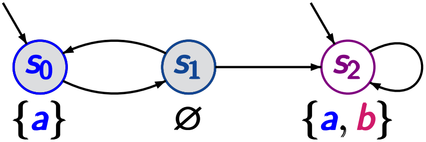
$\pi \models \bigcirc \bigcirc \neg b$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$?

Correct or wrong ?



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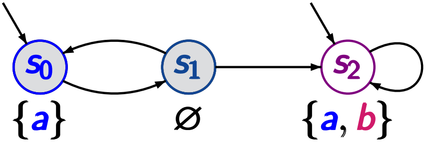
$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ $\qquad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often



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$\pi \models \bigcirc \bigcirc \neg b$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ as $s_1 \not\models a$

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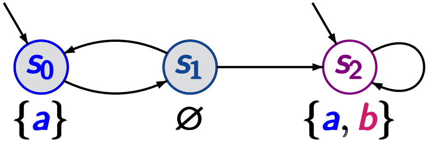
$\pi \models \Diamond\Box a$?

Correct or wrong ?



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$\pi \not\models a \,\mathsf{U}\, b$ $\qquad$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ $\qquad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

$\pi \not\models \Diamond\Box a$ $\qquad$ as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

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$\sigma \models \lozenge\varphi$ iff there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi$

$\sigma \models \square\varphi$ iff for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi$

LTL-semantics of derived operators

for $\sigma = A_0 A_1 A_2 \ldots \in \left(2^{AP}\right)^{\omega}$:

$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j A_{j+1} A_{j+2} \ldots \models \varphi$

LTL-semantics of derived operators

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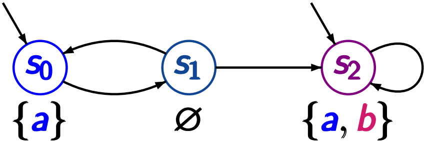
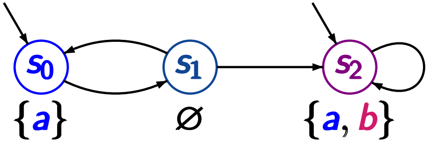
$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box\varphi$	iff	for almost all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$

LTL-semantics over paths



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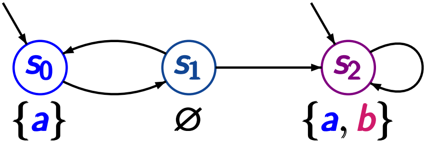
LTL-semantics over paths



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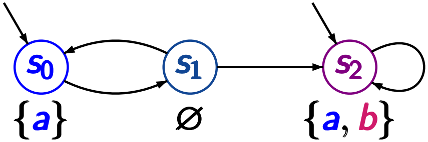
$\pi \models \bigcirc((\neg a \wedge \neg b) \, \mathsf{U} \, (a \wedge b))$?

LTL-semantics over paths



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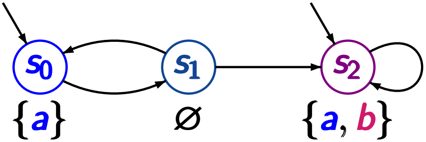
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b) \,\mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

LTL-semantics over paths



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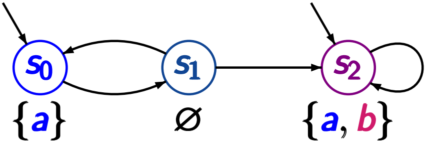
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$?

LTL-semantics over paths



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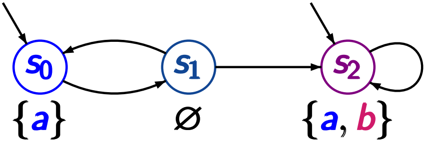
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U} (a \wedge b))$ $\quad$ as $s_1 \models \neg a \wedge \neg b$
$s_2 \models a \wedge b$

$\pi \models \bigcirc \Box (a \leftrightarrow b)$ $\quad$ as $s_1, s_2 \models a \leftrightarrow b$

LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b) \,\mathsf{U}\, (a \wedge b))$ $\quad$ as $s_1 \models \neg a \wedge \neg b$

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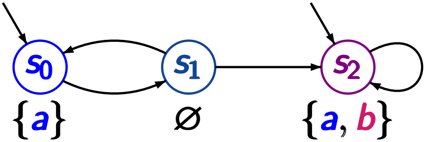

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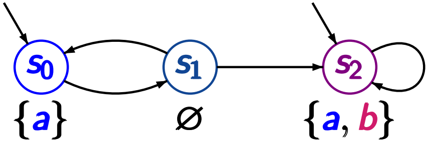

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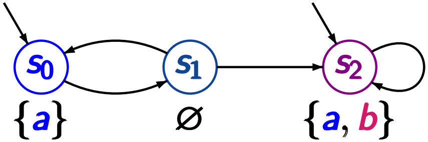

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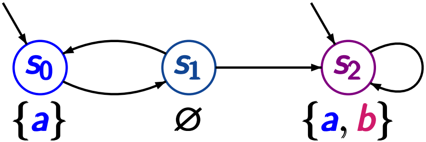

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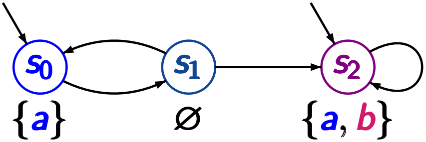

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LTL semantics over the states of a TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

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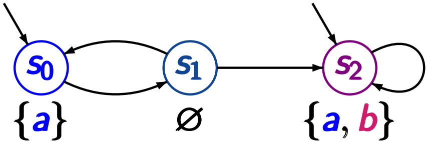

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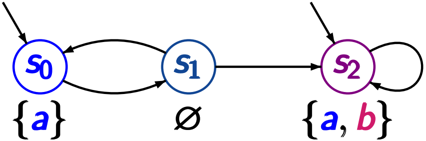

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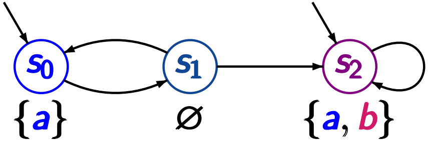

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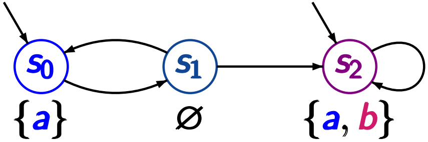

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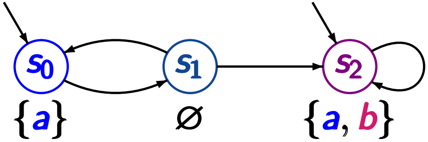

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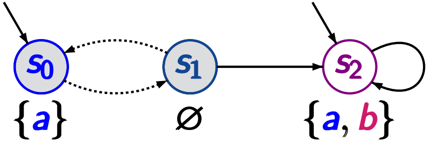

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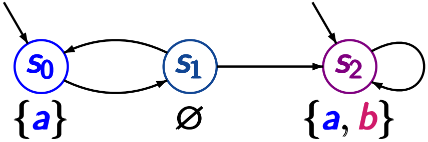

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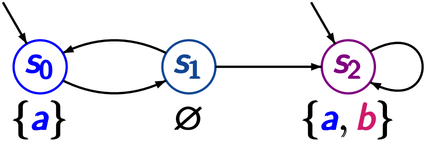

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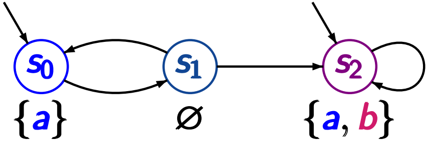

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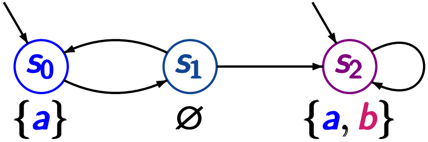

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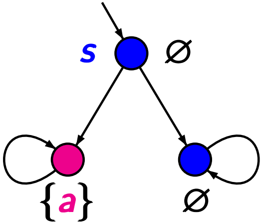

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LTL-formulas for MUTEX protocols

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- set of all words $A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$ such that:

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Self-duality of the next operator

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Claim: $\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ "self-duality of next"

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Proof: $A_0\, A_1\, A_2\, A_3 \ldots \models \neg \bigcirc \varphi$

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\end{array}
$$

Correct or wrong?

$$\Diamond(\varphi \vee \psi) \equiv \Diamond\varphi \vee \Diamond\psi$$

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wrong,
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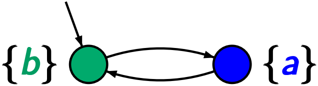

$\models \Diamond b \wedge \Diamond a$

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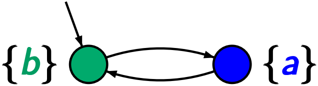

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---

similarly: $\Box(\varphi \wedge \psi) \;\equiv\; \Box\varphi \wedge \Box\psi$

$\Box(\varphi \vee \psi) \;\not\equiv\; \Box\varphi \vee \Box\psi$

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correct Analogous: $\Box\Box\varphi \;\equiv\; \Box\varphi$