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LEHRSTUHL FÜR INFORMATIK 2
RWTH Aachen · D-52056 Aachen
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Prof. Dr. Ir. J.-P. Katoen
N. Jansen & B. Kaminiski

Modeling and Verification of Probabilistic Systems
Summer term 2014

– Series 5 –

Hand in on June 5 before the exercise class.

Exercise 1 **(2 points)**

Construct an MDP $\mathcal{M}$ with initial state s_0, and an ω–regular property P, such that for any positional policy $\mathfrak{S}$ for $\mathcal{M}$ it holds that

$$\mathrm{Pr}^{\min}(s_0 \vDash P) \; < \; \mathrm{Pr}^{\mathcal{M}}_{\mathfrak{S}}(s_0 \vDash P) \; .$$

Exercise 2 **(3 points)**

Let $\mathcal{M} = (S,\, Act,\, \mathbf{P},\, \iota_{init},\, AP,\, L)$ be a finite MDP, and let $B \subseteq S$.

Prove or disprove: There exists a positional policy $\mathfrak{S}$, such that

$$\mathrm{Pr}^{\max}(s \vDash \Box\Diamond B) \; = \; \mathrm{Pr}^{\mathcal{M}}_{\mathfrak{S}}(s \vDash \Box\Diamond B) \; .$$

Exercise 3 **(5 points)**

a) Give a finite MDP $\mathcal{M} = (S,\, Act,\, \mathbf{P},\, \iota_{init},\, AP,\, L)$ with dedicated states $s_0, s \in S$, and a policy $\mathfrak{S}$, such that $\mathrm{Pr}^{\mathcal{M}}_{\mathfrak{S}}(s_0 \vDash \Diamond\{s\})$ is not computable!

b) Reconsider your MDP $\mathcal{M}$ and give a policy $\mathfrak{S}_{\min}$ such that $\mathrm{Pr}^{\mathcal{M}}_{\mathfrak{S}_{\min}}(s_0 \vDash \Diamond\{s\}) = \mathrm{Pr}^{\min}(s_0 \vDash \Diamond\{s\})$! What is $\mathrm{Pr}^{\min}(s_0 \vDash \Diamond\{s\})$?

c) Reconsider your MDP $\mathcal{M}$ and give a policy $\mathfrak{S}_{\max}$ such that $\mathrm{Pr}^{\mathcal{M}}_{\mathfrak{S}_{\max}}(s_0 \vDash \Diamond\{s\}) = \mathrm{Pr}^{\max}(s_0 \vDash \Diamond\{s\})$! What is $\mathrm{Pr}^{\max}(s_0 \vDash \Diamond\{s\})$?