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LEHRSTUHL FÜR INFORMATIK 2
RWTH Aachen · D-52056 Aachen
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Modeling and Verification of Probabilistic Systems
Summer term 2014

– Series 4 –

Hand in on May 22 before the exercise class.

Exercise 1 (3 points)

Prove the following theorem: For any finite DTMC it holds that

$$\mathbb{P}_{=1}(\Diamond a) \quad \equiv \quad \forall\big((\exists \Diamond a)\ \mathsf{W}\ a\big) \ ,$$

where W is the weak until operator defined by $\Phi\ \mathsf{W}\ \Psi = (\Phi\ \mathsf{U}\ \Psi) \vee \Box\Phi$.

Exercise 2 (2 points)

A PCTL formula is in *negation normal form*, if it adheres to the following syntax: PCTL state formulae adhere to the syntax

$$\Phi \ ::= \ \mathsf{true} \mid \mathsf{false} \mid a \mid \neg a \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid \mathbb{P}_J(\varphi) \ ,$$

where $a \in AP$, $J \subseteq [0,\, 1]$ is a non–empty interval, and φ is a PCTL path formula. PCTL path formulae adhere to the syntax

$$\varphi \ ::= \ \bigcirc \Phi \mid \Phi\ \mathsf{U}\ \Phi \mid \Phi\ \mathsf{U}^{\leq n}\ \Phi \ .$$

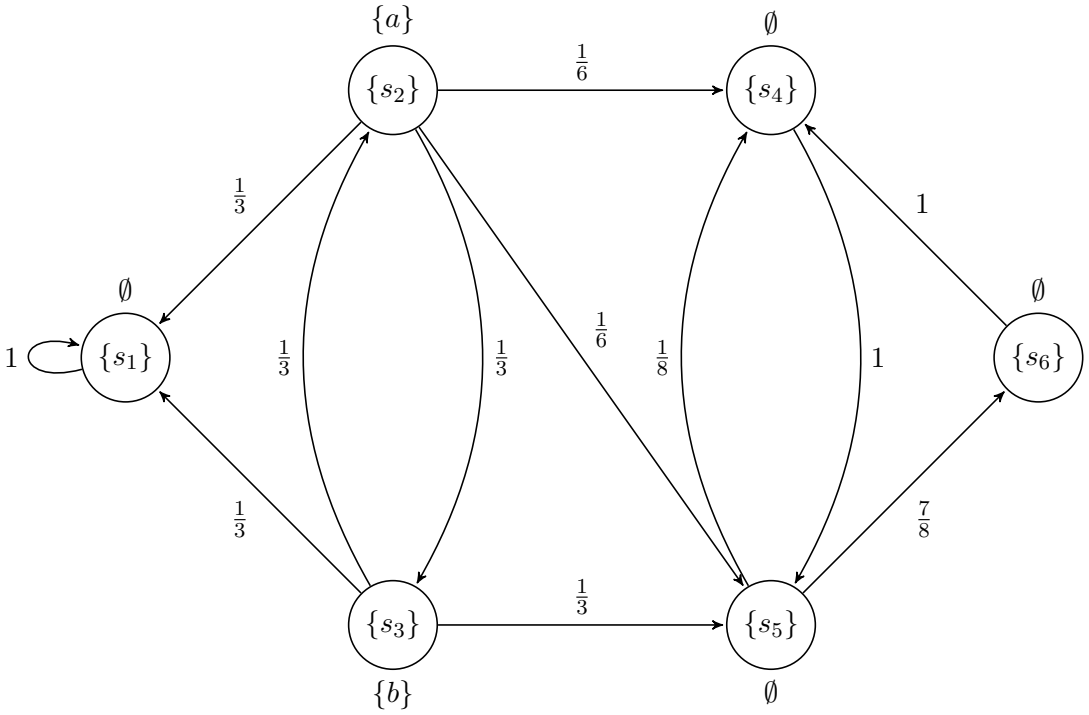
Prove or disprove: Every PCTL formula is equivalent to a PCTL formula in negation normal form.

Exercise 3 (3 points)

Prove that the alternative definition of strong bisimilarity (Lecture 8, Slide 12) and its traditional definition (Lecture 8, Slide 4) are equivalent.

Exercise 4 **(2 points)**

Consider the following DTMC $\mathcal{D}$:



Give $\mathcal{D}/\sim_p$, i.e. the quotient of $\mathcal{D}$ under probabilistic bisimulation!