Compiler Construction

Lecture 7: Syntax Analysis III ($LL(1)$ Parsing)

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Summer Semester 2014

Outline

1. Recap: $LL(k)$ Grammars
2. Characterization of $LL(1)$
3. Computing Lookahead Sets
4. Deterministic Top-Down Parsing

$LL(k)$ Grammars

$LL(k)$: reading of input from Left to right with k-lookahead, computing a Leftmost analysis

Definition ($LL(k)$ grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $k \in \mathbb{N}$. Then G has the $LL(k)$ property (notation: $G \in LL(k)$) if for all leftmost derivations of the form

$$S \Rightarrow_l^* wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \Rightarrow_l^* wx \\ \Rightarrow_l w\gamma\alpha \Rightarrow_l^* wy \end{cases}$$

such that $\beta \neq \gamma$, it follows that $\mathrm{first}_k(x) \neq \mathrm{first}_k(y)$
(i.e., different productions must not yield the same lookahead).

The Case $k = 1$

Motivation:

- $k = 1$ sufficient to resolve nondeterminism in "most" practical applications
- Implementation of $LL(k)$ parsers for $k > 1$ rather involved (cf. ANTLR [ANother Tool for Language Recognition; formerly PCCTS] at http://www.antlr.org/)

Abbreviations: $\mathrm{fi} := \mathrm{first}_1$, $\mathrm{fo} := \mathrm{follow}_1$, $\Sigma_\varepsilon := \Sigma \cup \{\varepsilon\}$

Corollary

1. *For every* $\alpha \in X^*$,

$$\mathrm{fi}(\alpha) = \{a \in \Sigma \mid \textit{ex. } w \in \Sigma^* : \alpha \Rightarrow^* aw\} \cup \{\varepsilon \mid \alpha \Rightarrow^* \varepsilon\} \subseteq \Sigma_\varepsilon$$

2. *For every* $A \in N$,

$$\mathrm{fo}(A) = \{x \in \mathrm{fi}(\alpha) \mid \textit{ex. } w \in \Sigma^*, \alpha \in X^* : S \Rightarrow_l^* wA\alpha\} \subseteq \Sigma_\varepsilon.$$

Lookahead Sets

Definition (Lookahead set)

Given $\pi = A \to \beta \in P$,

$$\mathrm{la}(\pi) := \mathrm{fi}(\beta \cdot \mathrm{fo}(A)) \subseteq \Sigma_\varepsilon$$

is called the lookahead set of π (where $\mathrm{fi}(\Gamma) := \bigcup_{\gamma \in \Gamma} \mathrm{fi}(\gamma)$).

Corollary

1. *For all* $a \in \Sigma$,

$$a \in \mathrm{la}(A \to \beta) \textit{ iff } a \in \mathrm{fi}(\beta) \textit{ or } (\beta \Rightarrow^* \varepsilon \textit{ and } a \in \mathrm{fo}(A))$$

2. $\varepsilon \in \mathrm{la}(A \to \beta)$ *iff* $\beta \Rightarrow^* \varepsilon$ *and* $\varepsilon \in \mathrm{fo}(A)$

Outline

Characterization of $LL(1)$

Theorem 7.1 (Characterization of $LL(1)$)

$G \in LL(1)$ iff for all pairs of rules $A \to \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$\mathrm{la}(A \to \beta) \cap \mathrm{la}(A \to \gamma) = \emptyset.$$

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Proof.

on the board □

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Proof.

on the board □

Remark: the above theorem generally does not hold if $k > 1$ (cf. exercises)

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3. Computing Lookahead Sets
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Computing Lookahead Sets I

(see Waite/Goos: *Compiler Construction*, p. 164f)

Lemma 7.2 (Computation of fi/fo)

The sets $\mathrm{fi}(\alpha) \subseteq \Sigma_\varepsilon$ *(for* $\alpha \in X^*$*) and* $\mathrm{fo}(A) \subseteq \Sigma_\varepsilon$ *(for* $A \in N$*) are the least sets such that:*

1. $\mathrm{fi}(Y)$ *for* $Y \in X$*:*
   - $Y \in \Sigma \implies \mathrm{fi}(Y) = \{Y\}$
   - $Y \to A_1 \ldots A_k Z\alpha \in P, k \in \mathbb{N}, Z \in X, \varepsilon \in \mathrm{fi}(A_1) \cap \ldots \cap \mathrm{fi}(A_k),$
     $a \in \mathrm{fi}(Z) \implies a \in \mathrm{fi}(Y)$
   - $Y \to A_1 \ldots A_k \in P, k \in \mathbb{N}, \varepsilon \in \mathrm{fi}(A_1) \cap \ldots \cap \mathrm{fi}(A_k) \implies \varepsilon \in \mathrm{fi}(Y)$

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   - $Y \to A_1 \ldots A_k \in P, k \in \mathbb{N}, \varepsilon \in \mathrm{fi}(A_1) \cap \ldots \cap \mathrm{fi}(A_k) \implies \varepsilon \in \mathrm{fi}(Y)$
2. $\mathrm{fi}(Y_1 \ldots Y_n)$ *for* $n \in \mathbb{N}, Y_i \in X$*:*
   - $\varepsilon \in \mathrm{fi}(Y_1) \cap \ldots \cap \mathrm{fi}(Y_{k-1}), a \in \mathrm{fi}(Y_k), k \in [n] \implies a \in \mathrm{fi}(Y_1 \ldots Y_n)$
   - $\varepsilon \in \mathrm{fi}(Y_1) \cap \ldots \cap \mathrm{fi}(Y_n) \implies \varepsilon \in \mathrm{fi}(Y_1 \ldots Y_n)$

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   - $\varepsilon \in \mathrm{fi}(Y_1) \cap \ldots \cap \mathrm{fi}(Y_n) \implies \varepsilon \in \mathrm{fi}(Y_1 \ldots Y_n)$
3. $\mathrm{fo}(A)$ *for* $A \in N$*:*
   - $\varepsilon \in \mathrm{fo}(S)$
   - $A \to \alpha B\beta \in P, a \in \mathrm{fi}(\beta) \implies a \in \mathrm{fo}(B)$
   - $A \to \alpha B\beta \in P, \varepsilon \in \mathrm{fi}(\beta), x \in \mathrm{fo}(A) \implies x \in \mathrm{fo}(B)$

Computing Lookahead Sets II

Corollary 7.3

1. $A \to a\alpha \in P \implies a \in \mathrm{fi}(A)$
2. $A \to B\alpha \in P, a \in \mathrm{fi}(B) \implies a \in \mathrm{fi}(A)$
3. $A \to \varepsilon \in P \implies \varepsilon \in \mathrm{fi}(A)$
4. $\mathrm{fi}(\varepsilon) = \{\varepsilon\}$
5. $a \in \mathrm{fi}(A) \implies a \in \mathrm{fi}(A\alpha)$
6. $A \to \alpha B \in P, x \in \mathrm{fo}(A) \implies x \in \mathrm{fo}(B)$

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6. $A \to \alpha B \in P, x \in \text{fo}(A) \implies x \in \text{fo}(B)$

Example 7.4

Grammar for arithmetic expressions (cf. Example 5.10):

$$
\begin{aligned}
G_{AE}: \quad E &\to E\texttt{+}T \mid T \\
T &\to T\texttt{*}F \mid F \\
F &\to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b}
\end{aligned}
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- $F \to \texttt{a} \in P \implies \texttt{a} \in \mathrm{fi}(F)$

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- $F \to \mathtt{a} \in P \implies \mathtt{a} \in \mathrm{fi}(F)$
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- $\mathtt{a} \in \mathrm{fi}(T)$

  $\implies \mathrm{la}(T \to T{*}F) = \mathrm{fi}(T{*}F \cdot \mathrm{fo}(T)) \ni \mathtt{a}$

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- $F \to \texttt{a} \in P \implies \texttt{a} \in \mathrm{fi}(F)$
- $T \to F \in P, \texttt{a} \in \mathrm{fi}(F) \implies \texttt{a} \in \mathrm{fi}(T)$
- $\texttt{a} \in \mathrm{fi}(T)$
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- $\implies \texttt{a} \in \mathrm{la}(T \to T\texttt{*}F) \cap \mathrm{la}(T \to F) \neq \emptyset$
- $\implies G_{AE} \notin LL(1)$

Fixing the Problem

(general methods later)

Example 7.5 (continuing Example 7.4)

Restructuring (such that $L(G'_{AE}) = L(G_{AE})$):

$$
\begin{array}{llll}
G'_{AE}: & E & \to & TE' \\
 & E' & \to & +TE' \mid \varepsilon \\
 & T & \to & FT' \\
 & T' & \to & *FT' \mid \varepsilon \\
 & F & \to & (E) \mid \mathtt{a} \mid \mathtt{b}
\end{array}
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\end{aligned}
$$

$A \in N$	$\mathrm{fi}(A)$
E	$\{(, a, b\}$
E'	$\{+, \varepsilon\}$
T	$\{(, a, b\}$
T'	$\{*, \varepsilon\}$
F	$\{(, a, b\}$

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$A \in N$	$\text{fi}(A)$	$\text{fo}(A)$
E	$\{(, \mathtt{a}, \mathtt{b}\}$	$\{\varepsilon,)\}$
E'	$\{+, \varepsilon\}$	$\{\varepsilon,)\}$
T	$\{(, \mathtt{a}, \mathtt{b}\}$	$\{+, \varepsilon,)\}$
T'	$\{*, \varepsilon\}$	$\{+, \varepsilon,)\}$
F	$\{(, \mathtt{a}, \mathtt{b}\}$	$\{*, +, \varepsilon,)\}$

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$A \to \beta \in P$	$\mathrm{la}(A \to \beta) = \mathrm{fi}(\beta \cdot \mathrm{fo}(A))$
$E \to TE'$	$\{(, \mathtt{a}, \mathtt{b}\}$
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$E' \to \varepsilon$	$\{\varepsilon,)\}$
$T \to FT'$	$\{(, \mathtt{a}, \mathtt{b}\}$
$T' \to *FT'$	$\{*\}$
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$F \to (E)$	$\{(\}$
$F \to \mathtt{a}$	$\{\mathtt{a}\}$
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$\Longrightarrow G'_{AE} \in LL(1)$

Outline

Deterministic Top-Down Parsing

Approach: given $G \in CFG_\Sigma$,

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1. Verify that $G \in LL(1)$ by computing the lookahead sets and checking alternatives for disjointness
2. Start with nondeterministic top-down parsing automaton $\mathrm{NTA}(G)$
3. Use 1-symbol lookahead to control the choice of expanding productions:
   - $(aw, A\alpha, z) \vdash (aw, \beta\alpha, zi)$
     if $\pi_i = A \to \beta$ and $a \in \mathrm{la}(\pi_i)$
   - $(\varepsilon, A\alpha, z) \vdash (\varepsilon, \beta\alpha, zi)$
     if $\pi_i = A \to \beta$ and $\varepsilon \in \mathrm{la}(\pi_i)$
   - [matching steps as before: $(aw, a\alpha, z) \vdash (w, \alpha, z)$]

$\Longrightarrow$ deterministic top-down parsing automaton $\mathrm{DTA}(G)$

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   - $(\varepsilon, A\alpha, z) \vdash (\varepsilon, \beta\alpha, zi)$
     if $\pi_i = A \to \beta$ and $\varepsilon \in \mathrm{la}(\pi_i)$
   - [matching steps as before: $(aw, a\alpha, z) \vdash (w, \alpha, z)$]

$\Longrightarrow$ deterministic top-down parsing automaton $\mathrm{DTA}(G)$

Remarks:

- $\mathrm{DTA}(G)$ is actually not a pushdown automaton (a is read but not consumed). But: can be simulated using the finite control.
- Advantage of using lookahead is twofold:
  - Removal of nondeterminism
  - Earlier detection of syntax errors
    (in configurations $(aw, A\alpha, z)$ where $a \notin \bigcup_{A\to\beta\in P} \mathrm{la}(A \to \beta)$)

The Deterministic Top-Down Automaton I

Definition 7.6 (Deterministic top-down parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LL(1)$. The deterministic top-down parsing automaton of G, $\mathrm{DTA}(G)$, is defined by the following components.

- Input alphabet Σ, pushdown alphabet X, output alphabet $[p]$
- Configurations $\Sigma^* \times X^* \times [p]^*$, initial configuration (w, S, ε), final configurations $\{\varepsilon\} \times \{\varepsilon\} \times [p]^*$ (as $\mathrm{NTA}(G)$)
- Action function

$$\begin{aligned} \mathrm{act} : \Sigma_\varepsilon \times X_\varepsilon &\to \{(\alpha, i) \mid \pi_i = A \to \alpha\} \cup \{\mathsf{pop}, \mathsf{accept}, \mathsf{error}\} \\ \text{with } \mathrm{act}(x, A) &:= (\alpha, i) \text{ if } \pi_i = A \to \alpha \text{ and } x \in \mathrm{la}(\pi_i) \\ \mathrm{act}(a, a) &:= \mathsf{pop} \\ \mathrm{act}(\varepsilon, \varepsilon) &:= \mathsf{accept} \\ \mathrm{act}(x, y) &:= \mathsf{error} \qquad \text{otherwise} \end{aligned}$$

- Transitions for $x \in \Sigma_\varepsilon$, $w \in \Sigma^*$, $Y \in X$, $\beta \in X^*$, and $z \in [p]^*$:

$$(xw, Y\beta, z) \vdash \begin{cases} (xw, \alpha\beta, zi) & \text{if } \mathrm{act}(x, Y) = (\alpha, i) \\ (w, \beta, z) & \text{if } \mathrm{act}(x, Y) = \mathsf{pop} \end{cases}$$

Example 7.7 (cf. Example 7.5)

$$
\begin{array}{llll}
G'_{AE}: & E & \rightarrow TE' & (1) \\
& E' & \rightarrow +TE' \mid \varepsilon & (2,3) \\
& T & \rightarrow FT' & (4) \\
& T' & \rightarrow *FT' \mid \varepsilon & (5,6) \\
& F & \rightarrow (E) \mid \mathtt{a} \mid \mathtt{b} & (7,8,9)
\end{array}
$$

$A \rightarrow \beta \in P$	$\text{la}(A \rightarrow \beta)$
$E \rightarrow TE'$	$\{(, \mathtt{a}, \mathtt{b}\}$
$E' \rightarrow +TE'$	$\{+\}$
$E' \rightarrow \varepsilon$	$\{\varepsilon,)\}$
$T \rightarrow FT'$	$\{(, \mathtt{a}, \mathtt{b}\}$
$T' \rightarrow *FT'$	$\{*\}$
$T' \rightarrow \varepsilon$	$\{+, \varepsilon,)\}$
$F \rightarrow (E)$	$\{(\}$
$F \rightarrow \mathtt{a}$	$\{\mathtt{a}\}$
$F \rightarrow \mathtt{b}$	$\{\mathtt{b}\}$

Example 7.7 (cf. Example 7.5)

$$
\begin{array}{lrcll}
G'_{AE}: & E & \to & TE' & (1) \\
 & E' & \to & +TE' \mid \varepsilon & (2,3) \\
 & T & \to & FT' & (4) \\
 & T' & \to & *FT' \mid \varepsilon & (5,6) \\
 & F & \to & (E) \mid \mathtt{a} \mid \mathtt{b} & (7,8,9)
\end{array}
$$

$A \to \beta \in P$	$\mathrm{la}(A \to \beta)$
$E \to TE'$	$\{(, \mathtt{a}, \mathtt{b}\}$
$E' \to +TE'$	$\{+\}$
$E' \to \varepsilon$	$\{\varepsilon,)\}$
$T \to FT'$	$\{(, \mathtt{a}, \mathtt{b}\}$
$T' \to *FT'$	$\{*\}$
$T' \to \varepsilon$	$\{+, \varepsilon,)\}$
$F \to (E)$	$\{(\}$
$F \to \mathtt{a}$	$\{\mathtt{a}\}$
$F \to \mathtt{b}$	$\{\mathtt{b}\}$

$\mathrm{act}: \Sigma_\varepsilon \times X_\varepsilon \to \{(\alpha, i) \mid \pi_i = A \to \alpha\} \cup \{\mathsf{pop}, \mathsf{accept}, \mathsf{error}\}$ (empty = error)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\text{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\text{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$(\mathtt{(}E\mathtt{)}, 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(\mathtt{*}FT', 5)$						pop		
+		$(\mathtt{+}TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$(\mathtt{(a)*b}, E \qquad , \varepsilon \qquad)$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of (a)*b:

$$
\begin{array}{ll}
 & (\mathtt{(a)*b},\ E \quad , \varepsilon \quad) \\
\vdash & (\mathtt{(a)*b},\ TE' \quad , 1 \quad)
\end{array}
$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		(a, 8)	pop						
b	$(TE', 1)$		$(FT', 4)$		(b, 9)		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of (a)*b:

$$
\begin{array}{ll}
 & ((a)*b,\ E,\ \varepsilon) \\
\vdash & ((a)*b,\ TE',\ 1) \\
\vdash & ((a)*b,\ FT'E',\ 14)
\end{array}
$$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$({*}FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{array}{ll}
 & (\mathtt{(a)*b},\ E \qquad , \varepsilon \quad) \\
\vdash & (\mathtt{(a)*b},\ TE' \qquad , 1 \quad) \\
\vdash & (\mathtt{(a)*b},\ FT'E' \qquad , 14 \quad) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E' \qquad , 147 \quad)
\end{array}
$$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$(\mathtt{(a)*b},\ E,\ \varepsilon)$$
$$\vdash (\mathtt{(a)*b},\ TE',\ 1)$$
$$\vdash (\mathtt{(a)*b},\ FT'E',\ 14)$$
$$\vdash (\mathtt{(a)*b},\ (E)T'E',\ 147)$$
$$\vdash (\mathtt{a)*b},\ E)T'E',\ 147)$$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of (a)*b:

$$
\begin{array}{ll}
 & (\mathtt{(a)*b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE',\ 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E',\ 14) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ TE')T'E',\ 1471)
\end{array}
$$

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Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{array}{ll}
& (\mathtt{(a)*b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE',\ 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E',\ 14) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ TE')T'E',\ 1471) \\
\vdash & (\mathtt{a)*b},\ FT'E')T'E',\ 14714)
\end{array}
$$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of (a)*b:

$$
\begin{array}{ll}
 & (\mathtt{(a)*b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE',\ 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E',\ 14) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ TE')T'E',\ 1471) \\
\vdash & (\mathtt{a)*b},\ FT'E')T'E',\ 14714) \\
\vdash & (\mathtt{a)*b},\ \mathtt{a}T'E')T'E',\ 147148)
\end{array}
$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		(a, 8)	pop						
b	$(TE', 1)$		$(FT', 4)$		(b, 9)		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of (a)*b:

$$
\begin{array}{ll}
& ((a)*b,\ E,\ \varepsilon) \\
\vdash & ((a)*b,\ TE',\ 1) \\
\vdash & ((a)*b,\ FT'E',\ 14) \\
\vdash & ((a)*b,\ (E)T'E',\ 147) \\
\vdash & (a)*b,\ E)T'E',\ 147) \\
\vdash & (a)*b,\ TE')T'E',\ 1471) \\
\vdash & (a)*b,\ FT'E')T'E',\ 14714) \\
\vdash & (a)*b,\ aT'E')T'E',\ 147148) \\
\vdash & ()*b,\ T'E')T'E',\ 147148)
\end{array}
$$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{aligned}
& (\mathtt{(a)*b},\ E, \varepsilon) \\
\vdash\ & (\mathtt{(a)*b},\ TE', 1) \\
\vdash\ & (\mathtt{(a)*b},\ FT'E', 14) \\
\vdash\ & (\mathtt{(a)*b},\ (E)T'E', 147) \\
\vdash\ & (\mathtt{a)*b},\ E)T'E', 147) \\
\vdash\ & (\mathtt{a)*b},\ TE')T'E', 1471) \\
\vdash\ & (\mathtt{a)*b},\ FT'E')T'E', 14714) \\
\vdash\ & (\mathtt{a)*b},\ \mathtt{a}T'E')T'E', 147148) \\
\vdash\ & (\mathtt{)*b},\ T'E')T'E', 147148) \\
\vdash\ & (\mathtt{)*b},\ E')T'E', 1471486)
\end{aligned}
$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{array}{lll}
 & (\mathtt{(a)*b},\ E & ,\ \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE' & ,\ 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E' & ,\ 14) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E' & ,\ 147) \\
\vdash & (\mathtt{a)*b},\ E)T'E' & ,\ 147) \\
\vdash & (\mathtt{a)*b},\ TE')T'E' & ,\ 1471) \\
\vdash & (\mathtt{a)*b},\ FT'E')T'E' & ,\ 14714) \\
\vdash & (\mathtt{a)*b},\ \mathtt{a}T'E')T'E' & ,\ 147148) \\
\vdash & (\mathtt{)*b},\ T'E')T'E' & ,\ 147148) \\
\vdash & (\mathtt{)*b},\ E')T'E' & ,\ 1471486) \\
\vdash & (\mathtt{)*b},\ )T'E' & ,\ 14714863)
\end{array}
$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{array}{ll}
 & (\mathtt{(a)*b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE',\ 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E',\ 14) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ TE')T'E',\ 1471) \\
\vdash & (\mathtt{a)*b},\ FT'E')T'E',\ 14714) \\
\vdash & (\mathtt{a)*b},\ \mathtt{a}T'E')T'E',\ 147148) \\
\vdash & (\mathtt{)*b},\ T'E')T'E',\ 147148) \\
\vdash & (\mathtt{)*b},\ E')T'E',\ 1471486) \\
\vdash & (\mathtt{)*b},\ )T'E',\ 14714863) \\
\vdash & (\mathtt{*b},\ T'E',\ 14714863)
\end{array}
$$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$(\mathtt{(a)*b},\ E,\ \varepsilon)$$
$$\vdash (\mathtt{(a)*b},\ TE',\ 1)$$
$$\vdash (\mathtt{(a)*b},\ FT'E',\ 14)$$
$$\vdash (\mathtt{(a)*b},\ (E)T'E',\ 147)$$
$$\vdash (\mathtt{a)*b},\ E)T'E',\ 147)$$
$$\vdash (\mathtt{a)*b},\ TE')T'E',\ 1471)$$
$$\vdash (\mathtt{a)*b},\ FT'E')T'E',\ 14714)$$
$$\vdash (\mathtt{a)*b},\ \mathtt{a}T'E')T'E',\ 147148)$$
$$\vdash (\mathtt{)*b},\ T'E')T'E',\ 147148)$$
$$\vdash (\mathtt{)*b},\ E')T'E',\ 1471486)$$
$$\vdash (\mathtt{)*b},\ )T'E',\ 14714863)$$
$$\vdash (\mathtt{*b},\ T'E',\ 14714863)$$
$$\vdash (\mathtt{*b},\ *FT'E',\ 147148635)$$

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Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	$*$	$+$	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
$*$				$(*FT', 5)$						pop		
$+$		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{array}{ll}
& (\mathtt{(a)*b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE',\ 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E',\ 14) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ TE')T'E',\ 1471) \\
\vdash & (\mathtt{a)*b},\ FT'E')T'E',\ 14714) \\
\vdash & (\mathtt{a)*b},\ \mathtt{a}T'E')T'E',\ 147148) \\
\vdash & (\mathtt{)*b},\ T'E')T'E',\ 147148) \\
\vdash & (\mathtt{)*b},\ E')T'E',\ 1471486) \\
\vdash & (\mathtt{)*b},\ )T'E',\ 14714863) \\
\vdash & (\mathtt{*b},\ T'E',\ 14714863) \\
\vdash & (\mathtt{*b},\ *FT'E',\ 147148635) \\
\vdash & (\mathtt{b},\ FT'E',\ 147148635)
\end{array}
$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$(\mathtt{(}E\mathtt{)}, 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of (a)*b:

$$
\begin{array}{ll}
& (\mathtt{(a)*b},\ E, \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE', 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E', 14) \\
\vdash & (\mathtt{(a)*b},\ \mathtt{(}E\mathtt{)}T'E', 147) \\
\vdash & (\mathtt{a)*b},\ E\mathtt{)}T'E', 147) \\
\vdash & (\mathtt{a)*b},\ TE'\mathtt{)}T'E', 1471) \\
\vdash & (\mathtt{a)*b},\ FT'E'\mathtt{)}T'E', 14714) \\
\vdash & (\mathtt{a)*b},\ \mathtt{a}T'E'\mathtt{)}T'E', 147148) \\
\vdash & (\mathtt{)*b},\ T'E'\mathtt{)}T'E', 147148) \\
\vdash & (\mathtt{)*b},\ E'\mathtt{)}T'E', 1471486) \\
\vdash & (\mathtt{)*b},\ \mathtt{)}T'E', 14714863) \\
\vdash & (\mathtt{*b},\ T'E', 14714863) \\
\vdash & (\mathtt{*b},\ *FT'E', 147148635) \\
\vdash & (\mathtt{b},\ FT'E', 147148635) \\
\vdash & (\mathtt{b},\ \mathtt{b}T'E', 1471486359)
\end{array}
$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	$*$	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
$*$				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{array}{ll}
& (\mathtt{(a)*b},\ E,\ \varepsilon) \\
\vdash & (\mathtt{(a)*b},\ TE',\ 1) \\
\vdash & (\mathtt{(a)*b},\ FT'E',\ 14) \\
\vdash & (\mathtt{(a)*b},\ (E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ E)T'E',\ 147) \\
\vdash & (\mathtt{a)*b},\ TE')T'E',\ 1471) \\
\vdash & (\mathtt{a)*b},\ FT'E')T'E',\ 14714) \\
\vdash & (\mathtt{a)*b},\ \mathtt{a}T'E')T'E',\ 147148) \\
\vdash & (\mathtt{)*b},\ T'E')T'E',\ 147148) \\
\vdash & (\mathtt{)*b},\ E')T'E',\ 1471486) \\
\vdash & (\mathtt{)*b},\ )T'E',\ 14714863) \\
\vdash & (\mathtt{*b},\ T'E',\ 14714863) \\
\vdash & (\mathtt{*b},\ *FT'E',\ 147148635) \\
\vdash & (\mathtt{b},\ FT'E',\ 147148635) \\
\vdash & (\mathtt{b},\ \mathtt{b}T'E',\ 1471486359) \\
\vdash & (\varepsilon,\ T'E',\ 1471486359)
\end{array}
$$

The Deterministic Top-Down Automaton III

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$$
\begin{array}{ll}
(\mathtt{(a)*b},\ E,\ \varepsilon) & \vdash (\mathtt{)*b},\ E')T'E',\ 1471486) \\
\vdash (\mathtt{(a)*b},\ TE',\ 1) & \vdash (\mathtt{)*b},\ )T'E',\ 14714863) \\
\vdash (\mathtt{(a)*b},\ FT'E',\ 14) & \vdash (\mathtt{*b},\ T'E',\ 14714863) \\
\vdash (\mathtt{(a)*b},\ (E)T'E',\ 147) & \vdash (\mathtt{*b},\ *FT'E',\ 147148635) \\
\vdash (\mathtt{a)*b},\ E)T'E',\ 147) & \vdash (\mathtt{b},\ FT'E',\ 147148635) \\
\vdash (\mathtt{a)*b},\ TE')T'E',\ 1471) & \vdash (\mathtt{b},\ \mathtt{b}T'E',\ 1471486359) \\
\vdash (\mathtt{a)*b},\ FT'E')T'E',\ 14714) & \vdash (\varepsilon,\ T'E',\ 1471486359) \\
\vdash (\mathtt{a)*b},\ \mathtt{a}T'E')T'E',\ 147148) & \vdash (\varepsilon,\ E',\ 14714863596) \\
\vdash (\mathtt{)*b},\ T'E')T'E',\ 147148) &
\end{array}
$$

Example 7.7 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE',1)$		$(FT',4)$		$(\mathtt{a},8)$	pop						
b	$(TE',1)$		$(FT',4)$		$(\mathtt{b},9)$		pop					
(	$(TE',1)$		$(FT',4)$		$((E),7)$			pop				
)		$(\varepsilon,3)$		$(\varepsilon,6)$					pop			
*				$(*FT',5)$						pop		
+		$(+TE',2)$		$(\varepsilon,6)$							pop	
ε		$(\varepsilon,3)$		$(\varepsilon,6)$								accept

Leftmost analysis of `(a)*b`:

$(\,(\mathtt{a})*\mathtt{b},\ E,\ \varepsilon\,)$
$\vdash (\,(\mathtt{a})*\mathtt{b},\ TE',\ 1\,)$
$\vdash (\,(\mathtt{a})*\mathtt{b},\ FT'E',\ 14\,)$
$\vdash (\,(\mathtt{a})*\mathtt{b},\ (E)T'E',\ 147\,)$
$\vdash (\,\mathtt{a})*\mathtt{b},\ E)T'E',\ 147\,)$
$\vdash (\,\mathtt{a})*\mathtt{b},\ TE')T'E',\ 1471\,)$
$\vdash (\,\mathtt{a})*\mathtt{b},\ FT'E')T'E',\ 14714\,)$
$\vdash (\,\mathtt{a})*\mathtt{b},\ \mathtt{a}T'E')T'E',\ 147148\,)$
$\vdash (\,)*\mathtt{b},\ T'E')T'E',\ 147148\,)$
$\vdash (\,)*\mathtt{b},\ E')T'E',\ 1471486\,)$
$\vdash (\,)*\mathtt{b},\ )T'E',\ 14714863\,)$
$\vdash (\,*\mathtt{b},\ T'E',\ 14714863\,)$
$\vdash (\,*\mathtt{b},\ *FT'E',\ 147148635\,)$
$\vdash (\,\mathtt{b},\ FT'E',\ 147148635\,)$
$\vdash (\,\mathtt{b},\ \mathtt{b}T'E',\ 1471486359\,)$
$\vdash (\,\varepsilon,\ T'E',\ 1471486359\,)$
$\vdash (\,\varepsilon,\ E',\ 14714863596\,)$
$\vdash (\,\varepsilon,\ \varepsilon,\ 147148635963\,)$