Compiler Construction

Lecture 2: Lexical Analysis I (Introduction)

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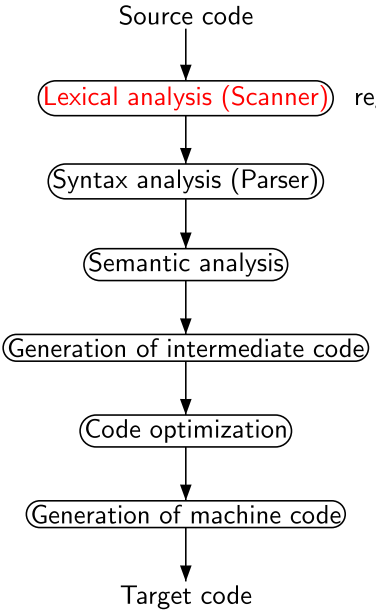
Summer Semester 2014

Exercise Class

Shift Fri 08:15–09:45 (AH 2) $\longrightarrow$ Fri 10:00–11:30 (AH 5)?

Conceptual Structure of a Compiler



`x1␣:=␣y2␣+␣1;`

regular expressions/finite automata

$(\text{id}, \texttt{x1})(\text{gets},)(\text{id}, \texttt{y2})(\text{plus},)(\text{int}, 1)$

Outline

Lexical Structures

From Merriam-Webster's Online Dictionary

Lexical: of or relating to words or the vocabulary of a language as distinguished from its grammar and construction

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- **Starting point:** source program P as a character sequence
  - Ω (finite) character set (e.g., ASCII, ISO Latin-1, Unicode, ...)
  - $a, b, c, \ldots \in \Omega$ characters (= lexical atoms)
  - $P \in \Omega^*$ source program
    (of course, not every $w \in \Omega^*$ is a valid program)

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  - $a, b, c, \ldots \in \Omega$ characters (= lexical atoms)
  - $P \in \Omega^*$ source program
    (of course, not every $w \in \Omega^*$ is a valid program)
- P exhibits lexical structures:
  - natural language for keywords, identifiers, ...
  - mathematical notation for numbers, formulae, ...
    (e.g., $x^2 \rightsquigarrow$ `x**2`)
  - spaces, linebreaks, indentation
  - comments and compiler directives (pragmas)
- Translation of P follows its hierarchical structure (later)

Observations

1. Syntactic atoms (called symbols) are represented as sequences of input characters, called lexemes

First goal of lexical analysis

Decomposition of program text into a sequence of lexemes

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Decomposition of program text into a sequence of lexemes

2. Differences between similar lexemes are (mostly) irrelevant (e.g., identifiers do not need to be distinguished)
   - lexemes grouped into symbol classes (e.g., identifiers, numbers, ...)
   - symbol classes abstractly represented by tokens
   - symbols identified by additional attributes (e.g., identifier names, numerical values, ...; required for semantic analysis and code generation)

$\Longrightarrow$ symbol = (token, attribute)

Second goal of lexical analysis

Transformation of a sequence of lexemes into a sequence of symbols

Lexical Analysis

Definition 2.1

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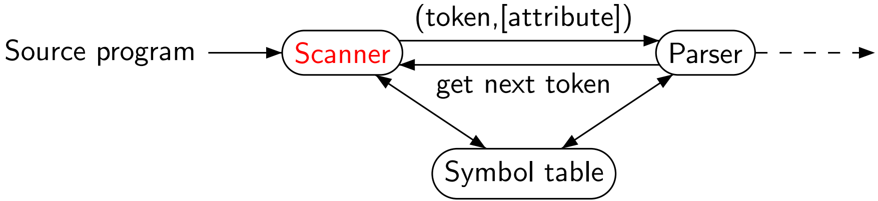
The corresponding program is called a scanner (or lexer):



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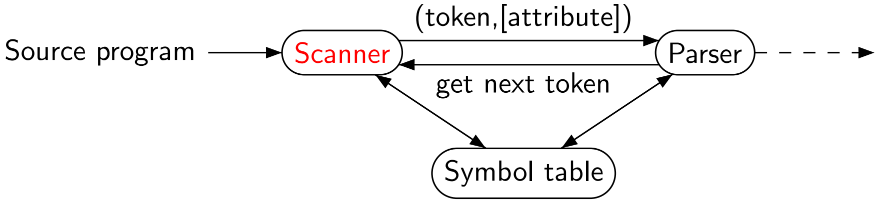

Example: `...␣x1␣:=y2+␣1␣;␣...`

$$\Downarrow$$

$$\ldots(\mathrm{id}, p_1)(\mathrm{gets},)(\mathrm{id}, p_2)(\mathrm{plus},)(\mathrm{int}, 1)(\mathrm{sem},)\ldots$$

Important Symbol Classes

Identifiers:

- for naming variables, constants, types, procedures, classes, ...
- usually a sequence of letters and digits (and possibly special symbols), starting with a letter
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Special symbols:
- one special character, e.g., `+`, `*`, `<`, `(`, `;`, ...
- ... or two or more special characters, e.g., `:=`, `**`, `<=`, ...
- each makes up a symbol class (plus, gets, ...)
- ... or several combined into one class (arithOp)

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White spaces:
- blanks, tabs, linebreaks, ...
- generally for separating symbols (exception: FORTRAN)
- usually not represented by token (but just removed)

Representation of symbols: symbol = (token, attribute)

Token: (binary) denotation of symbol class (id, gets, plus, ...)

Attribute: additional information required in later compilation phases

- reference to symbol table,
- value of numeral,
- concrete arithmetic/relational/Boolean operator, ...
- usually unused for singleton symbol classes

Specification and Implementation of Scanners

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Observation: symbol classes are regular sets

$\Longrightarrow$
- specification by regular expressions
- recognition by finite automata
- enables automatic generation of scanners (`[f]lex`)

Outline

Regular Expressions I

Definition 2.2 (Syntax of regular expressions)

Given some alphabet Ω, the set of regular expressions over Ω, RE_Ω, is the least set with

- $\emptyset \in RE_\Omega$,
- $\Omega \subseteq RE_\Omega$, and
- whenever $\alpha, \beta \in RE_\Omega$, also $\alpha \mid \beta, \alpha \cdot \beta, \alpha^* \in RE_\Omega$.

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Remarks:

- abbreviations: $\alpha^+ := \alpha \cdot \alpha^*$, $\varepsilon := \emptyset^*$
- $\alpha \cdot \beta$ often written as $\alpha\beta$
- Binding priority: $^* > \cdot > \mid$ (i.e., $a \mid b \cdot c^* := a \mid (b \cdot (c^*))$)

Regular expressions specify regular languages:

Definition 2.3 (Semantics of regular expressions)

The semantics of a regular expression is defined by the mapping

$$[\![.]\!] : RE_\Omega \to 2^{\Omega^*} \text{ where}$$

$$\begin{aligned}
[\![\emptyset]\!] &:= \emptyset \\
[\![a]\!] &:= \{a\} \\
[\![\alpha \mid \beta]\!] &:= [\![\alpha]\!] \cup [\![\beta]\!] \\
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Remarks: for formal languages $L, M \subseteq \Omega^*$, we have

- $L \cdot M := \{vw \mid v \in L, w \in M\}$
- $L^* := \bigcup_{n=0}^{\infty} L^n$ where $L^0 := \{\varepsilon\}$ and $L^{n+1} := L \cdot L^n$
  (thus $L^* = \{w_1 w_2 \ldots w_n \mid n \in \mathbb{N}, \forall 1 \leq i \leq n : w_i \in L\}$ and $\varepsilon \in L^*$)
- $[\![\emptyset^*]\!] = [\![\emptyset]\!]^* = \emptyset^* = \{\varepsilon\}$

Regular Expressions III

Example 2.4

1. A keyword: `begin`

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$$\texttt{(a} \mid \ldots \mid \texttt{z} \mid \texttt{A} \mid \ldots \mid \texttt{Z)(a} \mid \ldots \mid \texttt{z} \mid \texttt{A} \mid \ldots \mid \texttt{Z} \mid \texttt{0} \mid \ldots \mid \texttt{9} \mid \texttt{\$} \mid \texttt{\_} \mid \ldots\texttt{)}^*$$

Regular Expressions III

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Regular Expressions III

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3. (Unsigned) Integer numbers: $(\mathtt{0} \mid \ldots \mid \mathtt{9})^+$
4. (Unsigned) Fixed-point numbers:

$$((\mathtt{0} \mid \ldots \mid \mathtt{9})^+ \mathtt{.} (\mathtt{0} \mid \ldots \mid \mathtt{9})^*) \mid ((\mathtt{0} \mid \ldots \mid \mathtt{9})^* \mathtt{.} (\mathtt{0} \mid \ldots \mid \mathtt{9})^+)$$

Outline

The Simple Matching Problem I

Problem 2.5 (Simple matching problem)

Given $\alpha \in RE_\Omega$ *and* $w \in \Omega^*$, *decide whether* $w \in [\![\alpha]\!]$ *or not.*

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This problem can be solved using the following concept:

Definition 2.6 (Finite automaton)

A nondeterministic finite automaton (NFA) is of the form $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle$ where

- Q is a finite set of states
- Ω denotes the input alphabet
- $\delta : Q \times \Omega_\varepsilon \to 2^Q$ is the transition function where $\Omega_\varepsilon := \Omega \cup \{\varepsilon\}$ (notation: $q \xrightarrow{x} q'$ for $q' \in \delta(q, x)$)
- $q_0 \in Q$ is the initial state
- $F \subseteq Q$ is the set of final states

The set of all NFA over Ω is denoted by NFA_Ω.

If $\delta(q, \varepsilon) = \emptyset$ and $|\delta(q, a)| = 1$ for every $q \in Q$ and $a \in \Omega$ (i.e., $\delta : Q \times \Omega \to Q$), then $\mathfrak{A}$ is called deterministic (DFA). Notation: DFA_Ω

The Simple Matching Problem II

Definition 2.7 (Acceptance condition)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F\rangle \in NFA_\Omega$ and $w = a_1 \dots a_n \in \Omega^*$.

- A w-labeled $\mathfrak{A}$-run from q_1 to q_2 is a sequence of transitions
$$q_1 \xrightarrow{\varepsilon}{}^* \xrightarrow{a_1} \xrightarrow{\varepsilon}{}^* \xrightarrow{a_2} \xrightarrow{\varepsilon}{}^* \dots \xrightarrow{\varepsilon}{}^* \xrightarrow{a_n} \xrightarrow{\varepsilon}{}^* q_2$$
- $\mathfrak{A}$ accepts w if there is a w-labeled $\mathfrak{A}$-run from q_0 to some $q \in F$
- The language recognized by $\mathfrak{A}$ is
$$L(\mathfrak{A}) := \{w \in \Omega^* \mid \mathfrak{A} \text{ accepts } w\}$$
- A language $L \subseteq \Omega^*$ is called NFA-recognizable if there exists a NFA $\mathfrak{A}$ such that $L(\mathfrak{A}) = L$

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Example 2.8

NFA for $a^*b \mid a^*$ (on the board)

The Simple Matching Problem III

Remarks:

- NFA as specified in Definition 2.6 are sometimes called NFA with ε-transitions (ε-NFA).

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- For $\mathfrak{A} \in DFA_\Omega$, the acceptance condition yields $\delta^* : Q \times \Omega^* \to Q$ with $\delta^*(q, \varepsilon) = q$ and $\delta^*(q, aw) = \delta^*(\delta(q, a), w)$, and
$$L(\mathfrak{A}) = \{w \in \Omega^* \mid \delta^*(q_0, w) \in F\}.$$

The DFA Method I

Known from *Formal Systems, Automata and Processes*:

Algorithm 2.9 (DFA method)

Input: *regular expression* $\alpha \in RE_\Omega$, *input string* $w \in \Omega^*$

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Procedure:
1. *using Kleene's Theorem, construct* $\mathfrak{A}_\alpha \in NFA_\Omega$ *such that* $L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$
2. *apply powerset construction to obtain* $\mathfrak{A}'_\alpha = \langle Q', \Omega, \delta', q'_0, F' \rangle \in DFA_\Omega$ *with* $L(\mathfrak{A}'_\alpha) = L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$
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Output: *"yes" or "no"*

The DFA Method II

The powerset construction involves the following concept:

Definition 2.10 (ε-closure)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F\rangle \in NFA_\Omega$. The ε-closure $\varepsilon(T) \subseteq Q$ of a subset $T \subseteq Q$ is defined by

- $T \subseteq \varepsilon(T)$ and
- if $q \in \varepsilon(T)$, then $\delta(q, \varepsilon) \subseteq \varepsilon(T)$

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Example 2.11

1. Kleene's Theorem (on the board)
2. Powerset construction (on the board)

Outline

Complexity of DFA Method

1. in construction phase:
   - Kleene method: time and space $\mathcal{O}(|\alpha|)$
     (where $|\alpha| :=$ length of α)
   - Powerset construction: time and space $\mathcal{O}(2^{|\mathfrak{A}_\alpha|}) = \mathcal{O}(2^{|\alpha|})$
     (where $|\mathfrak{A}_\alpha| := \#$ of states of $\mathfrak{A}_\alpha$)

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$\Longrightarrow$ nice runtime behavior but memory requirements very high (and exponential time in construction phase)

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Procedure: $T := \varepsilon(\{q_0\})$;
while $w \neq \varepsilon$ **do**
$a :=$ **head**(w);
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Output: *if* $T \cap F \neq \emptyset$ *then "yes" else "no"*

Complexity Analysis

For NFA method at runtime:

- Space: $\mathcal{O}(|\alpha|)$ (for storing NFA and T)
- Time: $\mathcal{O}(|\alpha| \cdot |w|)$
  (in the loop's body, $|T|$ states need to be considered)

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In practice:

- Exponential blowup of DFA method usually does not occur in "real" applications ($\Longrightarrow$ used in `[f]lex`)
- Improvement of NFA method: caching of transitions $\delta'(T, a)$
  $\Longrightarrow$ combination of both methods