# Compiler Construction

## Lecture 10: Syntax Analysis VI ($LR(0)$ and $SLR(1)$ Parsing)

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

noll@cs.rwth-aachen.de

http://moves.rwth-aachen.de/teaching/ss-14/cc14/

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# Outline

1. Recap: $LR(0)$ Grammars
2. The $LR(0)$ Parsing Automaton
3. $SLR(1)$ Parsing
4. $LR(1)$ Parsing

# $LR(0)$ Grammars

The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, without any lookahead.

### Corollary ($LR(0)$ grammar)

*$G \in CFG_\Sigma$ has the $LR(0)$ property if for all rightmost derivations of the form*

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha\beta y \end{cases}$$

*it follows that $\alpha = \gamma$, $A = B$, and $x = y$.*

**Goal:** derive a finite information from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by $\mathrm{fo}$-sets)

# $LR(0)$ Items and Sets

## Definition ($LR(0)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta_1\beta_2 w$ (i.e., $A \to \beta_1\beta_2 \in P$).

- $[A \to \beta_1 \cdot \beta_2]$ is called an $LR(0)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all $LR(0)$ items for $\gamma$, called the $LR(0)$ set (or: $LR(0)$ information) of $\gamma$.
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

## Corollary

1. *For every* $\gamma \in X^*$, $LR(0)(\gamma)$ *is finite.*
2. $LR(0)(G)$ *is finite.*
3. *The item* $[A \to \beta\cdot] \in LR(0)(\gamma)$ *indicates the possible reduction* $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ *where* $\pi_i = A \to \beta$ *and* $\gamma = \alpha\beta$.
4. *The item* $[A \to \beta_1 \cdot Y\beta_2] \in LR(0)(\gamma)$ *indicates an incomplete handle* $\beta_1$ *(to be completed by shift operations or* $\varepsilon$*-steps).*

# $LR(0)$ Conflicts

## Definition ($LR(0)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(0)(G)$.

- $I$ has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ such that
$$[A \to \alpha_1 \cdot a\alpha_2], [B \to \beta \cdot] \in I.$$
- $I$ has a reduce/reduce conflict if there exist $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
$$[A \to \alpha \cdot], [B \to \beta \cdot] \in I.$$

## Lemma

*$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.*

## Proof.

omitted □

# The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the action function, which determines the shift/reduce decision.
(Reminder: $\pi_0 = S' \to S$)

## Definition ($LR(0)$ action function)

The $LR(0)$ action function

$$\mathrm{act} : LR(0)(G) \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$$

is defined by

$$\mathrm{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot] \in I \\ \text{shift} & \text{if } [A \to \alpha_1 \cdot a\alpha_2] \in I \\ \text{accept} & \text{if } [S' \to S\cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

## Corollary

*For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff $\mathrm{act}$ is well defined.*

# Outline

1. Recap: $LR(0)$ Grammars
2. The $LR(0)$ Parsing Automaton
3. $SLR(1)$ Parsing
4. $LR(1)$ Parsing

# The $LR(0)$ Parsing Table

## Example 10.1 (cf. Example 9.15)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

$$
\begin{array}{lll}
I_0 := LR(0)(\varepsilon): & [S' \rightarrow \cdot S] & \\
 & [S \rightarrow \cdot B] & [S \rightarrow \cdot C] \\
 & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] \\
I_1 := LR(0)(S): & [S' \rightarrow S \cdot] & \\
I_2 := LR(0)(B): & [S \rightarrow B \cdot] & \\
I_3 := LR(0)(C): & [S \rightarrow C \cdot] & \\
I_4 := LR(0)(a): & [B \rightarrow a \cdot B] & [C \rightarrow a \cdot C] \\
 & [B \rightarrow \cdot aB] & [B \rightarrow \cdot b] \\
 & [C \rightarrow \cdot aC] & [C \rightarrow \cdot c] \\
I_5 := LR(0)(b): & [B \rightarrow b \cdot] & \\
I_6 := LR(0)(c): & [C \rightarrow c \cdot] & \\
I_7 := LR(0)(aB): & [B \rightarrow aB \cdot] & \\
I_8 := LR(0)(aC): & [C \rightarrow aC \cdot] & \\
I_9 := \emptyset & &
\end{array}
$$

# The $LR(0)$ Parsing Table

## Example 10.1 (cf. Example 9.15)

$$
\begin{array}{llll}
G: & S' & \to S & (0) \\
 & S & \to B \mid C & (1,2) \\
 & B & \to aB \mid b & (3,4) \\
 & C & \to aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$$
\begin{array}{lll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & \\
 & [S \to \cdot B] & [S \to \cdot C] \\
 & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] \\
I_1 := LR(0)(S): & [S' \to S \cdot] & \\
I_2 := LR(0)(B): & [S \to B \cdot] & \\
I_3 := LR(0)(C): & [S \to C \cdot] & \\
I_4 := LR(0)(a): & [B \to a \cdot B] & [C \to a \cdot C] \\
 & [B \to \cdot aB] & [B \to \cdot b] \\
 & [C \to \cdot aC] & [C \to \cdot c] \\
I_5 := LR(0)(b): & [B \to b \cdot] & \\
I_6 := LR(0)(c): & [C \to c \cdot] & \\
I_7 := LR(0)(aB): & [B \to aB \cdot] & \\
I_8 := LR(0)(aC): & [C \to aC \cdot] & \\
I_9 := \emptyset & &
\end{array}
$$

# The $LR(0)$ Parsing Automaton I

## Definition 10.2 ($LR(0)$ parsing automaton)

Let $G = \langle N, \Sigma, P, S\rangle \in LR(0)$. The (deterministic) $LR(0)$ parsing automaton of $G$ is defined by the following components.

- Input alphabet $\Sigma$
- Pushdown alphabet $\Gamma := LR(0)(G)$
- Output alphabet $\Delta := [p] \cup \{0, \text{error}\}$
- Configurations $\Sigma^* \times \Gamma^* \times \Delta^*$
- Initial configuration $(w, I_0, \varepsilon)$ where $I_0 := LR(0)(\varepsilon)$
- Final configurations $\{\varepsilon\} \times \{\varepsilon\} \times \Delta^*$
- Transitions:

$$\begin{aligned}
\text{shift:}\quad & (aw, \alpha I, z) \vdash (w, \alpha I I', z) \text{ if } \mathrm{act}(I) = \text{shift and } \mathrm{goto}(I, a) = I' \\
\text{reduce:}\quad & (w, \alpha I I_1 \dots I_n, z) \vdash (w, \alpha I I', zi) \text{ if } \mathrm{act}(I_n) = \text{red } i,\ \pi_i = A \to Y_1 \dots Y_n, \\
& \text{and } \mathrm{goto}(I, A) = I' \\
\text{accept:}\quad & (\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0) \text{ if } \mathrm{act}(I) = \text{accept} \\
\text{error:}\quad & (w, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error}) \text{ if } \mathrm{act}(I) = \text{error}
\end{aligned}$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$\begin{array}{llll} G: & S' & \rightarrow S & (0) \\ & S & \rightarrow B \mid C & (1,2) \\ & B & \rightarrow aB \mid b & (3,4) \\ & C & \rightarrow aC \mid c & (5,6) \end{array}$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
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| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$(aac, I_0 \quad , \varepsilon \quad )$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{lllll}
G: & S' & \rightarrow & S & (0) \\
 & S & \rightarrow & B \mid C & (1,2) \\
 & B & \rightarrow & aB \mid b & (3,4) \\
 & C & \rightarrow & aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{ll}
 & (aac, I_0, \varepsilon) \\
\vdash & (ac, I_0 I_4, \varepsilon)
\end{array}
$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{llll}
G: & S' & \to S & (0) \\
 & S & \to B \mid C & (1,2) \\
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$$

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| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
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| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{ll}
 & (aac, I_0, \varepsilon) \\
\vdash & (ac, I_0 I_4, \varepsilon) \\
\vdash & (c, I_0 I_4 I_4, \varepsilon)
\end{array}
$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{llll}
G: & S' & \to S & (0) \\
 & S & \to B \mid C & (1,2) \\
 & B & \to aB \mid b & (3,4) \\
 & C & \to aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{ll}
 & (aac,\ I_0,\ \varepsilon) \\
\vdash & (ac,\ I_0 I_4,\ \varepsilon) \\
\vdash & (c,\ I_0 I_4 I_4,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon)
\end{array}
$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0) \\
 & S & \rightarrow B \mid C & (1,2) \\
 & B & \rightarrow aB \mid b & (3,4) \\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
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| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{rlll}
 & (aac, & I_0 & , \varepsilon) \\
\vdash & (ac, & I_0 I_4 & , \varepsilon) \\
\vdash & (c, & I_0 I_4 I_4 & , \varepsilon) \\
\vdash & (\varepsilon, & I_0 I_4 I_4 I_6, & \varepsilon) \\
\vdash & (\varepsilon, & I_0 I_4 I_4 I_8, & 6)
\end{array}
$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{lllll}
G: & S' & \to & S & (0) \\
 & S & \to & B \mid C & (1,2) \\
 & B & \to & aB \mid b & (3,4) \\
 & C & \to & aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{ll}
 & (aac,\ I_0,\ \varepsilon) \\
\vdash & (ac,\ I_0 I_4,\ \varepsilon) \\
\vdash & (c,\ I_0 I_4 I_4,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_8,\ 6) \\
\overset{(*)}{\vdash} & (\varepsilon,\ I_0 I_4 I_8,\ 65)
\end{array}
$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{llll}
G: & S' & \rightarrow S & (0)\\
 & S & \rightarrow B \mid C & (1,2)\\
 & B & \rightarrow aB \mid b & (3,4)\\
 & C & \rightarrow aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{lllll}
 & (aac, & I_0 & , \varepsilon & )\\
\vdash & (ac, & I_0 I_4 & , \varepsilon & )\\
\vdash & (c, & I_0 I_4 I_4 & , \varepsilon & )\\
\vdash & (\varepsilon, & I_0 I_4 I_4 I_6, & \varepsilon & )\\
\vdash & (\varepsilon, & I_0 I_4 I_4 I_8, & 6 & )\\
(*) & & & & \\
\vdash & (\varepsilon, & I_0 I_4 I_8 & , 65 & )\\
\vdash & (\varepsilon, & I_0 I_3 & , 655 & )
\end{array}
$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$\begin{array}{llll} G: & S' & \rightarrow S & (0) \\ & S & \rightarrow B \mid C & (1,2) \\ & B & \rightarrow aB \mid b & (3,4) \\ & C & \rightarrow aC \mid c & (5,6) \end{array}$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$\begin{array}{rl} & (aac, I_0, \varepsilon) \\ \vdash & (ac, I_0 I_4, \varepsilon) \\ \vdash & (c, I_0 I_4 I_4, \varepsilon) \\ \vdash & (\varepsilon, I_0 I_4 I_4 I_6, \varepsilon) \\ \vdash & (\varepsilon, I_0 I_4 I_4 I_8, 6) \\ (*) & \\ \vdash & (\varepsilon, I_0 I_4 I_8, 65) \\ \vdash & (\varepsilon, I_0 I_3, 655) \\ \vdash & (\varepsilon, I_0 I_1, 6552) \end{array}$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{lrcll}
G: & S' & \to & S & (0) \\
 & S & \to & B \mid C & (1,2) \\
 & B & \to & aB \mid b & (3,4) \\
 & C & \to & aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{ll}
 & (aac,\ I_0,\ \varepsilon) \\
\vdash & (ac,\ I_0 I_4,\ \varepsilon) \\
\vdash & (c,\ I_0 I_4 I_4,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon) \\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_8,\ 6) \\
(*) & \\
\vdash & (\varepsilon,\ I_0 I_4 I_8,\ 65) \\
\vdash & (\varepsilon,\ I_0 I_3,\ 655) \\
\vdash & (\varepsilon,\ I_0 I_1,\ 6552) \\
\vdash & (\varepsilon,\ \varepsilon,\ 65520)
\end{array}
$$

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$
\begin{array}{llll}
G: & S' & \to S & (0)\\
 & S & \to B \mid C & (1,2)\\
 & B & \to aB \mid b & (3,4)\\
 & C & \to aC \mid c & (5,6)
\end{array}
$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$
\begin{array}{rl}
 & (aac,\ I_0,\ \varepsilon)\\
\vdash & (ac,\ I_0 I_4,\ \varepsilon)\\
\vdash & (c,\ I_0 I_4 I_4,\ \varepsilon)\\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon)\\
\vdash & (\varepsilon,\ I_0 I_4 I_4 I_8,\ 6)\\
(*) & \\
\vdash & (\varepsilon,\ I_0 I_4 I_8,\ 65)\\
\vdash & (\varepsilon,\ I_0 I_3,\ 655)\\
\vdash & (\varepsilon,\ I_0 I_1,\ 6552)\\
\vdash & (\varepsilon,\ \varepsilon,\ 65520)
\end{array}
$$

Check by rightmost derivation (on the board)

# The $LR(0)$ Parsing Automaton II

## Example 10.3 (cf. Example 10.1)

$$\begin{array}{llll} G: & S' & \rightarrow S & (0) \\ & S & \rightarrow B \mid C & (1,2) \\ & B & \rightarrow aB \mid b & (3,4) \\ & C & \rightarrow aC \mid c & (5,6) \end{array}$$

| $LR(0)(G)$ | act | goto | | | | | |
|---|---|---|---|---|---|---|---|
| | | $S$ | $B$ | $C$ | $a$ | $b$ | $c$ |
| $I_0$ | shift | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ |
| $I_1$ | accept | | | | | | |
| $I_2$ | red 1 | | | | | | |
| $I_3$ | red 2 | | | | | | |
| $I_4$ | shift | | $I_7$ | $I_8$ | $I_4$ | $I_5$ | $I_6$ |
| $I_5$ | red 4 | | | | | | |
| $I_6$ | red 6 | | | | | | |
| $I_7$ | red 3 | | | | | | |
| $I_8$ | red 5 | | | | | | |
| $I_9$ | error | | | | | | |

(empty $= I_9$)

$LR(0)$ parsing of $aac$:

$$\begin{array}{rl} & (aac,\ I_0,\ \varepsilon) \\ \vdash & (ac,\ I_0 I_4,\ \varepsilon) \\ \vdash & (c,\ I_0 I_4 I_4,\ \varepsilon) \\ \vdash & (\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon) \\ \vdash & (\varepsilon,\ I_0 I_4 I_4 I_8,\ 6) \\ \overset{(*)}{\vdash} & (\varepsilon,\ I_0 I_4 I_8,\ 65) \\ \vdash & (\varepsilon,\ I_0 I_3,\ 655) \\ \vdash & (\varepsilon,\ I_0 I_1,\ 6552) \\ \vdash & (\varepsilon,\ \varepsilon,\ 65520) \end{array}$$

Check by rightmost derivation (on the board)

**Remark:** in the corresponding computation of $\mathrm{NBA}(G)$, $(*)$ is nondeterministic

# The $LR(0)$ Parsing Automaton III

**Theorem 10.4 (Correctness of $LR(0)$ Parsing Automaton)**

*If $G \in LR(0)$, then the $LR(0)$ parsing automaton of $G$ is deterministic, and for every $w \in \Sigma^*$ and $z \in \{0, \dots, p\}^*$:*

$$(w, I_0, \varepsilon) \vdash^* (\varepsilon, \varepsilon, z) \quad \textit{iff} \quad \overleftarrow{z} \textit{ is a rightmost analysis of } w$$

**Proof.**

omitted ☐

# Outline

1. Recap: $LR(0)$ Grammars
2. The $LR(0)$ Parsing Automaton
3. $SLR(1)$ Parsing
4. $LR(1)$ Parsing

# Removing Conflicts in $LR(0)$ Parsing

In practice: often $G \notin LR(0)$

**Example 10.5**

$$G_{AE}: \quad \begin{array}{lll} E' \to E & \qquad & E \to E\texttt{+}T \mid T \\ T \to T\texttt{*}F \mid F & \qquad & F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} \end{array}$$

# Removing Conflicts in $LR(0)$ Parsing

In practice: often $G \notin LR(0)$

### Example 10.5

$$G_{AE}: \quad E' \to E \qquad E \to E{+}T \mid T$$

$$T \to T{*}F \mid F \qquad F \to (E) \mid \mathtt{a} \mid \mathtt{b}$$

$LR(0)(G_{AE})$ with conflicts:

$I_0: [E' \to \cdot E] \quad [E \to \cdot E{+}T] \quad [E \to \cdot T] \quad [T \to \cdot T{*}F] \quad [T \to \cdot F] \quad [F \to \cdot (E)] \quad [F \to \cdot \mathtt{a}] \quad [F \to \cdot \mathtt{b}]$

$I_1: [E' \to E \cdot] \quad [E \to E \cdot {+}T]$

$I_2: [E \to T \cdot] \quad [T \to T \cdot {*}F]$

$I_3: [T \to F \cdot]$

$I_4: [F \to ( \cdot E)] \quad [E \to \cdot E{+}T] \quad [E \to \cdot T] \quad [T \to \cdot T{*}F] \quad [T \to \cdot F] \quad [F \to \cdot (E)] \quad [F \to \cdot \mathtt{a}] \quad [F \to \cdot \mathtt{b}]$

$I_5: [F \to \mathtt{a} \cdot]$

$I_6: [F \to \mathtt{b} \cdot]$

$I_7: [E \to E{+} \cdot T] \quad [T \to \cdot T{*}F] \quad [T \to \cdot F] \quad [F \to \cdot (E)] \quad [F \to \cdot \mathtt{a}] \quad [F \to \cdot \mathtt{b}]$

$I_8: [T \to T{*} \cdot F] \quad [F \to \cdot (E)] \quad [F \to \cdot \mathtt{a}] \quad [F \to \cdot \mathtt{b}]$

$I_9: [F \to (E \cdot )] \quad [E \to E \cdot {+}T]$

$I_{10}: [E \to E{+}T \cdot] \quad [T \to T \cdot {*}F]$

$I_{11}: [T \to T{*}F \cdot]$

$I_{12}: [F \to (E) \cdot]$

# Adding Lookahead I

**Goal:** resolving conflicts by considering next input symbol

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**Observations:**

- $[A \to \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$
  $$\Longrightarrow S' \Rightarrow_r^* \alpha A w \Rightarrow_r \underset{\underset{\text{pushdown}}{\nearrow}}{\alpha\beta_1}\,\underset{\underset{\text{next input symbol}}{\nwarrow}}{a}\beta_2 w$$

  **Thus:** shift only on lookahead $a$

# Adding Lookahead I

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**Observations:**

- $[A \to \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$
  $$\Longrightarrow\ S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\underset{\substack{\nearrow\\ \text{pushdown}}}{\beta_1}\underset{\substack{\nwarrow\\ \text{next input symbol}}}{a}\beta_2 w$$

  **Thus:** shift only on lookahead $a$
- $[A \to \beta\cdot] \in LR(0)(\alpha\beta)$
  $$\Longrightarrow\ S' \Rightarrow_r^* \alpha A x w \Rightarrow_r \alpha\underset{\substack{\nearrow\\ \text{pushdown}}}{\beta}\underset{\substack{\nwarrow\\ \text{input}}}{x}w$$
  $\Longrightarrow\ x \in \mathrm{fo}(A) \subseteq \Sigma_\varepsilon$ ($x = \varepsilon$ only if $w = \varepsilon$)

  **Thus:** reduce with $A \to \beta$ only if lookahead $x \in \mathrm{fo}(A)$

# Adding Lookahead II

## Example 10.6 (cf. Example 10.5)

$$G_{AE}: \begin{array}{lll} E' \to E & & (0) \\ E \to E\texttt{+}T \mid T & & (1,2) \\ T \to T\texttt{*}F \mid F & & (3,4) \\ F \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & & (5,6,7) \end{array}$$

| $A \in N$ | $\mathrm{fo}(A)$ |
|---|---|
| $E'$ | $\{\varepsilon\}$ |
| $E$ | $\{\texttt{+}, \texttt{)}, \varepsilon\}$ |

# Adding Lookahead II

## Example 10.6 (cf. Example 10.5)

$$G_{AE}: \begin{array}{lll} E' & \rightarrow E & (0) \\ E & \rightarrow E\texttt{+}T \mid T & (1,2) \\ T & \rightarrow T\texttt{*}F \mid F & (3,4) \\ F & \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

| $A \in N$ | $\mathrm{fo}(A)$ |
|---|---|
| $E'$ | $\{\varepsilon\}$ |
| $E$ | $\{\texttt{+}, \texttt{)}, \varepsilon\}$ |

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E \cdot \texttt{+} T]\}$:
  - accept on lookahead $\varepsilon$
  - shift on lookahead $\texttt{+}$

# Adding Lookahead II

## Example 10.6 (cf. Example 10.5)

$$\begin{array}{llll} G_{AE}: & E' & \rightarrow E & (0) \\ & E & \rightarrow E\texttt{+}T \mid T & (1,2) \\ & T & \rightarrow T\texttt{*}F \mid F & (3,4) \\ & F & \rightarrow \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7) \end{array}$$

| $A \in N$ | $\mathrm{fo}(A)$ |
|---|---|
| $E'$ | $\{\varepsilon\}$ |
| $E$ | $\{\texttt{+}, \texttt{)}, \varepsilon\}$ |

- $I_1 = \{[E' \to E\cdot], [E \to E \cdot \texttt{+}T]\}$:
  - accept on lookahead $\varepsilon$
  - shift on lookahead $\texttt{+}$
- $I_2 = \{[E \to T\cdot], [T \to T \cdot \texttt{*}F]\}$:
  - red 2 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead $\texttt{*}$

# Adding Lookahead II

## Example 10.6 (cf. Example 10.5)

$$G_{AE}: \begin{array}{lcll} E' & \to & E & (0) \\ E & \to & E{+}T \mid T & (1,2) \\ T & \to & T{*}F \mid F & (3,4) \\ F & \to & (E) \mid \mathtt{a} \mid \mathtt{b} & (5,6,7) \end{array}$$

| $A \in N$ | $\mathrm{fo}(A)$ |
|---|---|
| $E'$ | $\{\varepsilon\}$ |
| $E$ | $\{+, ), \varepsilon\}$ |

- $I_1 = \{[E' \to E\cdot], [E \to E \cdot {+} T]\}$:
  - accept on lookahead $\varepsilon$
  - shift on lookahead +
- $I_2 = \{[E \to T\cdot], [T \to T \cdot {*} F]\}$:
  - red 2 on lookahead $+/)/\varepsilon$
  - shift on lookahead $*$
- $I_{10} = \{[E \to E{+}T\cdot], [T \to T \cdot {*} F]\}$:
  - red 1 on lookahead $+/)/\varepsilon$
  - shift on lookahead $*$

# Adding Lookahead II

## Example 10.6 (cf. Example 10.5)

$$
\begin{array}{llll}
G_{AE}: & E' & \to E & (0) \\
 & E & \to E\texttt{+}T \mid T & (1,2) \\
 & T & \to T\texttt{*}F \mid F & (3,4) \\
 & F & \to \texttt{(}E\texttt{)} \mid \texttt{a} \mid \texttt{b} & (5,6,7)
\end{array}
$$

| $A \in N$ | $\mathrm{fo}(A)$ |
|---|---|
| $E'$ | $\{\varepsilon\}$ |
| $E$ | $\{\texttt{+}, \texttt{)}, \varepsilon\}$ |

- $I_1 = \{[E' \to E\cdot], [E \to E \cdot \texttt{+} T]\}$:
  - accept on lookahead $\varepsilon$
  - shift on lookahead +
- $I_2 = \{[E \to T\cdot], [T \to T \cdot \texttt{*} F]\}$:
  - red 2 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead *
- $I_{10} = \{[E \to E\texttt{+}T\cdot], [T \to T \cdot \texttt{*} F]\}$:
  - red 1 on lookahead $\texttt{+}/\texttt{)}/\varepsilon$
  - shift on lookahead *

$\Longrightarrow$ $SLR(1)$ parsing (Simple $LR(1)$)

# The $SLR(1)$ Action Function

## Definition 10.7 ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\mathrm{act} : LR(0)(G) \times \Sigma_\varepsilon \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \to \alpha, [A \to \alpha \cdot] \in I, \\ & \text{and } x \in \mathrm{fo}(A) \\ \text{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \to S \cdot] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

# The $SLR(1)$ Action Function

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## Definition 10.8 ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

# The $SLR(1)$ Action Function

## Definition 10.7 ($SLR(1)$ action function)

The $SLR(1)$ action function

$$\mathrm{act} : LR(0)(G) \times \Sigma_\varepsilon \to \{\mathsf{red}\ i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \mathsf{red}\ i & \text{if } i \neq 0, \pi_i = A \to \alpha, [A \to \alpha\cdot] \in I, \\ & \text{and } x \in \mathrm{fo}(A) \\ \mathsf{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2] \in I \text{ and } x \in \Sigma \\ \mathsf{accept} & \text{if } [S' \to S\cdot] \in I \text{ and } x = \varepsilon \\ \mathsf{error} & \text{otherwise} \end{cases}$$

## Definition 10.8 ($SLR(1)$ grammar)

A grammar $G \in CFG_\Sigma$ has the $SLR(1)$ property (notation: $G \in SLR(1)$) if its $SLR(1)$ action function is well defined.

Together, act and the $LR(0)$ goto function (cf. Definition 9.14) form the $SLR(1)$ parsing table of $G$.

# The $SLR(1)$ Parsing Table

## Example 10.9 (cf. Example 10.5)

$I_0 : [E' \to \cdot E]\ [E \to \cdot E{+}T]$
$\quad [E \to \cdot T]\ [T \to \cdot T{*}F]$
$\quad [T \to \cdot F]\ [F \to \cdot (E)]$
$\quad [F \to \cdot \mathrm{a}]\ [F \to \cdot \mathrm{b}]$

$I_4 : [F \to (\cdot E)]\ [E \to \cdot E{+}T]$
$\quad [E \to \cdot T]\ [T \to \cdot T{*}F]$
$\quad [T \to \cdot F]\ [F \to \cdot (E)]$
$\quad [F \to \cdot \mathrm{a}]\ [F \to \cdot \mathrm{b}]$

$I_8 : [T \to T{*} \cdot F]\ [F \to \cdot (E)]$
$\quad [F \to \cdot \mathrm{a}]\ [F \to \cdot \mathrm{b}]$

$I_{11} : [T \to T{*}F \cdot]$

$I_1 : [E' \to E \cdot]\ [E \to E \cdot {+}T]$
$I_2 : [E \to T \cdot]\ [T \to T \cdot {*}F]$
$I_3 : [T \to F \cdot]$

$I_5 : [F \to \mathrm{a} \cdot]$
$I_6 : [F \to \mathrm{b} \cdot]$
$I_7 : [E \to E{+} \cdot T]\ [T \to \cdot T{*}F]$
$\quad [T \to \cdot F]\ [F \to \cdot (E)]$
$\quad [F \to \cdot \mathrm{a}]\ [F \to \cdot \mathrm{b}]$

$I_9 : [F \to (E \cdot)]\ [E \to E \cdot {+}T]$
$I_{10} : [E \to E{+}T \cdot]\ [T \to T \cdot {*}F]$
$I_{12} : [F \to (E) \cdot]$

| $A \in N$ | $\mathrm{fo}(A)$ |
|---|---|
| $E'$ | $\{\varepsilon\}$ |
| $E$ | $\{+, ), \varepsilon\}$ |
| $T$ | $\{+, *, ), \varepsilon\}$ |
| $F$ | $\{+, *, ), \varepsilon\}$ |

# The $SLR(1)$ Parsing Table

## Example 10.9 (cf. Example 10.5)

$I_0 :$ $[E' \to \cdot E]$ $[E \to \cdot E{+}T]$ $[E \to \cdot T]$ $[T \to \cdot T{*}F]$ $[T \to \cdot F]$ $[F \to \cdot (E)]$ $[F \to \cdot \mathtt{a}]$ $[F \to \cdot \mathtt{b}]$

$I_1 :$ $[E' \to E \cdot]$ $[E \to E \cdot {+}T]$

$I_2 :$ $[E \to T \cdot]$ $[T \to T \cdot {*}F]$

$I_3 :$ $[T \to F \cdot]$

$I_4 :$ $[F \to ( \cdot E)]$ $[E \to \cdot E{+}T]$ $[E \to \cdot T]$ $[T \to \cdot T{*}F]$ $[T \to \cdot F]$ $[F \to \cdot (E)]$ $[F \to \cdot \mathtt{a}]$ $[F \to \cdot \mathtt{b}]$

$I_5 :$ $[F \to \mathtt{a} \cdot]$

$I_6 :$ $[F \to \mathtt{b} \cdot]$

$I_7 :$ $[E \to E{+} \cdot T]$ $[T \to \cdot T{*}F]$ $[T \to \cdot F]$ $[F \to \cdot (E)]$ $[F \to \cdot \mathtt{a}]$ $[F \to \cdot \mathtt{b}]$

$I_8 :$ $[T \to T{*} \cdot F]$ $[F \to \cdot (E)]$ $[F \to \cdot \mathtt{a}]$ $[F \to \cdot \mathtt{b}]$

$I_9 :$ $[F \to (E \cdot )]$ $[E \to E \cdot {+}T]$

$I_{10} :$ $[E \to E{+}T \cdot]$ $[T \to T \cdot {*}F]$

$I_{11} :$ $[T \to T{*}F \cdot]$

$I_{12} :$ $[F \to (E) \cdot]$

| $A \in N$ | $\mathrm{fo}(A)$ |
|---|---|
| $E'$ | $\{\varepsilon\}$ |
| $E$ | $\{+, ), \varepsilon\}$ |
| $T$ | $\{+, *, ), \varepsilon\}$ |
| $F$ | $\{+, *, ), \varepsilon\}$ |

| $LR(0)(G_{AE})$ | act | | | | | | | goto | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | + | * | ( | ) | a | b | $\varepsilon$ | $E$ | $T$ | $F$ | + | * | ( | ) | a | b |
| $I_0$ | | | shift | | shift | shift | | $I_1$ | $I_2$ | $I_3$ | | | $I_4$ | | $I_5$ | $I_6$ |
| $I_1$ | shift | | | | | | accept | | | | $I_7$ | | | | | |
| $I_2$ | red 2 | shift | | red 2 | | | red 2 | | | | | $I_8$ | | | | |
| $I_3$ | red 4 | red 4 | | red 4 | | | red 4 | | | | | | | | | |
| $I_4$ | | | shift | | shift | shift | | $I_9$ | $I_2$ | $I_3$ | | | $I_4$ | | $I_5$ | $I_6$ |
| $I_5$ | red 6 | red 6 | | red 6 | | | red 6 | | | | | | | | | |
| $I_6$ | red 7 | red 7 | | red 7 | | | red 7 | | | | | | | | | |
| $I_7$ | | | shift | | shift | shift | | | $I_{10}$ | $I_3$ | | | $I_4$ | | $I_5$ | $I_6$ |
| $I_8$ | | | shift | | shift | shift | | | | $I_{11}$ | | | $I_4$ | | $I_5$ | $I_6$ |
| $I_9$ | shift | | | shift | | | | | | | $I_7$ | | | $I_{12}$ | | |
| $I_{10}$ | red 1 | shift | | red 1 | | | red 1 | | | | | $I_8$ | | | | |
| $I_{11}$ | red 3 | red 3 | | red 3 | | | red 3 | | | | | | | | | |
| $I_{12}$ | red 5 | red 5 | | red 5 | | | red 5 | | | | | | | | | |

# The $SLR(1)$ Parsing Automaton

## Definition 10.10 ($SLR(1)$ parsing automaton)

The $SLR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 10.2), except for the transition relation:

shift: $(aw, \alpha I, z) \vdash (w, \alpha I I', z)$ if $\mathrm{act}(I, a) = \text{shift}$ and $\mathrm{goto}(I, a) = I'$

reduce$_a$: $(aw, \alpha I I_1 \ldots I_n, z) \vdash (aw, \alpha I I', zi)$ if $\mathrm{act}(I_n, a) = \text{red } i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

reduce$_\varepsilon$: $(\varepsilon, \alpha I I_1 \ldots I_n, z) \vdash (\varepsilon, \alpha I I', zi)$ if $\mathrm{act}(I_n, \varepsilon) = \text{red } i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$

accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0)$ if $\mathrm{act}(I, \varepsilon) = \text{accept}$

error$_a$: $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error})$ if $\mathrm{act}(I, a) = \text{error}$

error$_\varepsilon$: $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\,\text{error})$ if $\mathrm{act}(I, \varepsilon) = \text{error}$

# Outline

1. Recap: $LR(0)$ Grammars
2. The $LR(0)$ Parsing Automaton
3. $SLR(1)$ Parsing
4. $LR(1)$ Parsing

# $SLR(1)$ Conflicts

**Problem:** not all conflicts can be resolved using $\mathrm{fo}$ sets

**Example 10.11**

$G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathtt{a} \quad R \to L$

# $SLR(1)$ Conflicts

**Problem:** not all conflicts can be resolved using $\mathrm{fo}$ sets

## Example 10.11

$G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

$LR(0)(G_{LR})$:

$$
\begin{array}{llllll}
I_0 := LR(0)(\varepsilon): & [S' \to \cdot S] & [S \to \cdot L{=}R] & [S \to \cdot R] & \\
 & [L \to \cdot{*}R] & [L \to \cdot\mathrm{a}] & [R \to \cdot L] & \\
I_1 := LR(0)(S): & [S' \to S\cdot] & & & \\
I_2 := LR(0)(L): & [S \to L \cdot {=}R] & [R \to L\cdot] & & \\
I_3 := LR(0)(R): & [S \to R\cdot] & & & \\
I_4 := LR(0)({*}): & [L \to {*} \cdot R] & [R \to \cdot L] & [L \to \cdot{*}R] & [L \to \cdot\mathrm{a}] \\
I_5 := LR(0)(\mathrm{a}): & [L \to \mathrm{a}\cdot] & & & \\
I_6 := LR(0)(L{=}): & [S \to L{=} \cdot R] & [R \to \cdot L] & [L \to \cdot{*}R] & [L \to \cdot\mathrm{a}] \\
I_7 := LR(0)({*}R): & [L \to {*}R\cdot] & & & \\
I_8 := LR(0)({*}L): & [R \to L\cdot] & & & \\
I_9 := LR(0)(L{=}R): & [S \to L{=}R\cdot] & & &
\end{array}
$$

# $SLR(1)$ Conflicts

**Problem:** not all conflicts can be resolved using $\mathrm{fo}$ sets

### Example 10.11

$G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$LR(0)(G_{LR})$:

$I_0 := LR(0)(\varepsilon) : \quad [S' \to \cdot S] \quad [S \to \cdot L{=}R] \quad [S \to \cdot R]$
$\qquad [L \to \cdot *R] \quad [L \to \cdot \mathrm{a}] \quad [R \to \cdot L]$
$I_1 := LR(0)(S) : \quad [S' \to S \cdot]$
$I_2 := LR(0)(L) : \quad [S \to L \cdot {=}R] \quad [R \to L \cdot]$
$I_3 := LR(0)(R) : \quad [S \to R \cdot]$
$I_4 := LR(0)(*) : \quad [L \to * \cdot R] \quad [R \to \cdot L] \quad [L \to \cdot *R] \quad [L \to \cdot \mathrm{a}]$
$I_5 := LR(0)(\mathrm{a}) : \quad [L \to \mathrm{a} \cdot]$
$I_6 := LR(0)(L{=}) : \quad [S \to L{=} \cdot R] \quad [R \to \cdot L] \quad [L \to \cdot *R] \quad [L \to \cdot \mathrm{a}]$
$I_7 := LR(0)(*R) : \quad [L \to *R \cdot]$
$I_8 := LR(0)(*L) : \quad [R \to L \cdot]$
$I_9 := LR(0)(L{=}R) : \quad [S \to L{=}R \cdot]$

But: conflict in $I_2$ not $SLR(1)$-solvable since $= \in \mathrm{fo}(R)$

# $LR(1)$ Items and Sets I

**Observation:** not every element of $\mathrm{fo}(A)$ can follow every occurrence of $A$
$\Longrightarrow$ refinement of $LR(0)$ items by adding possible lookahead symbols

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### Definition 10.12 ($LR(1)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$.

- If $S' \Rightarrow_r^* \alpha A a w \Rightarrow_r \alpha\beta_1\beta_2 a w$, then $[A \to \beta_1 \cdot \beta_2, a]$ is called an $LR(1)$ item for $\alpha\beta_1$.

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- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$, then $[A \to \beta_1 \cdot \beta_2, \varepsilon]$ is called an $LR(1)$ item for $\alpha\beta_1$.

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**Observation:** not every element of $\mathrm{fo}(A)$ can follow every occurrence of $A$ $\Longrightarrow$ refinement of $LR(0)$ items by adding possible lookahead symbols

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- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha\beta_1\beta_2$, then $[A \to \beta_1 \cdot \beta_2, \varepsilon]$ is called an $LR(1)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for $\gamma$, called the $LR(1)$ set (or: $LR(1)$ information) of $\gamma$.

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**Observation:** not every element of $\mathrm{fo}(A)$ can follow every occurrence of $A$ $\Longrightarrow$ refinement of $LR(0)$ items by adding possible lookahead symbols

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- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha\beta_1\beta_2$, then $[A \to \beta_1 \cdot \beta_2, \varepsilon]$ is called an $LR(1)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for $\gamma$, called the $LR(1)$ set (or: $LR(1)$ information) of $\gamma$.
- $LR(1)(G) := \{LR(1)(\gamma) \mid \gamma \in X^*\}$.

# $LR(1)$ Items and Sets II

### Corollary 10.13

1. *For every* $\gamma \in X^*$, $LR(1)(\gamma)$ *is finite.*

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# $LR(1)$ Items and Sets II

## Corollary 10.13

1. *For every* $\gamma \in X^*$, $LR(1)(\gamma)$ *is finite.*
2. $LR(1)(G)$ *is finite.*
3. *For every* $\gamma \in X^*$, $LR(1)(\gamma)$ *"contains"* $LR(0)(\gamma)$, *i.e.,*

$$\{[A \to \beta_1 \cdot \beta_2] \mid [A \to \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

# $LR(1)$ Items and Sets II

**Corollary 10.13**

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$$\{[A \to \beta_1 \cdot \beta_2] \mid [A \to \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$
4. $[A \to \beta_1 \cdot \beta_2, x] \in I \in LR(1)(G) \implies x \in \mathrm{fo}(A)$

# $LR(1)$ Conflicts

## Definition 10.14 ($LR(1)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(1)(G)$.

- $I$ has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ and $x \in \Sigma_\varepsilon$ such that

$$[A \to \alpha_1 \cdot a\alpha_2, x], [B \to \beta\cdot, a] \in I.$$

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- $I$ has a reduce/reduce conflict if there exist $x \in \Sigma_\varepsilon$ and $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
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- $I$ has a reduce/reduce conflict if there exist $x \in \Sigma_\varepsilon$ and $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that
$$[A \to \alpha\cdot, x], [B \to \beta\cdot, x] \in I.$$

## Lemma 10.15

*$G \in LR(1)$ iff no $I \in LR(1)(G)$ contains conflicting items.*

# Computing $LR(1)$ Sets I

The computation of $LR(0)$ sets (cf. Theorem 9.10) can be extended to cover right contexts:

**Theorem 10.16 (Computing $LR(1)$ sets)**

*Let* $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ *be start separated by* $S' \to S$ *and reduced.*

1. $LR(1)(\varepsilon)$ *is the least set such that*
    - $[S' \to \cdot S, \varepsilon] \in LR(1)(\varepsilon)$ *and*
    - *if* $[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon)$, $B \to \beta \in P$, *and* $y \in \mathrm{fi}(\gamma x)$, *then* $[B \to \cdot\beta, y] \in LR(1)(\varepsilon)$.

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   - *if* $[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon)$, $B \to \beta \in P$, *and* $y \in \mathrm{fi}(\gamma x)$, *then* $[B \to \cdot\beta, y] \in LR(1)(\varepsilon)$.
2. $LR(1)(\alpha Y)$ $(\alpha \in X^*, Y \in X)$ *is the least set such that*
   - *if* $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$, *then* $[A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$ *and*
   - *if* $[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y)$, $B \to \beta \in P$, *and* $y \in \mathrm{fi}(\gamma_2 x)$, *then* $[B \to \cdot\beta, y] \in LR(1)(\alpha Y)$.

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L\texttt{=}R \mid R \quad L \to *R \mid \texttt{a} \quad R \to L$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L\texttt{=}R \mid R \quad L \to *R \mid \texttt{a} \quad R \to L$

$LR(1)(G_{LR})$: $[S' \to \cdot S, \varepsilon] \in LR(1)(\varepsilon)$

$I_0' := LR(1)(\varepsilon) : \qquad [S' \to \cdot S, \varepsilon]$

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$LR(1)(G_{LR})$: $[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon), B \to \beta \in P, y \in \mathrm{fi}(\gamma x)$
$\implies [B \to \cdot\beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon)$ : $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L\texttt{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$

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$G_{LR} : S' \to S \quad S \to L\texttt{=}R \mid R \quad L \to \ast R \mid \texttt{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \cdot B\gamma, x] \in LR(1)(\varepsilon), B \to \beta \in P, y \in \mathrm{fi}(\gamma x)$
$\implies [B \to \cdot\beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L\texttt{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot\ast R, \texttt{=}]$
$[L \to \cdot\texttt{a}, \texttt{=}]$

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$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathtt{a} \quad R \to L$

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$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot{*}R, =]$
$[L \to \cdot\mathtt{a}, =]$ $[R \to \cdot L, \varepsilon]$

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$\Longrightarrow [B \to \cdot\beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon):$

$[S' \to \cdot S, \varepsilon] \quad [S \to \cdot L\texttt{=}R, \varepsilon] \quad [S \to \cdot R, \varepsilon] \quad [L \to \cdot{*}R, \texttt{=}]$
$[L \to \cdot\texttt{a}, \texttt{=}] \quad [R \to \cdot L, \varepsilon] \quad [L \to \cdot{*}R, \varepsilon] \quad [L \to \cdot\texttt{a}, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

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$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I_0' := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot {*}R, {=}]$
$[L \to \cdot \mathtt{a}, {=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot {*}R, \varepsilon]$ $[L \to \cdot \mathtt{a}, \varepsilon]$

$I_1' := LR(1)(S):$ $[S' \to S \cdot, \varepsilon]$

# Computing $LR(1)$ Sets II

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$\implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$ : $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L\texttt{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, \texttt{=}]$
$[L \to \cdot \texttt{a}, \texttt{=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \texttt{a}, \varepsilon]$

$I'_1 := LR(1)(S)$ : $[S' \to S \cdot, \varepsilon]$

$I'_2 := LR(1)(L)$ : $[S \to L \cdot \texttt{=}R, \varepsilon]$

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$I'_0 := LR(1)(\varepsilon)$ : $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot {*}R, {=}]$
$[L \to \cdot \mathtt{a}, {=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot {*}R, \varepsilon]$ $[L \to \cdot \mathtt{a}, \varepsilon]$

$I'_1 := LR(1)(S)$ : $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$ : $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathtt{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$

$\Longrightarrow [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot {*}R, {=}]$ $[L \to \cdot \mathtt{a}, {=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot {*}R, \varepsilon]$ $[L \to \cdot \mathtt{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S \cdot, \varepsilon]$

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$I'_3 := LR(1)(R):$ $[S \to R \cdot, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

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$I'_0 := LR(1)(\varepsilon)$ : $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$ $[L \to \cdot \mathtt{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathtt{a}, \varepsilon]$

$I'_1 := LR(1)(S)$ : $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$ : $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R)$ : $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*)$ : $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

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$\implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot{*}R, {=}]$
$[L \to \cdot\mathrm{a}, {=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L):$ $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R):$ $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*):$ $[L \to * \cdot R, {=}]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, {=}]$ $[R \to \cdot L, \varepsilon]$

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| | | | | |
|---|---|---|---|---|
| $I'_0 := LR(1)(\varepsilon):$ | $[S' \to \cdot S, \varepsilon]$ | $[S \to \cdot L{=}R, \varepsilon]$ | $[S \to \cdot R, \varepsilon]$ | $[L \to \cdot {*}R, {=}]$ |
| | $[L \to \cdot \mathtt{a}, {=}]$ | $[R \to \cdot L, \varepsilon]$ | $[L \to \cdot {*}R, \varepsilon]$ | $[L \to \cdot \mathtt{a}, \varepsilon]$ |
| $I'_1 := LR(1)(S):$ | $[S' \to S \cdot, \varepsilon]$ | | | |
| $I'_2 := LR(1)(L):$ | $[S \to L \cdot {=}R, \varepsilon]$ | $[R \to L \cdot, \varepsilon]$ | | |
| $I'_3 := LR(1)(R):$ | $[S \to R \cdot, \varepsilon]$ | | | |
| $I'_4 := LR(1)({*}):$ | $[L \to {*} \cdot R, {=}]$ | $[L \to {*} \cdot R, \varepsilon]$ | $[R \to \cdot L, {=}]$ | $[R \to \cdot L, \varepsilon]$ |
| | $[L \to \cdot {*}R, {=}]$ | $[L \to \cdot \mathtt{a}, {=}]$ | $[L \to \cdot {*}R, \varepsilon]$ | $[L \to \cdot \mathtt{a}, \varepsilon]$ |

# Computing $LR(1)$ Sets II

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$G_{LR}$ : $S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

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\begin{array}{ll}
LR(1)(G_{LR}): & [A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha) \\
 & \Longrightarrow [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)
\end{array}
$$

$$
\begin{array}{lllll}
I'_0 := LR(1)(\varepsilon): & [S' \to \cdot S, \varepsilon] & [S \to \cdot L{=}R, \varepsilon] & [S \to \cdot R, \varepsilon] & [L \to \cdot{*}R, =] \\
 & [L \to \cdot\mathrm{a}, =] & [R \to \cdot L, \varepsilon] & [L \to \cdot{*}R, \varepsilon] & [L \to \cdot\mathrm{a}, \varepsilon] \\
I'_1 := LR(1)(S): & [S' \to S\cdot, \varepsilon] & & & \\
I'_2 := LR(1)(L): & [S \to L \cdot {=}R, \varepsilon] & [R \to L\cdot, \varepsilon] & & \\
I'_3 := LR(1)(R): & [S \to R\cdot, \varepsilon] & & & \\
I'_4 := LR(1)({*}): & [L \to {*} \cdot R, =] & [L \to {*} \cdot R, \varepsilon] & [R \to \cdot L, =] & [R \to \cdot L, \varepsilon] \\
 & [L \to \cdot{*}R, =] & [L \to \cdot\mathrm{a}, =] & [L \to \cdot{*}R, \varepsilon] & [L \to \cdot\mathrm{a}, \varepsilon] \\
I'_5 := LR(1)(\mathrm{a}): & [L \to \mathrm{a}\cdot, =] & [L \to \mathrm{a}\cdot, \varepsilon] & &
\end{array}
$$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$$\begin{array}{lllll}
I'_0 := LR(1)(\varepsilon) : & [S' \to \cdot S, \varepsilon] & [S \to \cdot L{=}R, \varepsilon] & [S \to \cdot R, \varepsilon] & [L \to \cdot {*}R, {=}] \\
 & [L \to \cdot \mathrm{a}, {=}] & [R \to \cdot L, \varepsilon] & [L \to \cdot {*}R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_1 := LR(1)(S) : & [S' \to S \cdot, \varepsilon] & & & \\
I'_2 := LR(1)(L) : & [S \to L \cdot {=}R, \varepsilon] & [R \to L \cdot, \varepsilon] & & \\
I'_3 := LR(1)(R) : & [S \to R \cdot, \varepsilon] & & & \\
I'_4 := LR(1)({*}) : & [L \to {*} \cdot R, {=}] & [L \to {*} \cdot R, \varepsilon] & [R \to \cdot L, {=}] & [R \to \cdot L, \varepsilon] \\
 & [L \to \cdot {*}R, {=}] & [L \to \cdot \mathrm{a}, {=}] & [L \to \cdot {*}R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_5 := LR(1)(\mathrm{a}) : & [L \to \mathrm{a} \cdot, {=}] & [L \to \mathrm{a} \cdot, \varepsilon] & & \\
I'_6 := LR(1)(L{=}) : & [S \to L{=} \cdot R, \varepsilon] & & &
\end{array}$$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x)$
$\implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$

$$\begin{array}{llllll}
I'_0 := LR(1)(\varepsilon): & [S' \to \cdot S, \varepsilon] & [S \to \cdot L{=}R, \varepsilon] & [S \to \cdot R, \varepsilon] & [L \to \cdot{*}R, {=}] \\
 & [L \to \cdot\mathrm{a}, {=}] & [R \to \cdot L, \varepsilon] & [L \to \cdot{*}R, \varepsilon] & [L \to \cdot\mathrm{a}, \varepsilon] \\
I'_1 := LR(1)(S): & [S' \to S\cdot, \varepsilon] & & & \\
I'_2 := LR(1)(L): & [S \to L \cdot {=}R, \varepsilon] & [R \to L\cdot, \varepsilon] & & \\
I'_3 := LR(1)(R): & [S \to R\cdot, \varepsilon] & & & \\
I'_4 := LR(1)(*): & [L \to * \cdot R, {=}] & [L \to * \cdot R, \varepsilon] & [R \to \cdot L, {=}] & [R \to \cdot L, \varepsilon] \\
 & [L \to \cdot{*}R, {=}] & [L \to \cdot\mathrm{a}, {=}] & [L \to \cdot{*}R, \varepsilon] & [L \to \cdot\mathrm{a}, \varepsilon] \\
I'_5 := LR(1)(\mathrm{a}): & [L \to \mathrm{a}\cdot, {=}] & [L \to \mathrm{a}\cdot, \varepsilon] & & \\
I'_6 := LR(1)(L{=}): & [S \to L{=} \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & &
\end{array}$$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L\texttt{=}R \mid R \quad L \to *R \mid \mathtt{a} \quad R \to L$

$$\begin{array}{ll} LR(1)(G_{LR}): & [A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x) \\ & \Longrightarrow [B \to \cdot\beta, y] \in LR(1)(\alpha Y) \end{array}$$

$$\begin{array}{lllll}
I'_0 := LR(1)(\varepsilon) : & [S' \to \cdot S, \varepsilon] & [S \to \cdot L\texttt{=}R, \varepsilon] & [S \to \cdot R, \varepsilon] & [L \to \cdot *R, \texttt{=}] \\
 & [L \to \cdot \mathtt{a}, \texttt{=}] & [R \to \cdot L, \varepsilon] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathtt{a}, \varepsilon] \\
I'_1 := LR(1)(S) : & [S' \to S \cdot, \varepsilon] & & & \\
I'_2 := LR(1)(L) : & [S \to L \cdot \texttt{=}R, \varepsilon] & [R \to L \cdot, \varepsilon] & & \\
I'_3 := LR(1)(R) : & [S \to R \cdot, \varepsilon] & & & \\
I'_4 := LR(1)(*) : & [L \to * \cdot R, \texttt{=}] & [L \to * \cdot R, \varepsilon] & [R \to \cdot L, \texttt{=}] & [R \to \cdot L, \varepsilon] \\
 & [L \to \cdot *R, \texttt{=}] & [L \to \cdot \mathtt{a}, \texttt{=}] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathtt{a}, \varepsilon] \\
I'_5 := LR(1)(\mathtt{a}) : & [L \to \mathtt{a} \cdot, \texttt{=}] & [L \to \mathtt{a} \cdot, \varepsilon] & & \\
I'_6 := LR(1)(L\texttt{=}) : & [S \to L\texttt{=} \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathtt{a}, \varepsilon]
\end{array}$$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L\texttt{=}R \mid R \quad L \to {*}R \mid \mathtt{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L\texttt{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot{*}R, \texttt{=}]$
$[L \to \cdot\mathtt{a}, \texttt{=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathtt{a}, \varepsilon]$
$I'_1 := LR(1)(S):$ $[S' \to S\cdot, \varepsilon]$
$I'_2 := LR(1)(L):$ $[S \to L \cdot \texttt{=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$
$I'_3 := LR(1)(R):$ $[S \to R\cdot, \varepsilon]$
$I'_4 := LR(1)({*}):$ $[L \to {*} \cdot R, \texttt{=}]$ $[L \to {*} \cdot R, \varepsilon]$ $[R \to \cdot L, \texttt{=}]$ $[R \to \cdot L, \varepsilon]$
$[L \to \cdot{*}R, \texttt{=}]$ $[L \to \cdot\mathtt{a}, \texttt{=}]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathtt{a}, \varepsilon]$
$I'_5 := LR(1)(\mathtt{a}):$ $[L \to \mathtt{a}\cdot, \texttt{=}]$ $[L \to \mathtt{a}\cdot, \varepsilon]$
$I'_6 := LR(1)(L\texttt{=}):$ $[S \to L\texttt{=} \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathtt{a}, \varepsilon]$
$I'_7 := LR(1)({*}R):$ $[L \to {*}R\cdot, \texttt{=}]$ $[L \to {*}R\cdot, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\Longrightarrow [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

| | | | | |
|---|---|---|---|---|
| $I'_0 := LR(1)(\varepsilon)$ : | $[S' \to \cdot S, \varepsilon]$ | $[S \to \cdot L{=}R, \varepsilon]$ | $[S \to \cdot R, \varepsilon]$ | $[L \to \cdot {*}R, {=}]$ |
| | $[L \to \cdot \mathrm{a}, {=}]$ | $[R \to \cdot L, \varepsilon]$ | $[L \to \cdot {*}R, \varepsilon]$ | $[L \to \cdot \mathrm{a}, \varepsilon]$ |
| $I'_1 := LR(1)(S)$ : | $[S' \to S \cdot, \varepsilon]$ | | | |
| $I'_2 := LR(1)(L)$ : | $[S \to L \cdot {=}R, \varepsilon]$ | $[R \to L \cdot, \varepsilon]$ | | |
| $I'_3 := LR(1)(R)$ : | $[S \to R \cdot, \varepsilon]$ | | | |
| $I'_4 := LR(1)({*})$ : | $[L \to {*} \cdot R, {=}]$ | $[L \to {*} \cdot R, \varepsilon]$ | $[R \to \cdot L, {=}]$ | $[R \to \cdot L, \varepsilon]$ |
| | $[L \to \cdot {*}R, {=}]$ | $[L \to \cdot \mathrm{a}, {=}]$ | $[L \to \cdot {*}R, \varepsilon]$ | $[L \to \cdot \mathrm{a}, \varepsilon]$ |
| $I'_5 := LR(1)(\mathrm{a})$ : | $[L \to \mathrm{a} \cdot, {=}]$ | $[L \to \mathrm{a} \cdot, \varepsilon]$ | | |
| $I'_6 := LR(1)(L{=})$ : | $[S \to L{=} \cdot R, \varepsilon]$ | $[R \to \cdot L, \varepsilon]$ | $[L \to \cdot {*}R, \varepsilon]$ | $[L \to \cdot \mathrm{a}, \varepsilon]$ |
| $I'_7 := LR(1)({*}R)$ : | $[L \to {*}R \cdot, {=}]$ | $[L \to {*}R \cdot, \varepsilon]$ | | |
| $I'_8 := LR(1)({*}L)$ : | $[R \to L \cdot, {=}]$ | $[R \to L \cdot, \varepsilon]$ | | |

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathtt{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot{*}R, {=}]$
$[L \to \cdot\mathtt{a}, {=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathtt{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L):$ $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R):$ $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*):$ $[L \to * \cdot R, {=}]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, {=}]$ $[R \to \cdot L, \varepsilon]$
$[L \to \cdot{*}R, {=}]$ $[L \to \cdot\mathtt{a}, {=}]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathtt{a}, \varepsilon]$

$I'_5 := LR(1)(\mathtt{a}):$ $[L \to \mathtt{a}\cdot, {=}]$ $[L \to \mathtt{a}\cdot, \varepsilon]$

$I'_6 := LR(1)(L{=}):$ $[S \to L{=} \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathtt{a}, \varepsilon]$

$I'_7 := LR(1)(*R):$ $[L \to {*}R\cdot, {=}]$ $[L \to {*}R\cdot, \varepsilon]$

$I'_8 := LR(1)(*L):$ $[R \to L\cdot, {=}]$ $[R \to L\cdot, \varepsilon]$

$I'_9 := LR(1)(L{=}R):$ $[S \to L{=}R\cdot, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon)$ : $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot{*}R, {=}]$
$[L \to \cdot\mathrm{a}, {=}]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S)$ : $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L)$ : $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R)$ : $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)({*})$ : $[L \to {*} \cdot R, {=}]$ $[L \to {*} \cdot R, \varepsilon]$ $[R \to \cdot L, {=}]$ $[R \to \cdot L, \varepsilon]$
$[L \to \cdot{*}R, {=}]$ $[L \to \cdot\mathrm{a}, {=}]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathrm{a}, \varepsilon]$

$I'_5 := LR(1)(\mathrm{a})$ : $[L \to \mathrm{a}\cdot, {=}]$ $[L \to \mathrm{a}\cdot, \varepsilon]$

$I'_6 := LR(1)(L{=})$ : $[S \to L{=} \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot{*}R, \varepsilon]$ $[L \to \cdot\mathrm{a}, \varepsilon]$

$I'_7 := LR(1)({*}R)$ : $[L \to {*}R\cdot, {=}]$ $[L \to {*}R\cdot, \varepsilon]$

$I'_8 := LR(1)({*}L)$ : $[R \to L\cdot, {=}]$ $[R \to L\cdot, \varepsilon]$

$I'_9 := LR(1)(L{=}R)$ : $[S \to L{=}R\cdot, \varepsilon]$

$I'_{10} := LR(1)(L{=}L)$ : $[R \to L\cdot, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathtt{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\Longrightarrow [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$
$[L \to \cdot \mathtt{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathtt{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S \cdot, \varepsilon]$

$I'_2 := LR(1)(L):$ $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L \cdot, \varepsilon]$

$I'_3 := LR(1)(R):$ $[S \to R \cdot, \varepsilon]$

$I'_4 := LR(1)(*):$ $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, =]$ $[R \to \cdot L, \varepsilon]$
$[L \to \cdot *R, =]$ $[L \to \cdot \mathtt{a}, =]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathtt{a}, \varepsilon]$

$I'_5 := LR(1)(\mathtt{a}):$ $[L \to \mathtt{a} \cdot, =]$ $[L \to \mathtt{a} \cdot, \varepsilon]$

$I'_6 := LR(1)(L{=}):$ $[S \to L{=} \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathtt{a}, \varepsilon]$

$I'_7 := LR(1)(*R):$ $[L \to *R \cdot, =]$ $[L \to *R \cdot, \varepsilon]$

$I'_8 := LR(1)(*L):$ $[R \to L \cdot, =]$ $[R \to L \cdot, \varepsilon]$

$I'_9 := LR(1)(L{=}R):$ $[S \to L{=}R \cdot, \varepsilon]$

$I'_{10} := LR(1)(L{=}L):$ $[R \to L \cdot, \varepsilon]$

$I'_{11} := LR(1)(L{=}*):$ $[L \to * \cdot R, \varepsilon]$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x)$
$\implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$

$$
\begin{array}{llllll}
I'_0 := LR(1)(\varepsilon): & [S' \to \cdot S, \varepsilon] & [S \to \cdot L{=}R, \varepsilon] & [S \to \cdot R, \varepsilon] & [L \to \cdot *R, {=}] \\
 & [L \to \cdot \mathrm{a}, {=}] & [R \to \cdot L, \varepsilon] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_1 := LR(1)(S): & [S' \to S\cdot, \varepsilon] & & & \\
I'_2 := LR(1)(L): & [S \to L \cdot {=}R, \varepsilon] & [R \to L\cdot, \varepsilon] & & \\
I'_3 := LR(1)(R): & [S \to R\cdot, \varepsilon] & & & \\
I'_4 := LR(1)(*): & [L \to * \cdot R, {=}] & [L \to * \cdot R, \varepsilon] & [R \to \cdot L, {=}] & [R \to \cdot L, \varepsilon] \\
 & [L \to \cdot *R, {=}] & [L \to \cdot \mathrm{a}, {=}] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_5 := LR(1)(\mathrm{a}): & [L \to \mathrm{a}\cdot, {=}] & [L \to \mathrm{a}\cdot, \varepsilon] & & \\
I'_6 := LR(1)(L{=}): & [S \to L{=} \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & [L \to \cdot *R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_7 := LR(1)(*R): & [L \to *R\cdot, {=}] & [L \to *R\cdot, \varepsilon] & & \\
I'_8 := LR(1)(*L): & [R \to L\cdot, {=}] & [R \to L\cdot, \varepsilon] & & \\
I'_9 := LR(1)(L{=}R): & [S \to L{=}R\cdot, \varepsilon] & & & \\
I'_{10} := LR(1)(L{=}L): & [R \to L\cdot, \varepsilon] & & & \\
I'_{11} := LR(1)(L{=}*): & [L \to * \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & &
\end{array}
$$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L{=}R \mid R \quad L \to {*}R \mid \mathrm{a} \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \to \beta \in P, y \in \mathrm{fi}(\gamma_2 x)$

$\implies [B \to \cdot\beta, y] \in LR(1)(\alpha Y)$

$$\begin{array}{lllll}
I'_0 := LR(1)(\varepsilon) : & [S' \to \cdot S, \varepsilon] & [S \to \cdot L{=}R, \varepsilon] & [S \to \cdot R, \varepsilon] & [L \to \cdot {*}R, {=}] \\
 & [L \to \cdot \mathrm{a}, {=}] & [R \to \cdot L, \varepsilon] & [L \to \cdot {*}R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_1 := LR(1)(S) : & [S' \to S \cdot, \varepsilon] & & & \\
I'_2 := LR(1)(L) : & [S \to L \cdot {=}R, \varepsilon] & [R \to L \cdot, \varepsilon] & & \\
I'_3 := LR(1)(R) : & [S \to R \cdot, \varepsilon] & & & \\
I'_4 := LR(1)({*}) : & [L \to {*} \cdot R, {=}] & [L \to {*} \cdot R, \varepsilon] & [R \to \cdot L, {=}] & [R \to \cdot L, \varepsilon] \\
 & [L \to \cdot {*}R, {=}] & [L \to \cdot \mathrm{a}, {=}] & [L \to \cdot {*}R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_5 := LR(1)(\mathrm{a}) : & [L \to \mathrm{a} \cdot, {=}] & [L \to \mathrm{a} \cdot, \varepsilon] & & \\
I'_6 := LR(1)(L{=}) : & [S \to L{=} \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & [L \to \cdot {*}R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon] \\
I'_7 := LR(1)({*}R) : & [L \to {*}R \cdot, {=}] & [L \to {*}R \cdot, \varepsilon] & & \\
I'_8 := LR(1)({*}L) : & [R \to L \cdot, {=}] & [R \to L \cdot, \varepsilon] & & \\
I'_9 := LR(1)(L{=}R) : & [S \to L{=}R \cdot, \varepsilon] & & & \\
I'_{10} := LR(1)(L{=}L) : & [R \to L \cdot, \varepsilon] & & & \\
I'_{11} := LR(1)(L{=}{*}) : & [L \to {*} \cdot R, \varepsilon] & [R \to \cdot L, \varepsilon] & [L \to \cdot {*}R, \varepsilon] & [L \to \cdot \mathrm{a}, \varepsilon]
\end{array}$$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid a \quad R \to L$

$LR(1)(G_{LR})$: $[A \to \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\implies [A \to \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

| | | | | |
|---|---|---|---|---|
| $I'_0 := LR(1)(\varepsilon):$ | $[S' \to \cdot S, \varepsilon]$ | $[S \to \cdot L{=}R, \varepsilon]$ | $[S \to \cdot R, \varepsilon]$ | $[L \to \cdot *R, =]$ |
| | $[L \to \cdot a, =]$ | $[R \to \cdot L, \varepsilon]$ | $[L \to \cdot *R, \varepsilon]$ | $[L \to \cdot a, \varepsilon]$ |
| $I'_1 := LR(1)(S):$ | $[S' \to S \cdot, \varepsilon]$ | | | |
| $I'_2 := LR(1)(L):$ | $[S \to L \cdot {=}R, \varepsilon]$ | $[R \to L \cdot, \varepsilon]$ | | |
| $I'_3 := LR(1)(R):$ | $[S \to R \cdot, \varepsilon]$ | | | |
| $I'_4 := LR(1)(*):$ | $[L \to * \cdot R, =]$ | $[L \to * \cdot R, \varepsilon]$ | $[R \to \cdot L, =]$ | $[R \to \cdot L, \varepsilon]$ |
| | $[L \to \cdot *R, =]$ | $[L \to \cdot a, =]$ | $[L \to \cdot *R, \varepsilon]$ | $[L \to \cdot a, \varepsilon]$ |
| $I'_5 := LR(1)(a):$ | $[L \to a \cdot, =]$ | $[L \to a \cdot, \varepsilon]$ | | |
| $I'_6 := LR(1)(L{=}):$ | $[S \to L{=} \cdot R, \varepsilon]$ | $[R \to \cdot L, \varepsilon]$ | $[L \to \cdot *R, \varepsilon]$ | $[L \to \cdot a, \varepsilon]$ |
| $I'_7 := LR(1)(*R):$ | $[L \to *R \cdot, =]$ | $[L \to *R \cdot, \varepsilon]$ | | |
| $I'_8 := LR(1)(*L):$ | $[R \to L \cdot, =]$ | $[R \to L \cdot, \varepsilon]$ | | |
| $I'_9 := LR(1)(L{=}R):$ | $[S \to L{=}R \cdot, \varepsilon]$ | | | |
| $I'_{10} := LR(1)(L{=}L):$ | $[R \to L \cdot, \varepsilon]$ | | | |
| $I'_{11} := LR(1)(L{=}*):$ | $[L \to * \cdot R, \varepsilon]$ | $[R \to \cdot L, \varepsilon]$ | $[L \to \cdot *R, \varepsilon]$ | $[L \to \cdot a, \varepsilon]$ |
| $I'_{12} := LR(1)(L{=}a):$ | $[L \to a \cdot, \varepsilon]$ | | | |

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \rightarrow S \quad S \rightarrow L{=}R \mid R \quad L \rightarrow *R \mid \mathrm{a} \quad R \rightarrow L$

$LR(1)(G_{LR})$: $[A \rightarrow \gamma_1 \cdot Y\gamma_2, x] \in LR(1)(\alpha)$
$\implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

| | | | | |
|---|---|---|---|---|
| $I'_0 := LR(1)(\varepsilon)$ : | $[S' \rightarrow \cdot S, \varepsilon]$ | $[S \rightarrow \cdot L{=}R, \varepsilon]$ | $[S \rightarrow \cdot R, \varepsilon]$ | $[L \rightarrow \cdot *R, =]$ |
| | $[L \rightarrow \cdot \mathrm{a}, =]$ | $[R \rightarrow \cdot L, \varepsilon]$ | $[L \rightarrow \cdot *R, \varepsilon]$ | $[L \rightarrow \cdot \mathrm{a}, \varepsilon]$ |
| $I'_1 := LR(1)(S)$ : | $[S' \rightarrow S \cdot, \varepsilon]$ | | | |
| $I'_2 := LR(1)(L)$ : | $[S \rightarrow L \cdot {=}R, \varepsilon]$ | $[R \rightarrow L \cdot, \varepsilon]$ | | |
| $I'_3 := LR(1)(R)$ : | $[S \rightarrow R \cdot, \varepsilon]$ | | | |
| $I'_4 := LR(1)(*)$ : | $[L \rightarrow * \cdot R, =]$ | $[L \rightarrow * \cdot R, \varepsilon]$ | $[R \rightarrow \cdot L, =]$ | $[R \rightarrow \cdot L, \varepsilon]$ |
| | $[L \rightarrow \cdot *R, =]$ | $[L \rightarrow \cdot \mathrm{a}, =]$ | $[L \rightarrow \cdot *R, \varepsilon]$ | $[L \rightarrow \cdot \mathrm{a}, \varepsilon]$ |
| $I'_5 := LR(1)(\mathrm{a})$ : | $[L \rightarrow \mathrm{a} \cdot, =]$ | $[L \rightarrow \mathrm{a} \cdot, \varepsilon]$ | | |
| $I'_6 := LR(1)(L{=})$ : | $[S \rightarrow L{=} \cdot R, \varepsilon]$ | $[R \rightarrow \cdot L, \varepsilon]$ | $[L \rightarrow \cdot *R, \varepsilon]$ | $[L \rightarrow \cdot \mathrm{a}, \varepsilon]$ |
| $I'_7 := LR(1)(*R)$ : | $[L \rightarrow *R \cdot, =]$ | $[L \rightarrow *R \cdot, \varepsilon]$ | | |
| $I'_8 := LR(1)(*L)$ : | $[R \rightarrow L \cdot, =]$ | $[R \rightarrow L \cdot, \varepsilon]$ | | |
| $I'_9 := LR(1)(L{=}R)$ : | $[S \rightarrow L{=}R \cdot, \varepsilon]$ | | | |
| $I'_{10} := LR(1)(L{=}L)$ : | $[R \rightarrow L \cdot, \varepsilon]$ | | | |
| $I'_{11} := LR(1)(L{=}*)$ : | $[L \rightarrow * \cdot R, \varepsilon]$ | $[R \rightarrow \cdot L, \varepsilon]$ | $[L \rightarrow \cdot *R, \varepsilon]$ | $[L \rightarrow \cdot \mathrm{a}, \varepsilon]$ |
| $I'_{12} := LR(1)(L{=}\mathrm{a})$ : | $[L \rightarrow \mathrm{a} \cdot, \varepsilon]$ | | | |
| $I'_{13} := LR(1)(L{=}*R)$ : | $[L \rightarrow *R \cdot, \varepsilon]$ | | | |

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR}: S' \to S \quad S \to L{=}R \mid R \quad L \to *R \mid \mathrm{a} \quad R \to L$

$LR(1)(G_{LR})$:

$I'_0 := LR(1)(\varepsilon):$ $[S' \to \cdot S, \varepsilon]$ $[S \to \cdot L{=}R, \varepsilon]$ $[S \to \cdot R, \varepsilon]$ $[L \to \cdot *R, =]$
$[L \to \cdot \mathrm{a}, =]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_1 := LR(1)(S):$ $[S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L):$ $[S \to L \cdot {=}R, \varepsilon]$ $[R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R):$ $[S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*):$ $[L \to * \cdot R, =]$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, =]$ $[R \to \cdot L, \varepsilon]$
$[L \to \cdot *R, =]$ $[L \to \cdot \mathrm{a}, =]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_5 := LR(1)(\mathrm{a}):$ $[L \to \mathrm{a}\cdot, =]$ $[L \to \mathrm{a}\cdot, \varepsilon]$

$I'_6 := LR(1)(L{=}):$ $[S \to L{=} \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_7 := LR(1)(*R):$ $[L \to *R\cdot, =]$ $[L \to *R\cdot, \varepsilon]$

$I'_8 := LR(1)(*L):$ $[R \to L\cdot, =]$ $[R \to L\cdot, \varepsilon]$

$I'_9 := LR(1)(L{=}R):$ $[S \to L{=}R\cdot, \varepsilon]$

$I'_{10} := LR(1)(L{=}L):$ $[R \to L\cdot, \varepsilon]$

$I'_{11} := LR(1)(L{=}*):$ $[L \to * \cdot R, \varepsilon]$ $[R \to \cdot L, \varepsilon]$ $[L \to \cdot *R, \varepsilon]$ $[L \to \cdot \mathrm{a}, \varepsilon]$

$I'_{12} := LR(1)(L{=}\mathrm{a}):$ $[L \to \mathrm{a}\cdot, \varepsilon]$

$I'_{13} := LR(1)(L{=}*R):$ $[L \to *R\cdot, \varepsilon]$

$I'_{14} := \emptyset$

# Computing $LR(1)$ Sets II

## Example 10.17 (cf. Example 10.11)

$G_{LR} : S' \to S \quad S \to L\texttt{=}R \mid R \quad L \to *R \mid \texttt{a} \quad R \to L$

$LR(1)(G_{LR})$:

$I'_0 := LR(1)(\varepsilon) : \quad [S' \to \cdot S, \varepsilon] \quad [S \to \cdot L\texttt{=}R, \varepsilon] \quad [S \to \cdot R, \varepsilon] \quad [L \to \cdot *R, \texttt{=}]$
$\quad [L \to \cdot \texttt{a}, \texttt{=}] \quad [R \to \cdot L, \varepsilon] \quad [L \to \cdot *R, \varepsilon] \quad [L \to \cdot \texttt{a}, \varepsilon]$

$I'_1 := LR(1)(S) : \quad [S' \to S\cdot, \varepsilon]$

$I'_2 := LR(1)(L) : \quad [S \to L \cdot \texttt{=}R, \varepsilon] \quad [R \to L\cdot, \varepsilon]$

$I'_3 := LR(1)(R) : \quad [S \to R\cdot, \varepsilon]$

$I'_4 := LR(1)(*) : \quad [L \to * \cdot R, \texttt{=}] \quad [L \to * \cdot R, \varepsilon] \quad [R \to \cdot L, \texttt{=}] \quad [R \to \cdot L, \varepsilon]$
$\quad [L \to \cdot *R, \texttt{=}] \quad [L \to \cdot \texttt{a}, \texttt{=}] \quad [L \to \cdot *R, \varepsilon] \quad [L \to \cdot \texttt{a}, \varepsilon]$

$I'_5 := LR(1)(\texttt{a}) : \quad [L \to \texttt{a}\cdot, \texttt{=}] \quad [L \to \texttt{a}\cdot, \varepsilon]$

$I'_6 := LR(1)(L\texttt{=}) : \quad [S \to L\texttt{=} \cdot R, \varepsilon] \quad [R \to \cdot L, \varepsilon] \quad [L \to \cdot *R, \varepsilon] \quad [L \to \cdot \texttt{a}, \varepsilon]$

$I'_7 := LR(1)(*R) : \quad [L \to *R\cdot, \texttt{=}] \quad [L \to *R\cdot, \varepsilon]$

$I'_8 := LR(1)(*L) : \quad [R \to L\cdot, \texttt{=}] \quad [R \to L\cdot, \varepsilon]$

$I'_9 := LR(1)(L\texttt{=}R) : \quad [S \to L\texttt{=}R\cdot, \varepsilon]$

$I'_{10} := LR(1)(L\texttt{=}L) : \quad [R \to L\cdot, \varepsilon]$

$I'_{11} := LR(1)(L\texttt{=}*) : \quad [L \to * \cdot R, \varepsilon] \quad [R \to \cdot L, \varepsilon] \quad [L \to \cdot *R, \varepsilon] \quad [L \to \cdot \texttt{a}, \varepsilon]$

$I'_{12} := LR(1)(L\texttt{=}\texttt{a}) : \quad [L \to \texttt{a}\cdot, \varepsilon]$

$I'_{13} := LR(1)(L\texttt{=}*R) : \quad [L \to *R\cdot, \varepsilon]$

$I'_{14} := \emptyset$

In $I'_2$: shift on =/reduce on $\varepsilon \implies G_{LR} \in LR(1)$

# The $LR(1)$ Action Function

### Definition 10.18 ($LR(1)$ action function)

The $LR(1)$ action function

$$\mathrm{act} : LR(1)(G) \times \Sigma_\varepsilon \to \{\mathsf{red}\ i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \mathsf{red}\ i & \text{if } i \neq 0, \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot, x] \in I \\ \mathsf{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2, y] \in I \text{ and } x \in \Sigma \\ \mathsf{accept} & \text{if } [S' \to S\cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \mathsf{error} & \text{otherwise} \end{cases}$$

# The $LR(1)$ Action Function

## Definition 10.18 ($LR(1)$ action function)

The $LR(1)$ action function

$$\mathrm{act} : LR(1)(G) \times \Sigma_\varepsilon \to \{\mathsf{red}\ i \mid i \in [p]\} \cup \{\mathsf{shift}, \mathsf{accept}, \mathsf{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \mathsf{red}\ i & \text{if } i \neq 0, \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot, x] \in I \\ \mathsf{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2, y] \in I \text{ and } x \in \Sigma \\ \mathsf{accept} & \text{if } [S' \to S\cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \mathsf{error} & \text{otherwise} \end{cases}$$

## Corollary 10.19

*For every $G \in CFG_\Sigma$, $G \in LR(1)$ iff its $LR(1)$ action function is well defined.*

# The $LR(1)$ $\mathrm{goto}$ Function

The $\mathrm{goto}$ function is defined in analogy to the $LR(0)$ case (Definition 9.14).

**Definition 10.20 ($LR(1)$ $\mathrm{goto}$ function)**

The function $\mathrm{goto} : LR(1)(G) \times X \to LR(1)(G)$ is determined by

$$\mathrm{goto}(I, Y) = I' \quad \text{iff} \quad \begin{array}{l} \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y). \end{array}$$

# The $LR(1)$ $\mathrm{goto}$ Function

The $\mathrm{goto}$ function is defined in analogy to the $LR(0)$ case (Definition 9.14).

**Definition 10.20 ($LR(1)$ $\mathrm{goto}$ function)**

The function $\mathrm{goto} : LR(1)(G) \times X \to LR(1)(G)$ is determined by

$$\mathrm{goto}(I, Y) = I' \quad \text{iff} \quad \begin{array}{l}\text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y).\end{array}$$

Again, $\mathrm{act}$ and $\mathrm{goto}$ form the $LR(1)$ parsing table of $G$.

# The $LR(1)$ Parsing Table

## Example 10.21 (cf. Example 10.17)

| $LR(1)(G_{LR})$ | $\mathrm{act/goto}\vert_\Sigma$ | | | | $\mathrm{goto}\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | * | = | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty = error/$\emptyset$)

# The $LR(1)$ Parsing Automaton I

## Definition 10.22 ($LR(1)$ parsing automaton)

The $LR(1)$ parsing automaton is defined as in the $LR(0)$ case (see Definition 10.2), except for the transition relation:

- shift: $(aw, \alpha I, z) \vdash (w, \alpha I I', z)$ if $\mathrm{act}(I, a) = \mathsf{shift}$ and $\mathrm{goto}(I, a) = I'$
- reduce$_a$: $(aw, \alpha I I_1 \ldots I_n, z) \vdash (aw, \alpha I I', zi)$ if $\mathrm{act}(I_n, a) = \mathsf{red}\ i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$
- reduce$_\varepsilon$: $(\varepsilon, \alpha I I_1 \ldots I_n, z) \vdash (\varepsilon, \alpha I I', zi)$ if $\mathrm{act}(I_n, \varepsilon) = \mathsf{red}\ i$, $\pi_i = A \to Y_1 \ldots Y_n$, and $\mathrm{goto}(I, A) = I'$
- accept: $(\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\, 0)$ if $\mathrm{act}(I, \varepsilon) = \mathsf{accept}$
- error$_a$: $(aw, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\, \mathsf{error})$ if $\mathrm{act}(I, a) = \mathsf{error}$
- error$_\varepsilon$: $(\varepsilon, \alpha I, z) \vdash (\varepsilon, \varepsilon, z\, \mathsf{error})$ if $\mathrm{act}(I, \varepsilon) = \mathsf{error}$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to *R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty = error/$\emptyset$)

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I_0'$ | shift/$I_4'$ | | shift/$I_5'$ | | $I_1'$ | $I_2'$ | $I_3'$ |
| $I_1'$ | | | | accept | | | |
| $I_2'$ | | shift/$I_6'$ | | red 5 | | | |
| $I_3'$ | | | | red 2 | | | |
| $I_4'$ | shift/$I_4'$ | | shift/$I_5'$ | | | $I_8'$ | $I_7'$ |
| $I_5'$ | | red 4 | | red 4 | | | |
| $I_6'$ | shift/$I_{11}'$ | | shift/$I_{12}'$ | | | $I_{10}'$ | $I_9'$ |
| $I_7'$ | | red 3 | | red 3 | | | |
| $I_8'$ | | red 5 | | | | | |
| $I_9'$ | | | | red 1 | | | |
| $I_{10}'$ | | | | red 5 | | | |
| $I_{11}'$ | shift/$I_{11}'$ | | shift/$I_{12}'$ | | | $I_{10}'$ | $I_{13}'$ |
| $I_{12}'$ | | | | red 4 | | | |
| $I_{13}'$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of a=*a:

(a=*a, $I_0'$ , $\varepsilon$ )

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \to S \ (0) \quad S \to L{=}R \mid R \ (1,2) \quad L \to *R \mid \mathtt{a} \ (3,4) \quad R \to L \ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | = | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$(\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon)$

$\vdash (\ \mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon)$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \to S \ (0) \quad S \to L\texttt{=}R \mid R \ (1,2) \quad L \to \texttt{*}R \mid \texttt{a} \ (3,4) \quad R \to L \ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $\texttt{*}$ | $\texttt{=}$ | $\texttt{a}$ | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of $\texttt{a=*a}$:

$$
\begin{aligned}
& (\texttt{a=*a},\ I'_0 &&, \varepsilon &&) \\
\vdash & (\ \texttt{=*a},\ I'_0 I'_5 &&, \varepsilon &&) \\
\vdash & (\ \texttt{=*a},\ I'_0 I'_2 &&, 4 &&)
\end{aligned}
$$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \to S\ (0) \quad S \to L\texttt{=}R \mid R\ (1,2) \quad L \to {*}R \mid \texttt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | = | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty = error/$\emptyset$)

$LR(1)$ parsing of `a=*a`:

$(\texttt{a=*a},\ I'_0,\ \varepsilon)$
$\vdash (\texttt{=*a},\ I'_0 I'_5,\ \varepsilon)$
$\vdash (\texttt{=*a},\ I'_0 I'_2,\ 4)$
$\vdash (\texttt{*a},\ I'_0 I'_2 I'_6,\ 4)$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$(\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon)$

$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon)$

$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4)$

$\vdash (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4)$

$\vdash (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4)$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \texttt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I_0'$ | shift/$I_4'$ | | shift/$I_5'$ | | $I_1'$ | $I_2'$ | $I_3'$ |
| $I_1'$ | | | | accept | | | |
| $I_2'$ | | shift/$I_6'$ | | red 5 | | | |
| $I_3'$ | | | | red 2 | | | |
| $I_4'$ | shift/$I_4'$ | | shift/$I_5'$ | | | $I_8'$ | $I_7'$ |
| $I_5'$ | | red 4 | | red 4 | | | |
| $I_6'$ | shift/$I_{11}'$ | | shift/$I_{12}'$ | | | $I_{10}'$ | $I_9'$ |
| $I_7'$ | | red 3 | | red 3 | | | |
| $I_8'$ | | red 5 | | | | | |
| $I_9'$ | | | | red 1 | | | |
| $I_{10}'$ | | | | red 5 | | | |
| $I_{11}'$ | shift/$I_{11}'$ | | shift/$I_{12}'$ | | | $I_{10}'$ | $I_{13}'$ |
| $I_{12}'$ | | | | red 4 | | | |
| $I_{13}'$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$(\texttt{a=*a},\ I_0',\ \varepsilon)$
$\vdash (\texttt{=*a},\ I_0'I_5',\ \varepsilon)$
$\vdash (\texttt{=*a},\ I_0'I_2',\ 4)$
$\vdash (\texttt{*a},\ I_0'I_2'I_6',\ 4)$
$\vdash (\texttt{a},\ I_0'I_2'I_6'I_{11}',\ 4)$
$\vdash (\varepsilon,\ I_0'I_2'I_6'I_{11}'I_{12}',\ 4)$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR}$ : $S' \to S$ (0) $S \to L{=}R \mid R$ (1, 2) $L \to {*}R \mid \mathtt{a}$ (3, 4) $R \to L$ (5)

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$\begin{array}{ll}
 & (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44)
\end{array}$$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty $=$ error$/\emptyset$)

$LR(1)$ parsing of a=*a:

$$\begin{array}{lrll}
 & (\mathtt{a{=}{*}a}, & I'_0 & , \varepsilon) \\
\vdash & (\mathtt{{=}{*}a}, & I'_0 I'_5 & , \varepsilon) \\
\vdash & (\mathtt{{=}{*}a}, & I'_0 I'_2 & , 4) \\
\vdash & (\mathtt{{*}a}, & I'_0 I'_2 I'_6 & , 4) \\
\vdash & (\mathtt{a}, & I'_0 I'_2 I'_6 I'_{11} & , 4) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_{11} I'_{12} & , 4) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_{11} I'_{10} & , 44) \\
\vdash & (\varepsilon, & I'_0 I'_2 I'_6 I'_{11} I'_{13} & , 445)
\end{array}$$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty = error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$$
\begin{array}{ll}
 & (\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon) \\
\vdash & (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4) \\
\vdash & (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4) \\
\vdash & (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445) \\
\vdash & (\varepsilon,\ I'_0 I'_2 I'_6 I'_{10},\ 4453)
\end{array}
$$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR}: S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to *R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of a=*a:

$(\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon)$
$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon)$
$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4)$
$\vdash (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4)$
$\vdash (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{10},\ 4453)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_9,\ 44535)$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \to S \ (0) \quad S \to L\texttt{=}R \mid R \ (1,2) \quad L \to \texttt{*}R \mid \texttt{a} \ (3,4) \quad R \to L \ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $\texttt{*}$ | $\texttt{=}$ | $\texttt{a}$ | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift$/I'_4$ | | shift$/I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift$/I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift$/I'_4$ | | shift$/I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift$/I'_{11}$ | | shift$/I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift$/I'_{11}$ | | shift$/I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty = error$/\emptyset$)

$LR(1)$ parsing of $\texttt{a=*a}$:

$(\texttt{a=*a},\ I'_0,\ \varepsilon)$

$\vdash (\texttt{=*a},\ I'_0 I'_5,\ \varepsilon)$

$\vdash (\texttt{=*a},\ I'_0 I'_2,\ 4)$

$\vdash (\texttt{*a},\ I'_0 I'_2 I'_6,\ 4)$

$\vdash (\texttt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4)$

$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4)$

$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44)$

$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445)$

$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{10},\ 4453)$

$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_9,\ 44535)$

$\vdash (\varepsilon,\ I'_0 I'_1,\ 445351)$

# The $LR(1)$ Parsing Automaton II

## Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \to S\ (0) \quad S \to L{=}R \mid R\ (1,2) \quad L \to {*}R \mid \mathtt{a}\ (3,4) \quad R \to L\ (5)$

| $LR(1)(G_{LR})$ | act/goto$\vert_\Sigma$ | | | | goto$\vert_N$ | | |
|---|---|---|---|---|---|---|---|
| | $*$ | $=$ | a | $\varepsilon$ | $S$ | $L$ | $R$ |
| $I'_0$ | shift/$I'_4$ | | shift/$I'_5$ | | $I'_1$ | $I'_2$ | $I'_3$ |
| $I'_1$ | | | | accept | | | |
| $I'_2$ | | shift/$I'_6$ | | red 5 | | | |
| $I'_3$ | | | | red 2 | | | |
| $I'_4$ | shift/$I'_4$ | | shift/$I'_5$ | | | $I'_8$ | $I'_7$ |
| $I'_5$ | | red 4 | | red 4 | | | |
| $I'_6$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_9$ |
| $I'_7$ | | red 3 | | red 3 | | | |
| $I'_8$ | | red 5 | | | | | |
| $I'_9$ | | | | red 1 | | | |
| $I'_{10}$ | | | | red 5 | | | |
| $I'_{11}$ | shift/$I'_{11}$ | | shift/$I'_{12}$ | | | $I'_{10}$ | $I'_{13}$ |
| $I'_{12}$ | | | | red 4 | | | |
| $I'_{13}$ | | | | red 3 | | | |

(empty $=$ error/$\emptyset$)

$LR(1)$ parsing of a=∗a:

$(\mathtt{a{=}{*}a},\ I'_0,\ \varepsilon)$
$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_5,\ \varepsilon)$
$\vdash (\mathtt{{=}{*}a},\ I'_0 I'_2,\ 4)$
$\vdash (\mathtt{{*}a},\ I'_0 I'_2 I'_6,\ 4)$
$\vdash (\mathtt{a},\ I'_0 I'_2 I'_6 I'_{11},\ 4)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{12},\ 4)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{10},\ 44)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{11} I'_{13},\ 445)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_{10},\ 4453)$
$\vdash (\varepsilon,\ I'_0 I'_2 I'_6 I'_9,\ 44535)$
$\vdash (\varepsilon,\ I'_0 I'_1,\ 445351)$
$\vdash (\varepsilon,\ \varepsilon,\ 4453510)$