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## Exercise 1 (Characterisation of LL$(1)$): (3 Points)

In the lecture two characterizations of $LL(1)$ have been given:

- $G \in LL(1)$ iff for all leftmost derivations of the form

$$S \Rightarrow_l^* wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \\ \Rightarrow_l w\gamma\alpha \end{cases}$$

  such that $\beta \neq \gamma$, it follows that $\mathrm{fi}(\beta\alpha) \cap \mathrm{fi}(\gamma\alpha) = \emptyset$.

- $G \in LL(1)$ iff for all pairs of rules $A \to \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$\mathrm{la}(A \to \beta) \cap \mathrm{la}(A \to \gamma) = \emptyset$$

a) Lift the second definition to $LL(k)$ for $k \in \mathbb{N}^+$. (The first definition was given for $k \in \mathbb{N}^+$ in the lecture.)

b) Show that the definitions are not equivalent by showing that the following grammar is in $LL(2)$ according to the first definition but not according to the second definition (also refered to as *strong* $LL(2)$ property).

$$\begin{array}{lcl} S & \to & aAab \mid bAbb \\ A & \to & a \mid \varepsilon \end{array}$$

c) Explain (in a few words) why the definitions are not equivalent.

## Exercise 2 (Find an equivalent LL$(1)$ grammar): (3 Points)

Consider the following grammar $G$:

$$\begin{array}{lcl} S & \to & (L) \mid a \\ L & \to & L, S \mid L, SS \mid S \mid SS \end{array}$$

**a)** Show that $G$ is not an $LL(1)$ grammar.

**b)** Transform $G$ into an equivalent grammar satisfying the $LL(1)$ property.

**c)** Prove that $G$ has the $LL(1)$ property.

## Exercise 3 (LL$(1)$ grammars are never ambiguous): (1 Points)

Show that for every context-free grammar $G$ the following holds:

$$G \text{ ambiguous } \Rightarrow G \notin LL(1)$$

## Exercise 4 (Deterministic Top-Down Automaton): (3 Points)

Consider the grammar $G = (N, \Sigma, P, start)$ covering some boolean expressions:

- $N := \{start, guard, rel\}$
- $\Sigma := \{AND, OR, ID, EQ, LEQ\}$

- $$\begin{aligned} start &\rightarrow guard \\ guard &\rightarrow rel \mid guard\ AND\ guard \mid guard\ OR\ guard \\ rel &\rightarrow ID\ EQ\ ID \mid ID\ LEQ\ ID \end{aligned}$$

**a)** Construct $NTA(G)$. (Either give a transition table or depict the automaton and specify what the edge labelling means. Do not forget to give a numbering to the grammar rules.)

**b)** Provide a run of $NTA(G)$ on the input $ID\ EQ\ ID\ AND\ ID\ LEQ\ ID$.

**c)** Construct an equivalent grammar $G'$ with $G' \in LL(1)$.

**d)** Specify the deterministic top-down parsing automaton of $DTA(G')$. (Again, either give a transition table as in the lecture or depict the automaton and specify what the edge labelling means. As before, do not forget to give a numbering to the grammar rules of $G'$.)

**e)** Provide a run of $DTA(G')$ on the input $ID\ EQ\ ID\ AND\ ID\ LEQ\ ID$.