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Exercise 1 (Yes/No): (0 Points)

1. Deterministic finite automata (DFA) are strictly less expressive than regular expressions.
2. Non-deterministic finite automata (NFA) are strictly more expressive than deterministic finite automata.
3. The languages of regular expressions are closed under...
   a) union,
   b) intersection,
   c) complement,
   d) concatenation,
   e) Kleene closure.
4. Context Free Languages (CFL) are closed under...
   a) union,
   b) intersection,
   c) complement,
   d) concatenation,
   e) Kleene closure.
5. DCFL is the set of context free languages that are accepted by deterministic push down automata. Is DCFL = CFL?

Exercise 2 (Regular Expressions and Grammars): (0 Points)

1. Describe the language of the following regular expression in words:

$$r = (0 + 1)^*0(0 + 1)^*0(0 + 1)^*.$$

2. Construct the regular expression for...
   a) the set of all strings with at most one pair of consecutive 0's and at the most one pair of consecutive 1's,
   b) the set of all strings with equal number of 0s and 1s such that no prefix has two more 0s than 1s nor two more 1s than 0s.
3. Construct a context free grammar (CFG) for a set of strings of $\{(,)\}^*$ such that every string of the set has equal number of left and right parenthesis, and every prefix has at least as many left parenthesis as right parenthesis.

Exercise 3 (Proofs): (0 Points)

1. Let r and s be regular expressions. Consider the equation $X = r.X + s$. Under the assumption that the language of r ($L(r)$) does not contain ϵ, find X and show that it is unique. What happens if $\epsilon \in L(r)$?

2. One can construct a regular expression form a finite automaton by solving a system of linear equation of the form

$$\begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix} = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ . & \dots & . \\ . & \dots & . \\ a_{n1} & \dots & a_{nn} \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{pmatrix}$$

   Given an NFA provide suitable definition of the vectors above. It might be helpful to assume an annihilator ε for regular expression, i.e., $\alpha.\varepsilon = \varepsilon, \varepsilon.\alpha = \varepsilon$ and $\varepsilon + \alpha = \alpha$

3. a) An equivalence relation $R \subseteq \Sigma^* \times \Sigma^*$ is said to be *right invariant* when,

$$\forall z \in \Sigma^* \quad (x, y) \in R \iff (xz, yz) \in R \ .$$

   Show that if $L \subseteq \Sigma^*$ is regular then it is the union of some of the equivalence classes of R.

   b) Let R_L be an relation defined as: $(x, y) \in R_L$ if and only if for all $z \in \Sigma^*$ $xz \in L$ exactly when $yz \in L$. Show that R_L is an equivalence relation. Show that number of equivalence classes of R_L is finite if and only if L is regular.

4. a) Show that the language $L = \{0^{i^2} \mid i \in \mathbb{N}\}$ is not regular.

   b) Show that the language $L = \{a^i b^i c^i \mid i \in \mathbb{N}\}$ is not a CFL.