Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

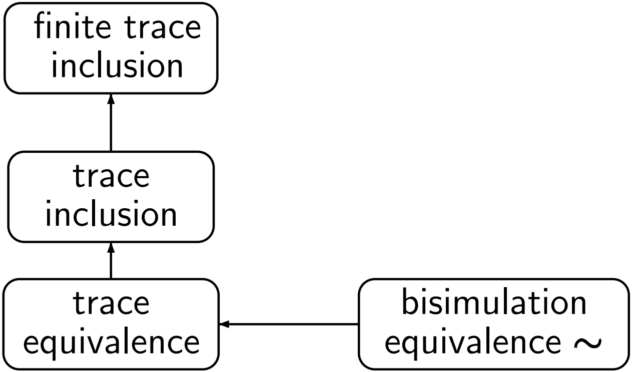
Linear Temporal Logic (LTL)

Computation-Tree Logic

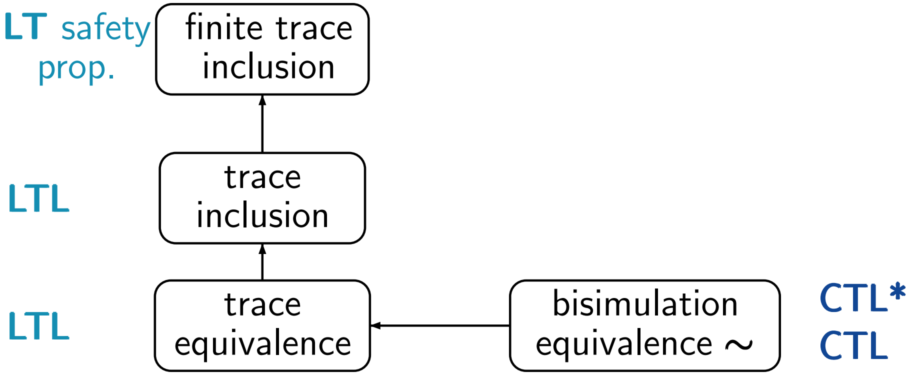
Equivalences and Abstraction

- bisimulation
- CTL, CTL*-equivalence
- computing the bisimulation quotient
- abstraction stutter steps
- simulation relations ⟵

Equivalences and preorders



Logical characterizations



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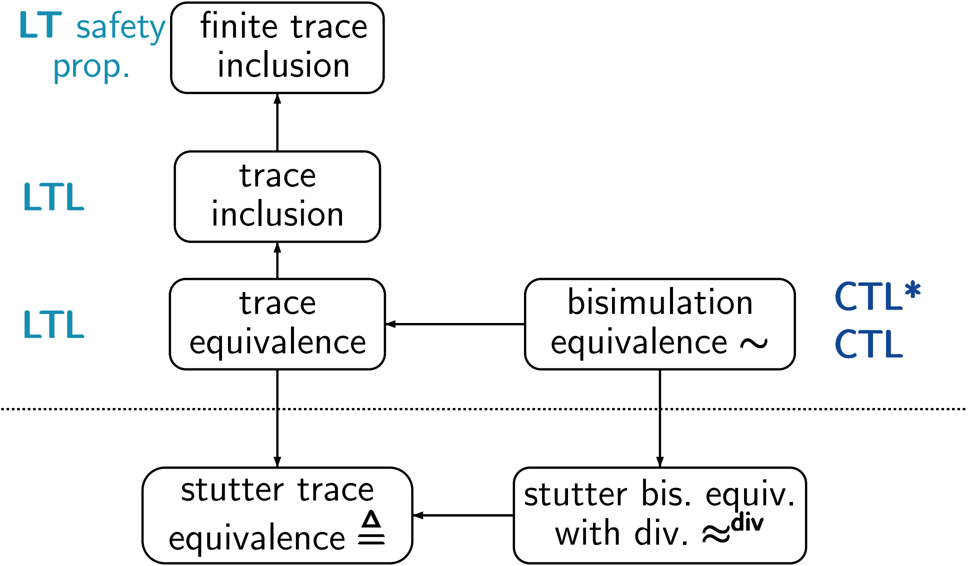

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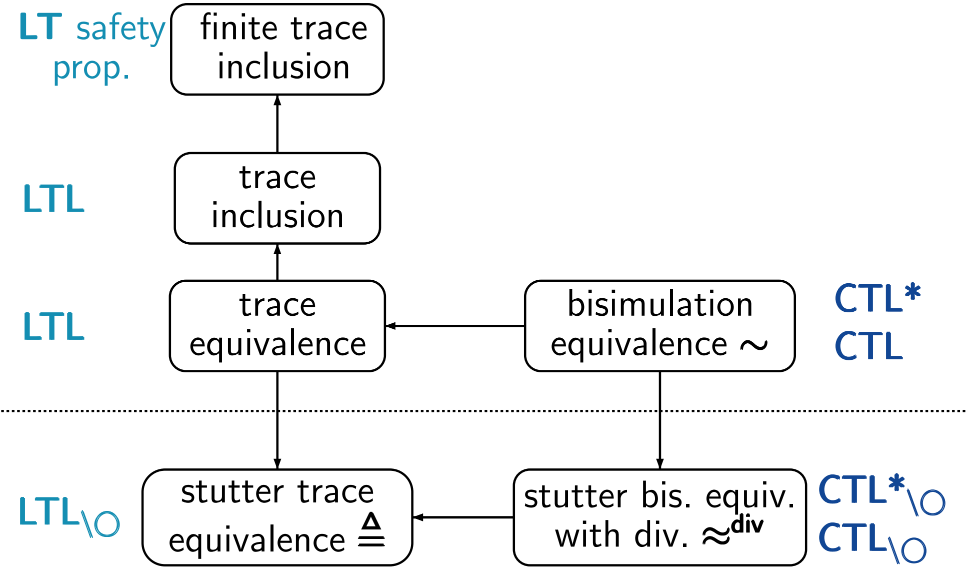

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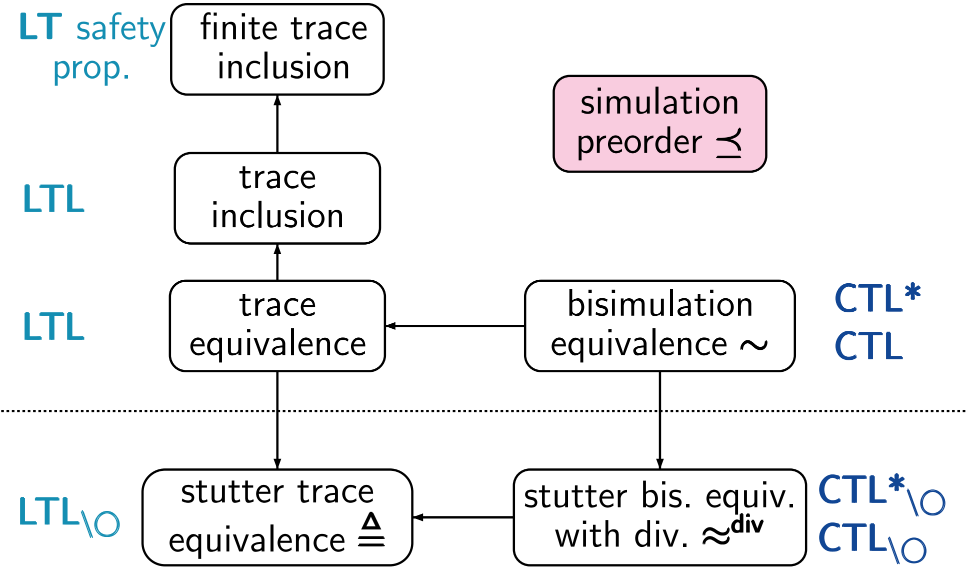

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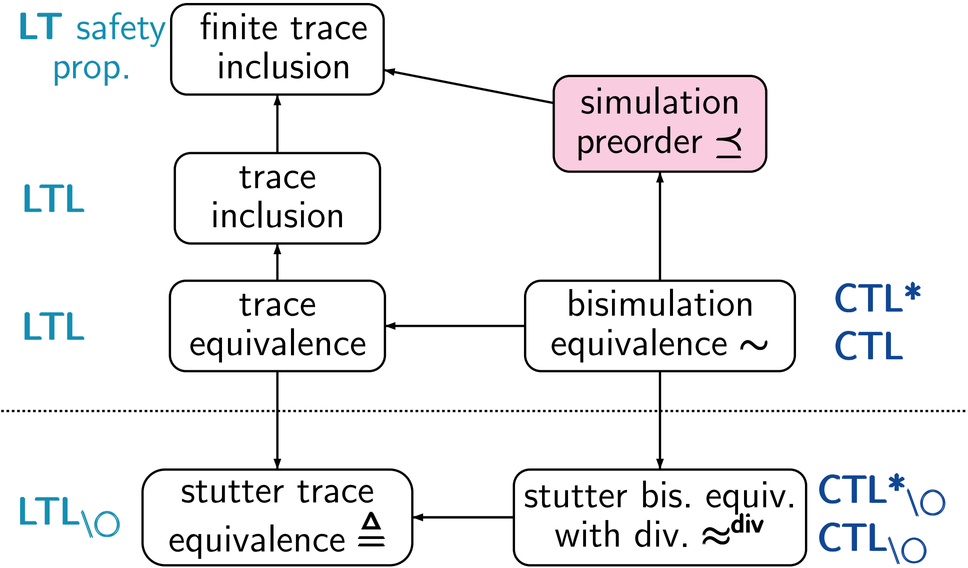

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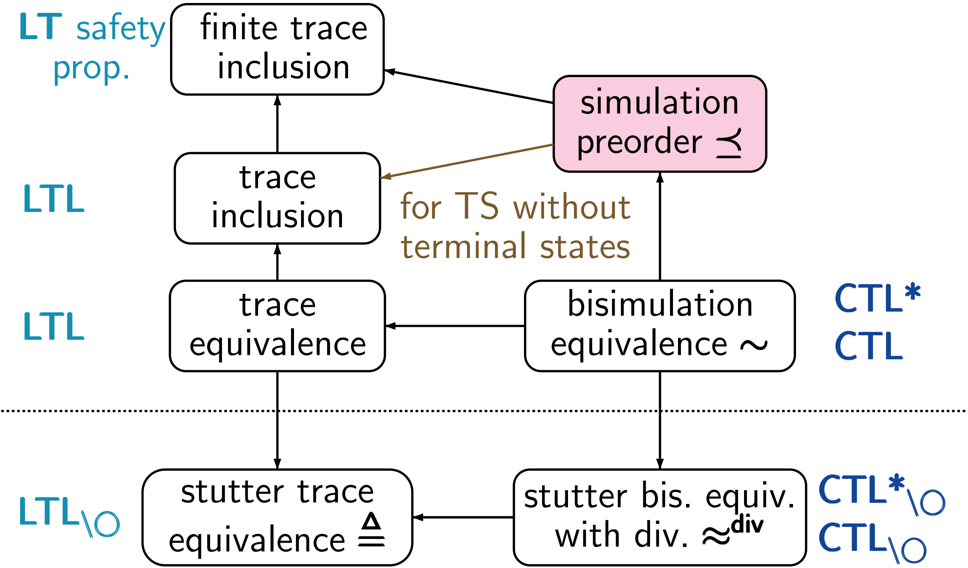

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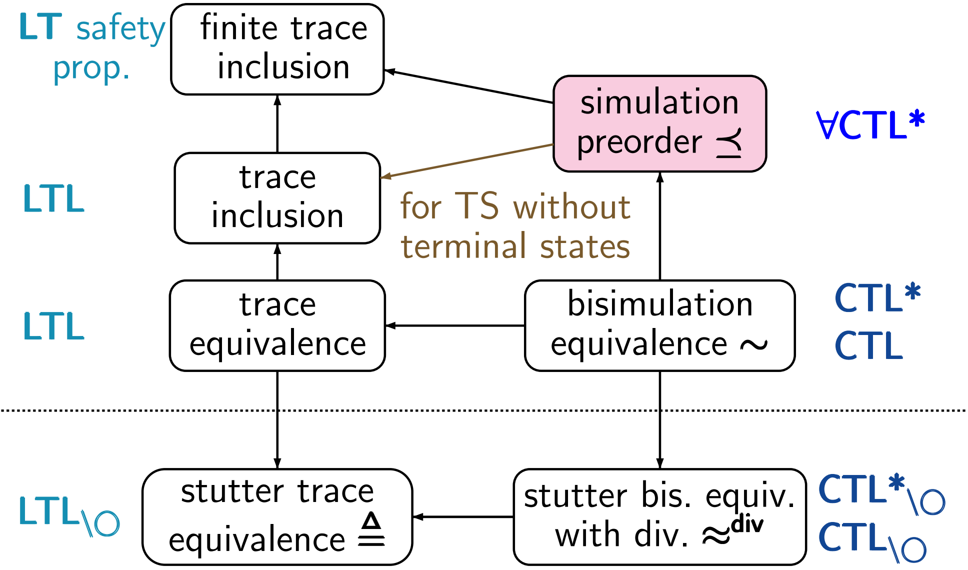

Logical characterization

for bisimulation equivalence $\sim_{\mathcal{T}}$:

$s_1 \sim_{\mathcal{T}} s_2$	iff	s_1, s_2 satisfy the same **CTL*** formulas
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observation: $\mathbb{L}$ <u>cannot</u> be closed under negation

The universal fragment ∀CTL* of CTL*

CTL* formulas in positive normal form, without ∃

Syntax of ∀CTL*

∀CTL* state formulas:

$$\Phi \quad ::= \quad \mathit{true} \;\mid\; \mathit{false} \;\mid\; a \;\mid\; \neg a \;\mid$$
$$\Phi_1 \wedge \Phi_2 \;\mid\; \Phi_1 \vee \Phi_2 \;\mid\; \forall \varphi$$

∀CTL* path formulas:

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always: $\Box \varphi \stackrel{\text{def}}{=} \varphi \, \mathsf{W} \, \mathit{false}$

Embedding of LTL in ∀CTL*

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but $\forall\Diamond\forall\Box a$ cannot be expressed in **LTL**

The universal fragments of CTL* and CTL

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$\forall$**CTL**: sublogic of $\forall$**CTL***

- no Boolean operators for paths formulas
- the arguments of the temporal modalities $\bigcirc$, **U** and **W** are state formulas

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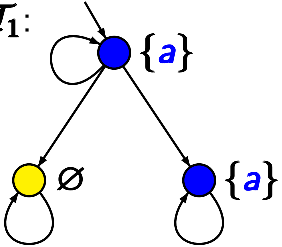
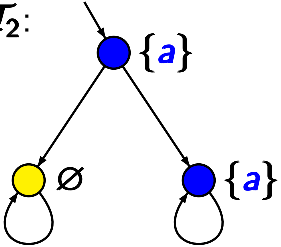
∀CTL and simulation

$\mathcal{T}_1$:

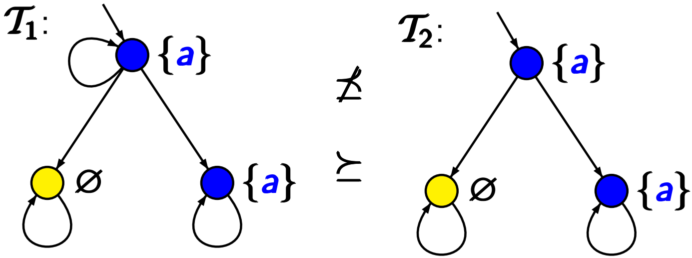


$\mathcal{T}_2$:



$AP = \{a\}$

∀CTL and simulation



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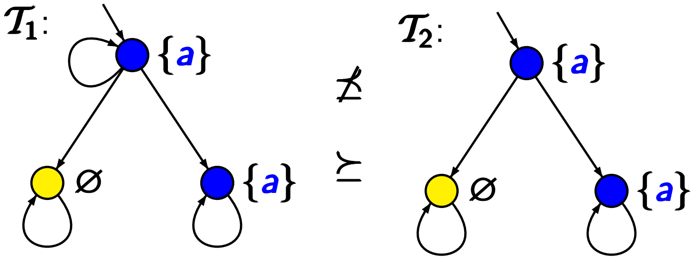

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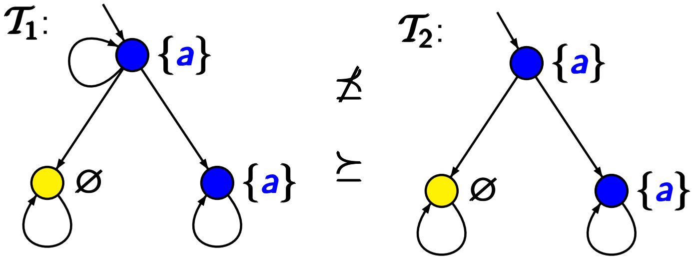

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↑

proof by structural induction

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is a simulation.

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dual to ∀**CTL***, i.e., **CTL*** formulas in **PNF**, without ∀

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$$\Psi \quad ::= \quad \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \Psi_1 \wedge \Psi_2 \mid \Psi_1 \vee \Psi_2 \mid \exists \varphi$$

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analogous: **∃CTL**

Duality of $\forall$CTL* and $\exists$CTL*

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For each $\forall$**CTL*** formula Φ there is a $\exists$**CTL*** formula Ψ s.t. $\Phi \equiv \neg\Psi$

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Duality of ∀CTL* and ∃CTL*

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GRM5.5-20A

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(2∃) for all ∃**CTL** formulas Ψ:

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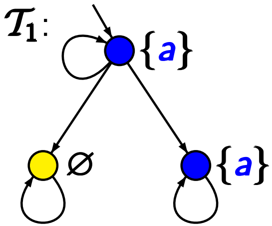
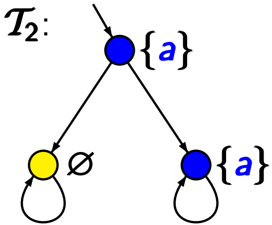
Example: ∀CTL/∃CTL and simulation





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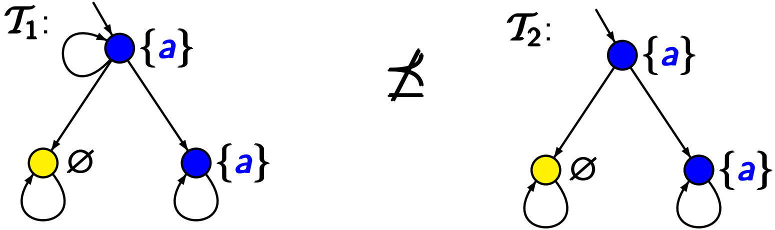
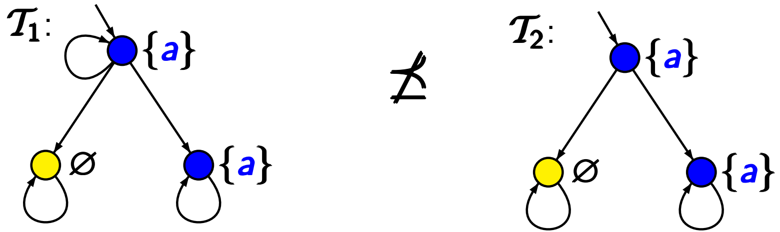


Example: ∀CTL/∃CTL and simulation



$\mathcal{T}_1 \not\models \forall\bigcirc(\forall\bigcirc\neg a \vee \forall\bigcirc a)$

$\mathcal{T}_2 \models \forall\bigcirc(\forall\bigcirc\neg a \vee \forall\bigcirc a)$

∀CTL formula

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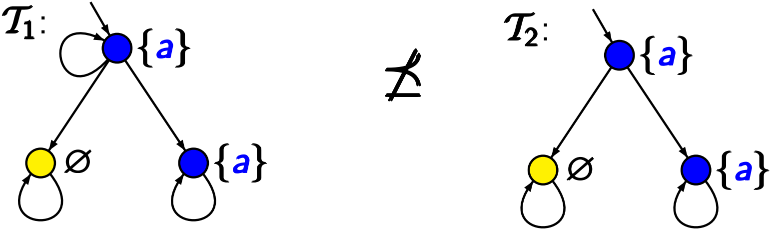

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∀CTL formula

$\mathcal{T}_1 \models \exists\bigcirc(\exists\bigcirc\neg a \wedge \exists\bigcirc a)$

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∃CTL formula

Characterizations of simulation equivalence

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for finite TS without terminal states:

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iff $\mathcal{T}_1, \mathcal{T}_2$ satisfy the same $\forall$CTL* formulas

iff $\mathcal{T}_1, \mathcal{T}_2$ satisfy the same $\forall$CTL formulas

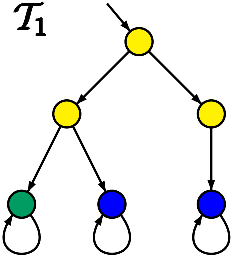
iff $\mathcal{T}_1, \mathcal{T}_2$ satisfy the same $\exists$CTL* formulas

iff $\mathcal{T}_1, \mathcal{T}_2$ satisfy the same $\exists$CTL formulas

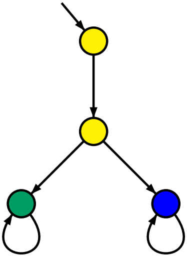
... even holds for $\forall\text{CTL*}_{\setminus \text{U,W}}$, $\forall\text{CTL}_{\setminus \text{U,W}}$, $\exists\text{CTL*}_{\setminus \text{U,W}}$, $\exists\text{CTL}_{\setminus \text{U,W}}$

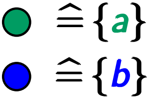
Simulation equivalence

$\mathcal{T}_1$

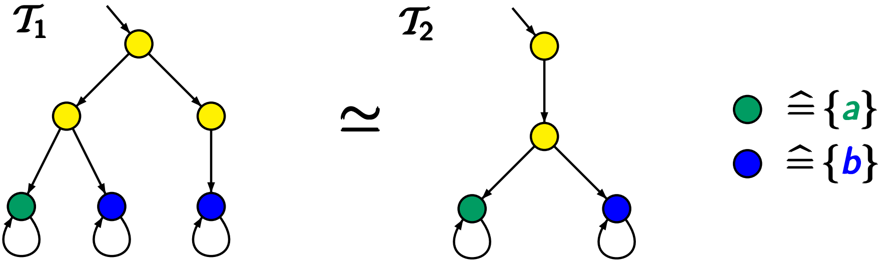


$\mathcal{T}_2$

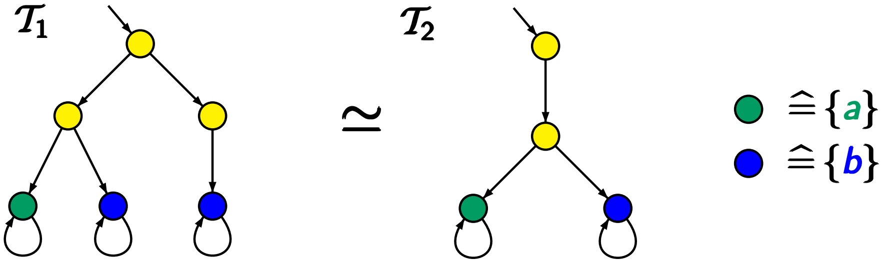




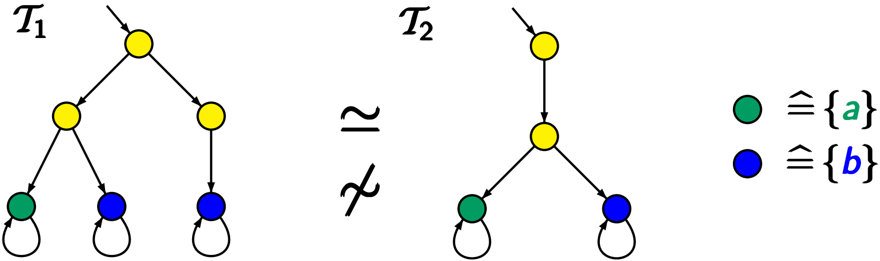
Simulation equivalence



Simulation equivalence



$\mathcal{T}_1$, $\mathcal{T}_2$ cannot be distinguished by the temporal logics
∀CTL, **∀CTL***, **∃CTL**, or **∃CTL***,



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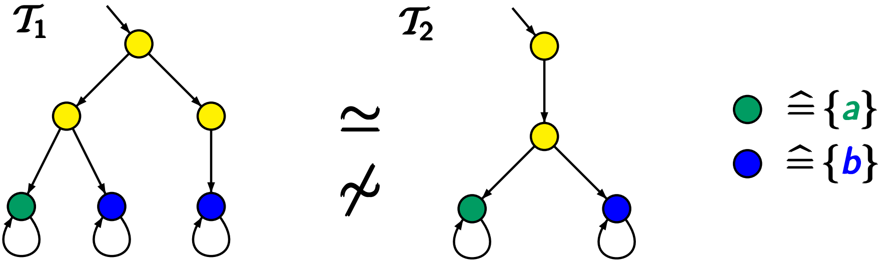

$\mathcal{T}_1$, $\mathcal{T}_2$ cannot be distinguished by the temporal logics **∀CTL**, **∀CTL***, **∃CTL**, or **∃CTL***,

but by **CTL**:

$$\mathcal{T}_1 \not\models \forall\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$$

$$\mathcal{T}_2 \models \forall\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$$

Correct or wrong?

In finite TS without terminal states:

If s_1, s_2 satisfy the same $\exists$CTL formulas
then s_1, s_2 satisfy the same LTL formulas

Correct or wrong?

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If s_1, s_2 satisfy the same $\exists$CTL formulas
then s_1, s_2 satisfy the same LTL formulas

correct

Correct or wrong?

In finite TS without terminal states:

If s_1, s_2 satisfy the same ∃CTL formulas
then s_1, s_2 satisfy the same LTL formulas

correct

$$\begin{aligned} & \exists\text{CTL equivalence} \\ = \; & \text{simulation equivalence} \\ = \; & \forall\text{CTL* equivalence} \end{aligned}$$

Correct or wrong?

In finite TS without terminal states:

If s_1, s_2 satisfy the same $\exists$CTL formulas
then s_1, s_2 satisfy the same LTL formulas

correct

$\exists$CTL equivalence
= simulation equivalence
= $\forall$CTL* equivalence

and LTL is a sublogic of $\forall$CTL*

Correct or wrong?

In finite TS without terminal states:

If s_1, s_2 satisfy the same $\exists$CTL formulas
then s_1, s_2 satisfy the same LTL formulas

correct

If s_1, s_2 satisfy the same LTL formulas
then s_1, s_2 satisfy the same $\forall$CTL formulas

In finite TS without terminal states:

If s_1, s_2 satisfy the same $\exists$CTL formulas
then s_1, s_2 satisfy the same LTL formulas

correct

If s_1, s_2 satisfy the same LTL formulas
then s_1, s_2 satisfy the same $\forall$CTL formulas

wrong

Correct or wrong?

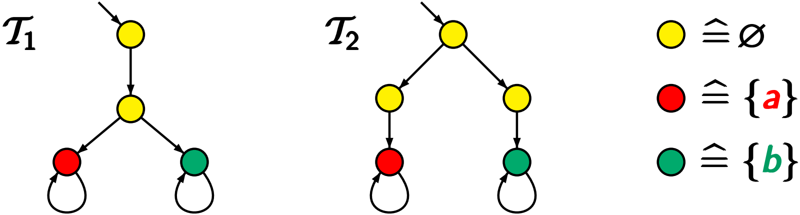
In finite TS without terminal states:

If s_1, s_2 satisfy the same $\exists$CTL formulas
then s_1, s_2 satisfy the same LTL formulas

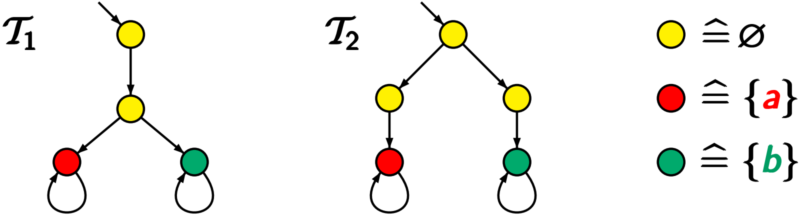
correct

If s_1, s_2 satisfy the same LTL formulas
then s_1, s_2 satisfy the same $\forall$CTL formulas

wrong, as trace equivalence does not imply simulation equivalence



Does there exist a **∃CTL** formula Φ s.t.
$\mathcal{T}_1 \models \Phi$ and $\mathcal{T}_2 \not\models \Phi$?



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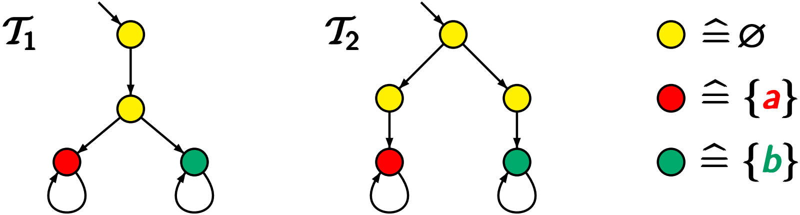
yes

Does there exist ...?



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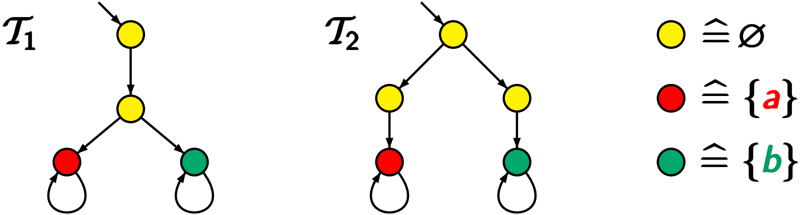

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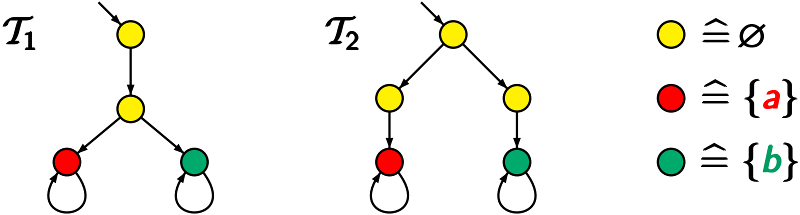
Does there exist ...?



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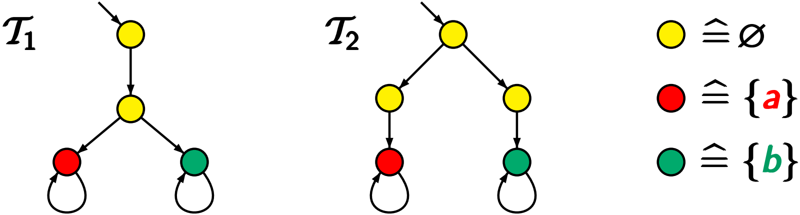

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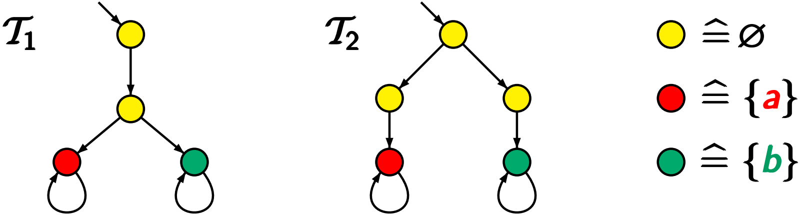
Does there exist ...?



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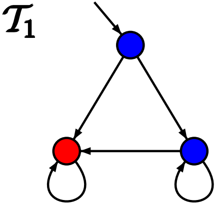
Does there exist a $\forall$**CTL** formula Φ s.t.
$\mathcal{T}_1 \models \Phi$ and $\mathcal{T}_2 \not\models \Phi$?

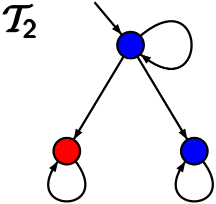
no, as $\mathcal{T}_2 \preceq \mathcal{T}_1$

Does there exist ...?

Does there exist a **∃CTL** formula Φ s.t.
$\mathcal{T}_1 \models \Phi$ and $\mathcal{T}_2 \not\models \Phi$?

Does there exist ...?





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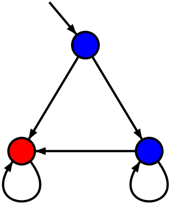
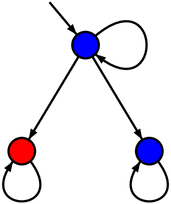
no

Does there exist ...?

$\mathcal{T}_1$



$\mathcal{T}_2$



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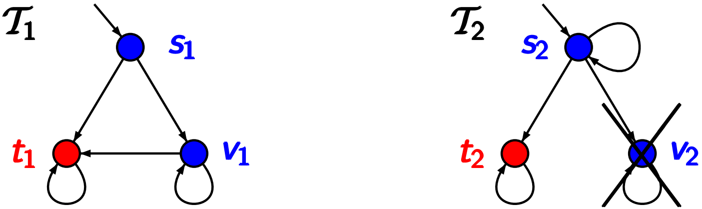

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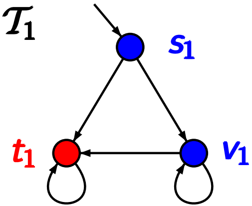
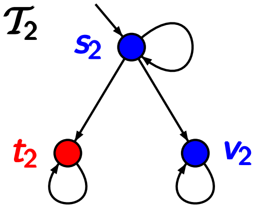
simulation for $(\mathcal{T}_1, \mathcal{T}_2)$: $\{(s_1, s_2), (v_1, s_2), (t_1, t_2)\}$

Does there exist ...?





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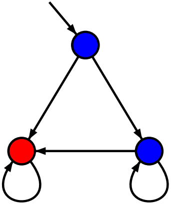
simulation for $(\mathcal{T}_1, \mathcal{T}_2)$: $\{(s_1, s_2), (v_1, s_2), (t_1, t_2)\}$

simulation for $(\mathcal{T}_2, \mathcal{T}_1)$:

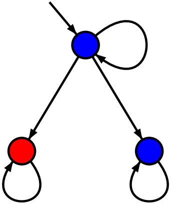
$\{(s_2, s_1), (s_2, v_1), (v_2, v_1), (t_1, t_2)\}$

Does there exist ...?

$\mathcal{T}_1$



$\mathcal{T}_2$



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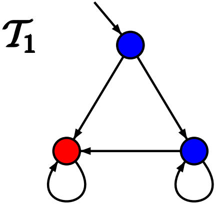
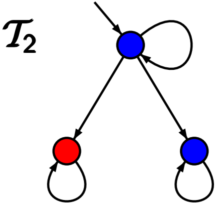
Does there exist ...?





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Does there exist ...?

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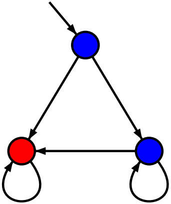
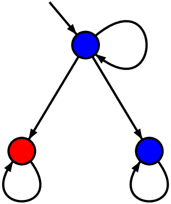
yes, as $\mathcal{T}_1 \not\sim \mathcal{T}_2$

Does there exist ...?

$\mathcal{T}_1$



$\mathcal{T}_2$



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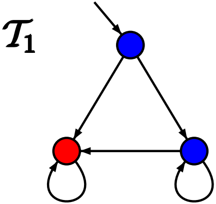
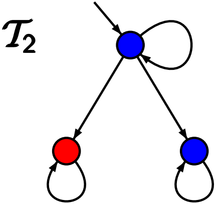
yes, as $\mathcal{T}_1 \not\sim \mathcal{T}_2$, e.g., $\Phi = \exists \bigcirc \forall \Box \mathit{blue}$

Does there exist ...?





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Does there exist a **LTL** formula φ s.t.
$\mathcal{T}_1 \not\models \varphi$ and $\mathcal{T}_2 \models \varphi$?

Does there exist a **CTL** formula Φ s.t.
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Does there exist a **LTL** formula φ s.t.
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no

Does there exist ...?

Does there exist a **CTL** formula Φ s.t.
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yes, as $\mathcal{T}_1 \not\sim \mathcal{T}_2$, e.g., $\Phi = \exists \bigcirc \forall \Box \textit{blue}$

Does there exist a **LTL** formula φ s.t.
$\mathcal{T}_1 \not\models \varphi$ and $\mathcal{T}_2 \models \varphi$?

no, as $\mathcal{T}_1$, $\mathcal{T}_2$ are simulation equivalent

Simulation quotient

Simulation quotient

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

simulation quotient $\mathcal{T}/\simeq$:

transition system that arises from $\mathcal{T}$ by collapsing all simulation equivalent states

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

$$\mathcal{T}/\simeq \ \stackrel{\text{def}}{=} \ (S/\simeq, Act', \rightarrow_{\simeq}, S'_0, AP', L')$$

Simulation quotient

Let $\mathcal{T} = (S, Act, \to, S_0, AP, L)$ be a TS. Then:

$$\mathcal{T}/\!\simeq \;\stackrel{\text{def}}{=}\; (S/\!\simeq, Act', \to_{\simeq}, S_0', AP', L')$$

- state space $S/\!\simeq$ $\longleftarrow$ set of all simulation equivalence classes

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

$$\mathcal{T}/\simeq \;\stackrel{\text{def}}{=}\; (S/\simeq, Act', \rightarrow_{\simeq}, S_0', AP', L')$$

- state space $S/\simeq$ ⟵ set of all simulation equivalence classes
- initial states: $S_0' = \{[s] : s \in S_0\}$

$$[s] = \{s' \in S : s \simeq_{\mathcal{T}} s'\}$$

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

$$\mathcal{T}/\!\simeq \;\stackrel{\text{def}}{=}\; (S/\!\simeq, Act', \rightarrow_{\simeq}, S_0', AP', L')$$

- state space $S/\!\simeq$ ⟵ set of all simulation equivalence classes
- initial states: $S_0' = \{[s] : s \in S_0\}$
- labeling: $AP' = AP$ and $L'([s]) = L(s)$

$$[s] = \{s' \in S : s \simeq_{\mathcal{T}} s'\}$$

Simulation quotient

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

$$\mathcal{T}/\simeq \ \stackrel{\text{def}}{=} \ (S/\simeq, Act', \rightarrow_{\simeq}, S_0', AP', L')$$

- state space $S/\simeq$ ⟵ set of all simulation equivalence classes
- initial states: $S_0' = \{[s] : s \in S_0\}$
- labeling: $AP' = AP$ and $L'([s]) = L(s)$
- transition relation: $\dfrac{s \longrightarrow s'}{[s] \longrightarrow_{\simeq} [s']}$

Simulation quotient

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

$$\mathcal{T}/\!\simeq \;\stackrel{\text{def}}{=}\; (S/\!\simeq, Act', \rightarrow_{\simeq}, S_0', AP', L')$$

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- labeling: $AP' = AP$ and $L'([s]) = L(s)$
- transition relation: $\dfrac{s \longrightarrow s'}{[s] \longrightarrow_{\simeq} [s']}$

  action labels: irrelevant

Similarity of $\mathcal{T}$ and $\mathcal{T}/\simeq$

Similarity of $\mathcal{T}$ and $\mathcal{T}/\simeq$

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS. Then:

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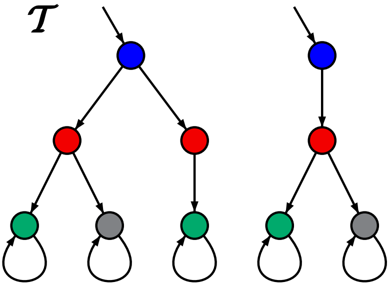
Example: simulation quotient



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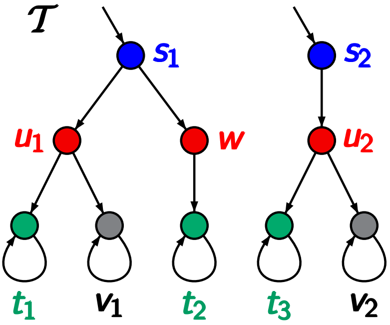

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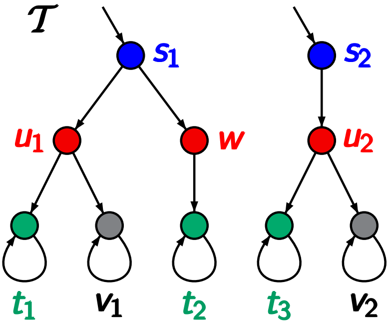

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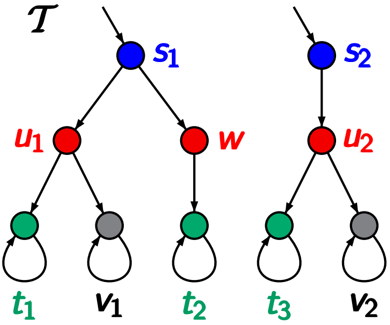

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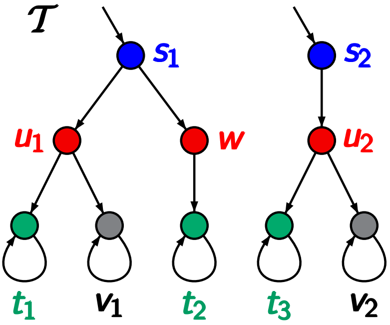

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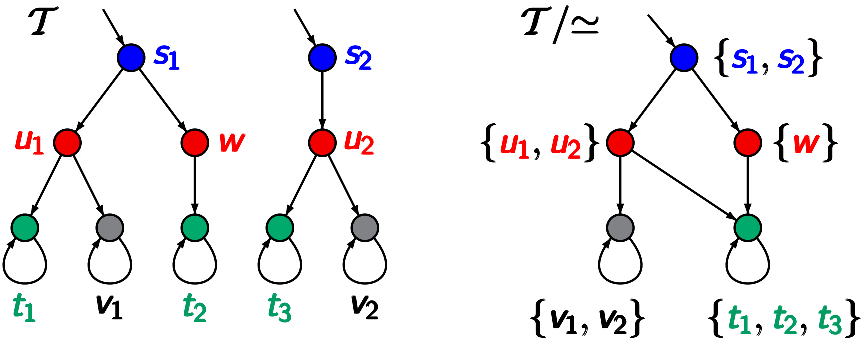

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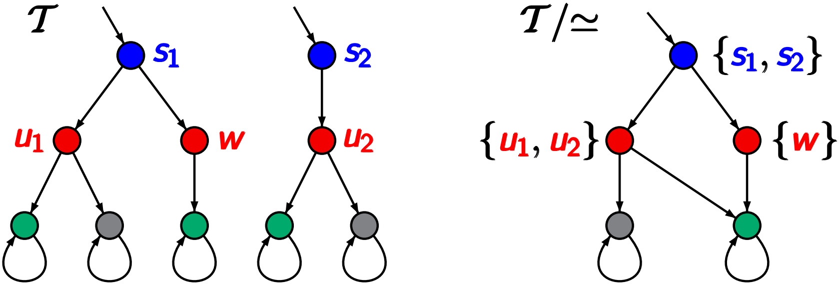

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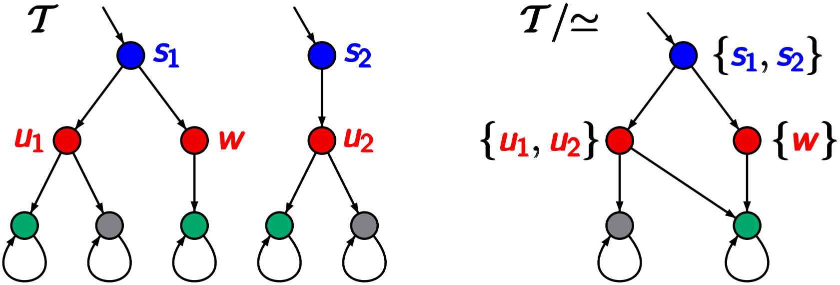

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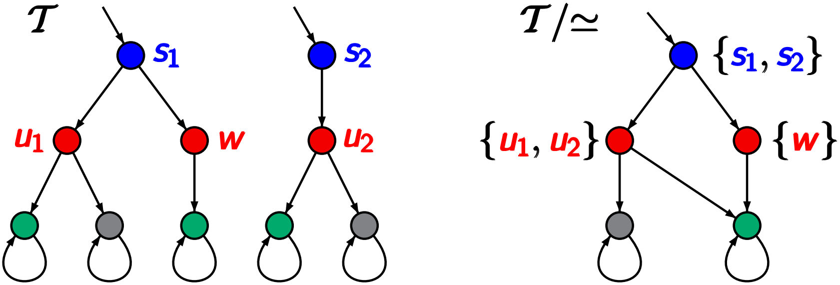

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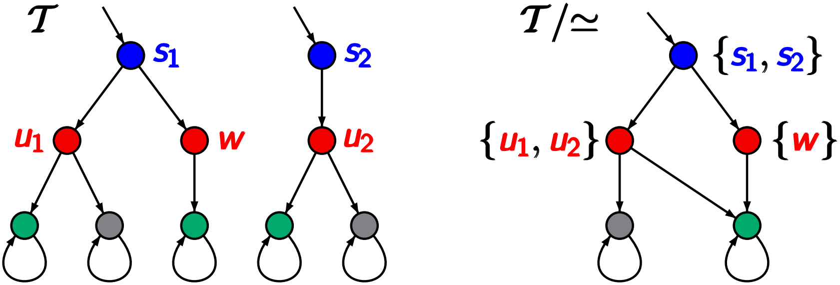

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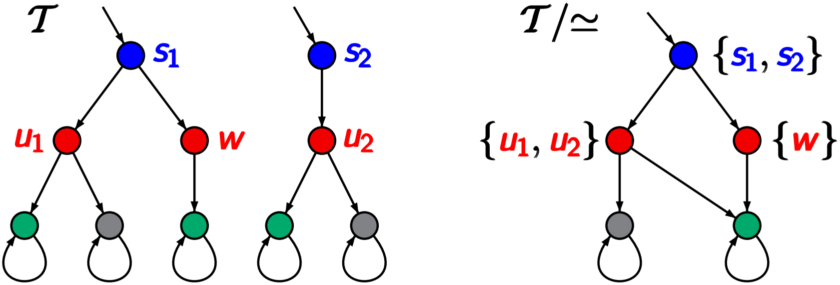

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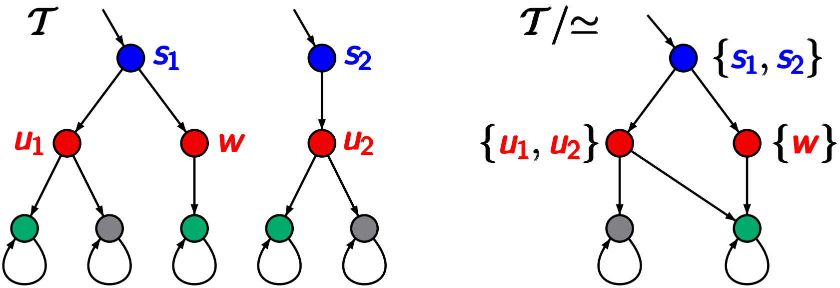

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