Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation
- CTL, CTL*-equivalence
- computing the bisimulation quotient
- abstraction stutter steps ⟵
- simulation relations

Divergent states

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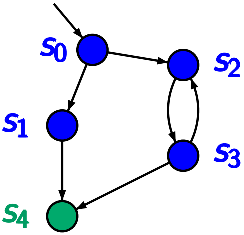
Which states are $\approx_{\mathcal{T}}$-divergent?



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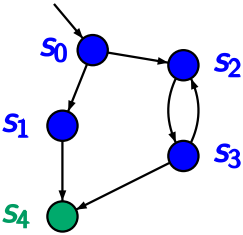

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$\{s_0, s_1, s_2, s_3\}$ $\{s_4\}$

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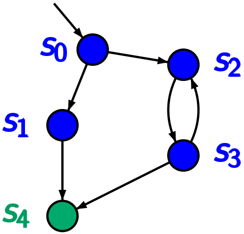

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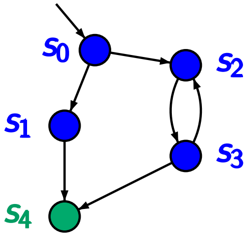

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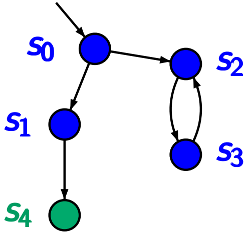

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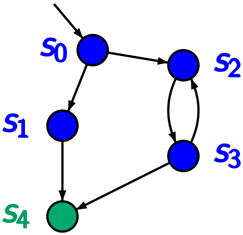

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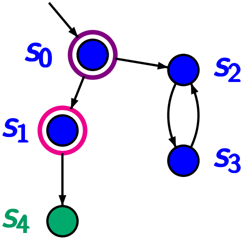

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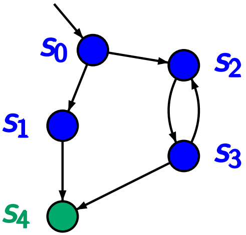

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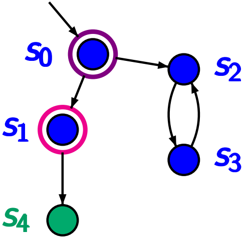

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Divergence-sensitivity

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$\mathcal{T}$ is called *divergence-sensitive* if for all states s_1 and s_2 in $\mathcal{T}$:

if $s_1 \approx_{\mathcal{T}} s_2$ and s_1 is $\approx_{\mathcal{T}}$-divergent then s_2 is $\approx_{\mathcal{T}}$-divergent

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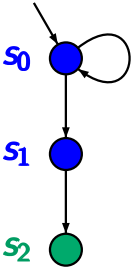

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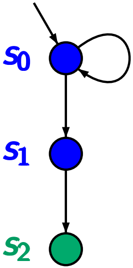

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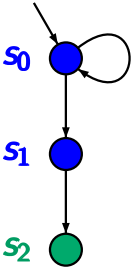

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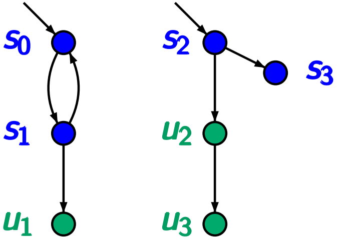
Example: divergence-sensitivity



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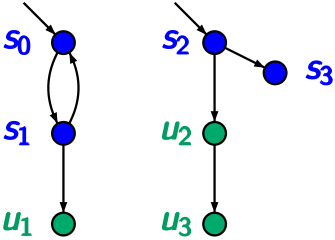

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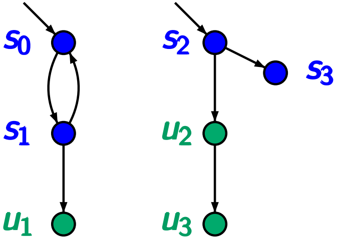

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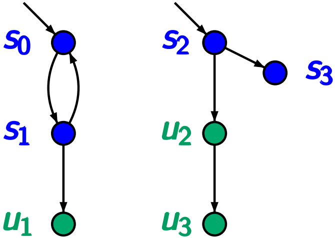

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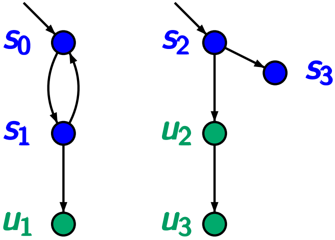

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We will see ...

If $\mathcal{T}$ is finite and divergence-sensitive then for all states s_1, s_2 and **CTL*** $_{\setminus \bigcirc}$ formulas Φ:

$$\text{if } s_1 \approx_{\mathcal{T}} s_2 \text{ and } s_1 \models \Phi \text{ then } s_2 \models \Phi$$

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to prove this we show:

stutter bisimulation equivalence with divergence $=$ $\mathbf{CTL}^*_{\setminus\bigcirc}$ equivalence

Divergence-sensitive equivalences

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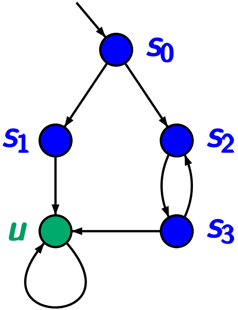

● $\widehat{=}$ $\{a\}$

● $\widehat{=}$ $\varnothing$

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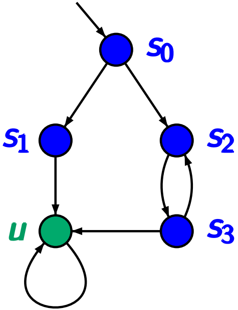

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$\mathcal{R}_2$: $\{s_0\}$ $\{s_1\}$ $\{s_2, s_3\}$ $\{u\}$

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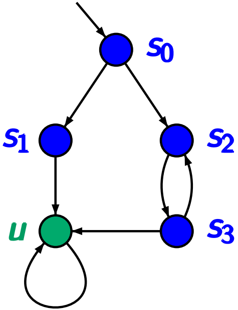

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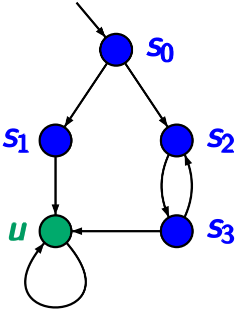

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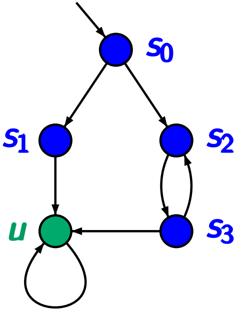

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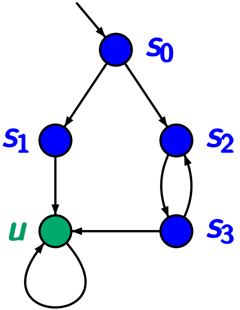

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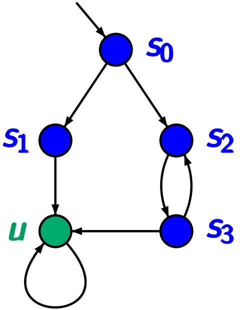

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Stutter bisimulation with divergence

Let $\mathcal{T} = (S, \ldots)$ be a TS and $\mathcal{R}$ an equivalence on S.

$\mathcal{R}$ is called *divergence-sensitive* if for all $s_1, s_2 \in S$:

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$\uparrow$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ is an equivalence relation on S and the coarsest divergence-sensitive stutter bisimulation for $\mathcal{T}$

$\approx_{\mathcal{T}}^{\text{div}}$ = coarsest equivalence on the state space S of $\mathcal{T}$ such that for all $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$:

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(1) $L(s_1) = L(s_2)$

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$\approx_{\mathcal{T}}^{\mathsf{div}}$ = coarsest equivalence on the state space S of $\mathcal{T}$ such that for all $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$:

(1) $L(s_1) = L(s_2)$

(2) $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$, $s_1 \to s_1'$ with $s_1' \not\approx_{\mathcal{T}}^{\mathsf{div}} s_1$

$\rightsquigarrow$ $s_2 \to u_1 \to \ldots \to u_n \to s_2'$ with $s_2, u_1, \ldots, u_n \approx_{\mathcal{T}}^{\mathsf{div}} s_1$ and $s_1' \approx_{\mathcal{T}}^{\mathsf{div}} s_2'$

Stutter bis. equivalence with divergence

$\approx_{\mathcal{T}}^{\text{div}}$ = coarsest equivalence on the state space S of $\mathcal{T}$ such that for all $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$:

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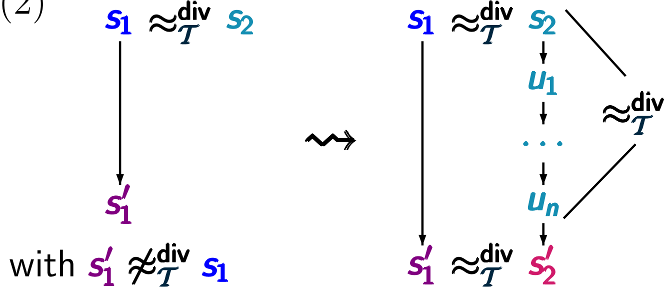
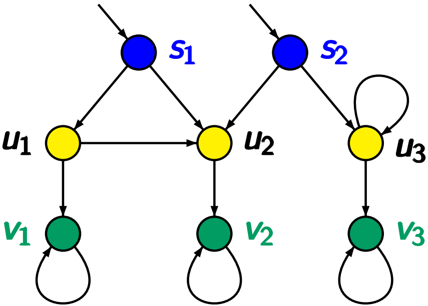
(2)

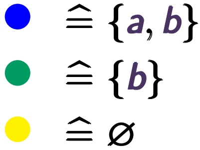


(3) s_1 $\approx_{\mathcal{T}}^{\text{div}}$-divergent iff s_2 $\approx_{\mathcal{T}}^{\text{div}}$-divergent

Example: $\approx_{\mathcal{T}}$ vs. $\approx_{\mathcal{T}}^{\mathbf{div}}$



$AP = \{a, b\}$



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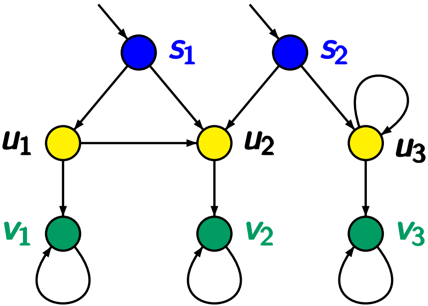

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stutter bisimulation equivalence classes:

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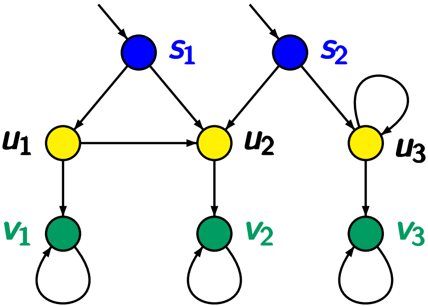
Example: $\approx_{\mathcal{T}}$ vs. $\approx_{\mathcal{T}}^{\text{div}}$



$AP = \{a, b\}$

- blue $\widehat{=} \{a, b\}$
- green $\widehat{=} \{b\}$
- yellow $\widehat{=} \varnothing$

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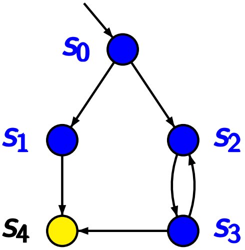
stutter bisimulation equiv. classes with divergence:

$\{v_1, v_2, v_3\}$ $\{u_1, u_2\}$ $\{u_3\}$ $\{s_1\}$ $\{s_2\}$

Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\mathbf{div}}$



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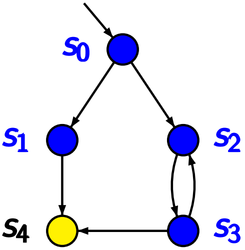

stutter bis. equivalence classes:

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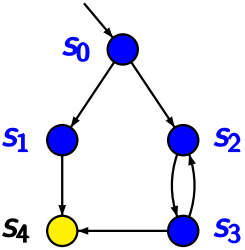
Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\text{div}}$



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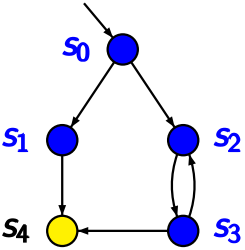

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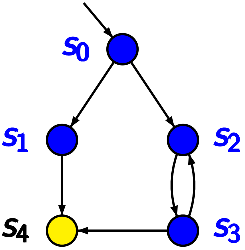
Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\mathsf{div}}$



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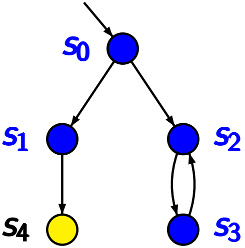
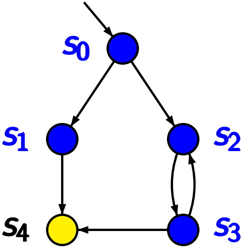


Equivalence classes under $\approx_{\mathcal{T}}$ and $\approx_{\mathcal{T}}^{\mathsf{div}}$



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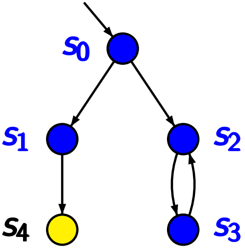

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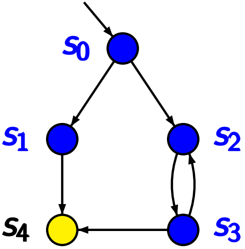

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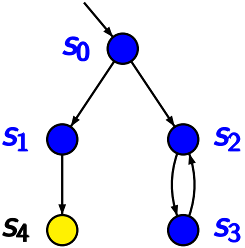

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$\approx^{\mathsf{div}}$ for two transition systems

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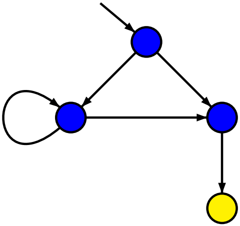
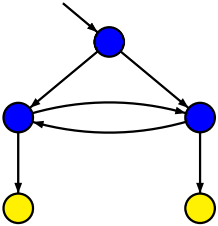
Correct or wrong?



$\approx^{\mathsf{div}}$



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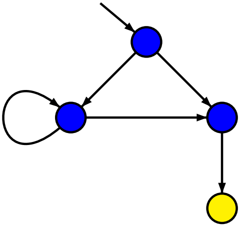

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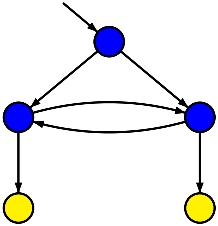

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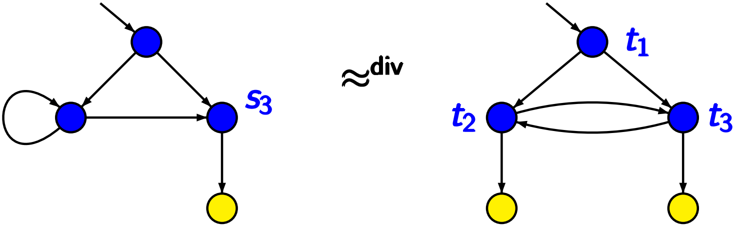
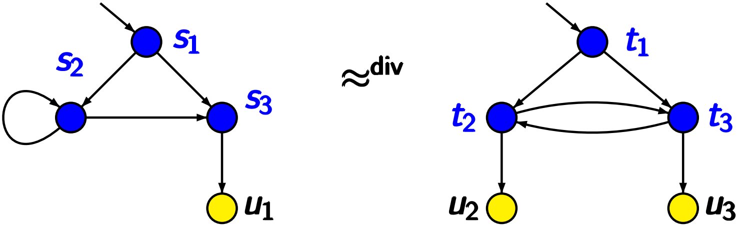


wrong

because s_3 is not divergent,
while t_1, t_2, t_3 are divergent

Correct or wrong?



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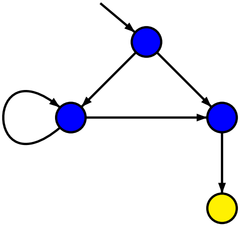
$\approx_{\mathcal{T}}^{\text{div}}$-equivalence classes:

$$\{\{t_1, t_2, t_3\}, \{s_1, s_2\}, \{s_3\}, \{u_1, u_2, u_3\}\}$$

Correct or wrong?



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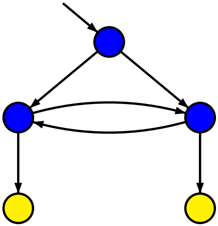

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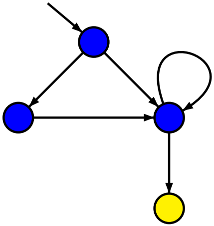

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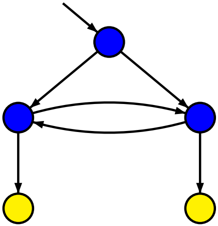

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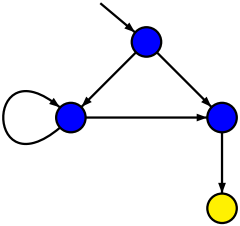

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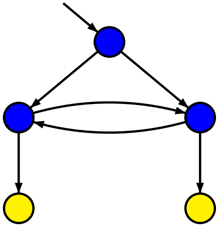

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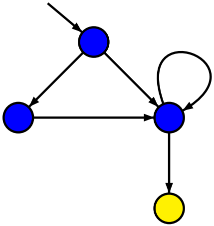

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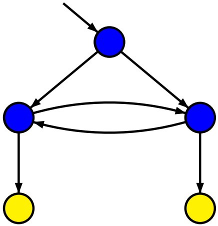

correct

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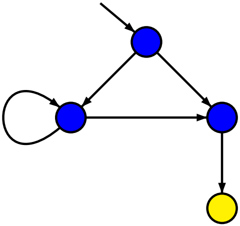

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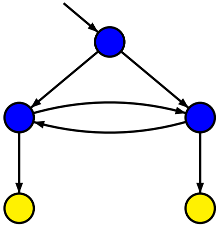

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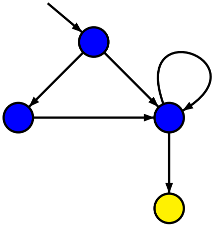

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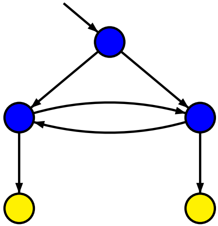
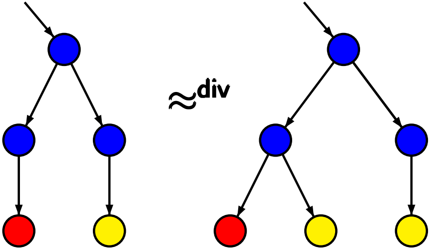


correct

all blue states are $\approx^{\textsf{div}}$-equivalent and divergent

Correct or wrong?



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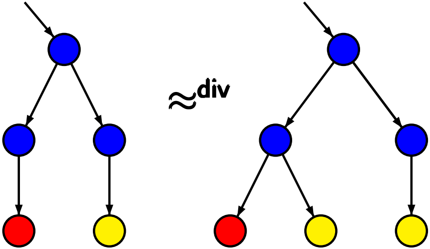

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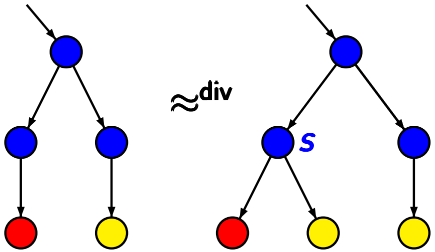
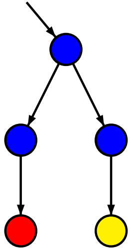


wrong

even not $\approx$-equivalent, since s has no equivalent state

Correct or wrong?



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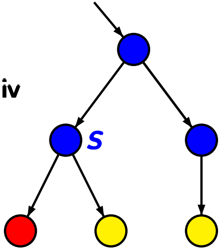

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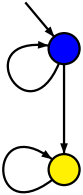

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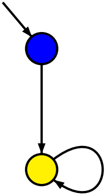

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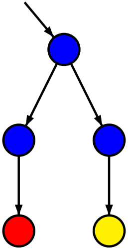

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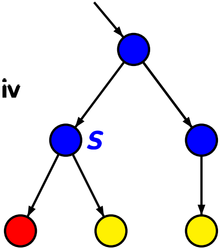

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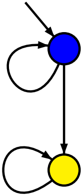

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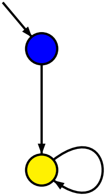

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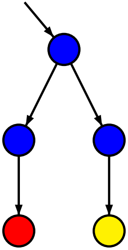

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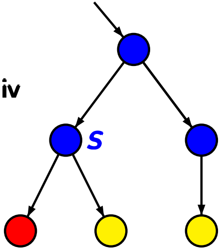

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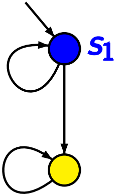

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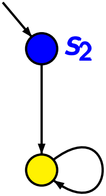
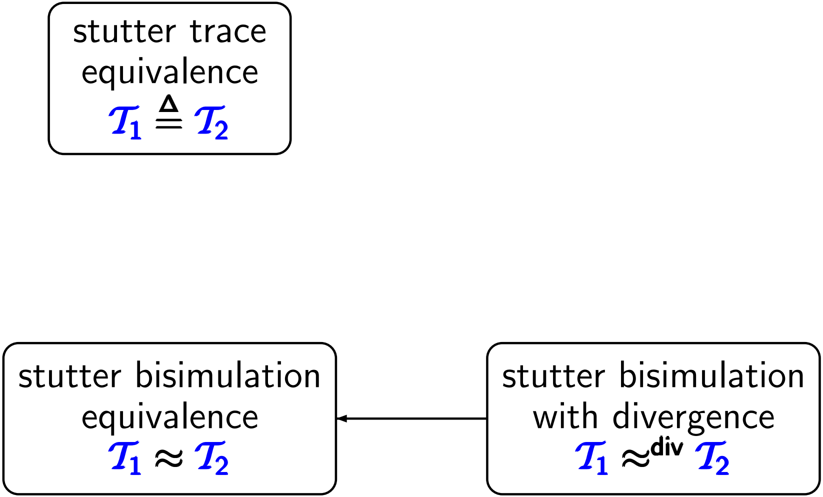


wrong

$s_1 \not\approx^{\text{div}} s_2$, as s_1 is $\approx^{\text{div}}$-divergent, while s_2 is not

Stutter abstract equivalences



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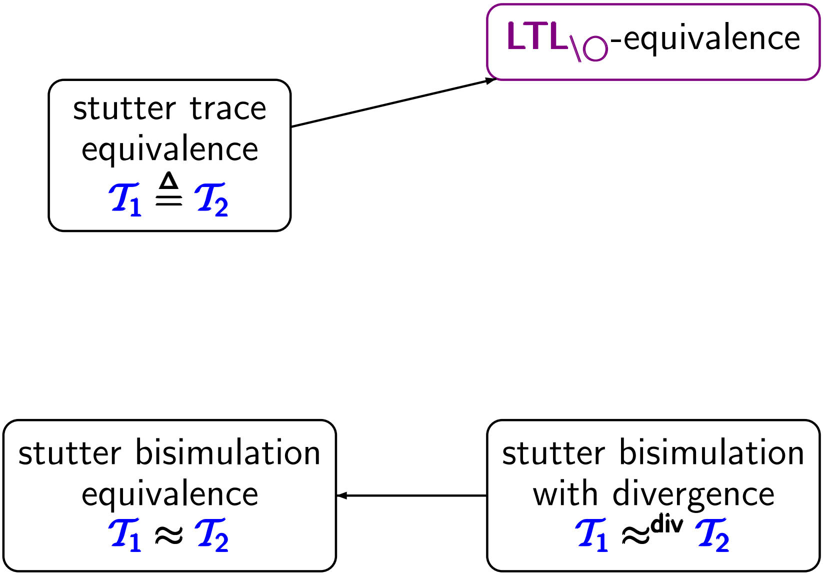

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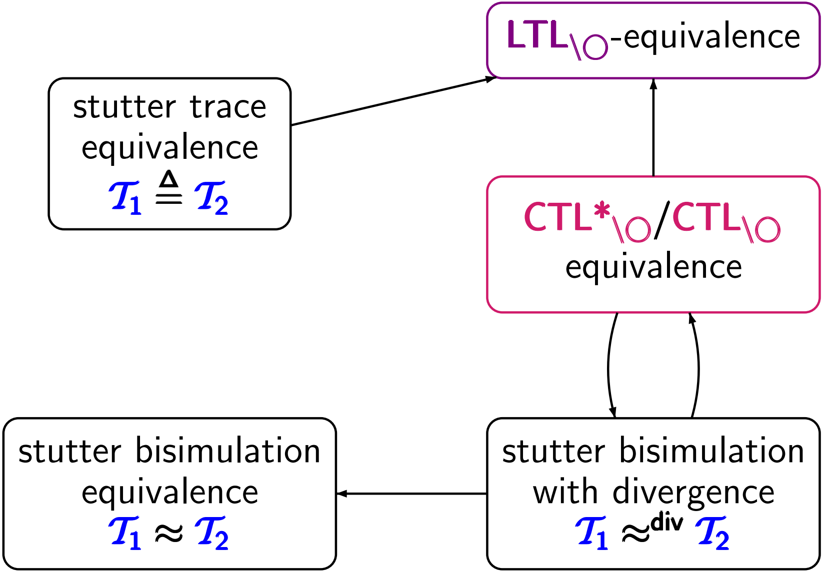

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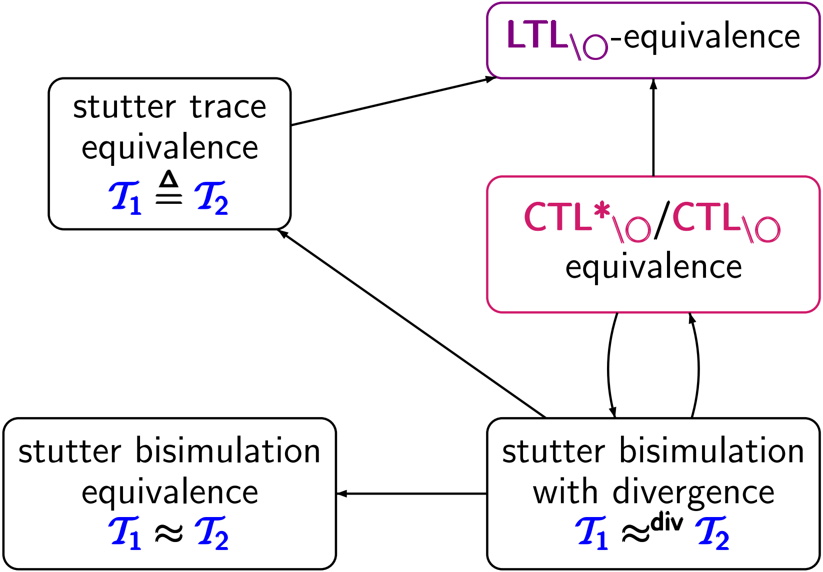

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(2) s_1, s_2 satisfy the same $\text{CTL}^*_{\setminus \bigcirc}$ formulas

$\text{CTL}^*_{\setminus \bigcirc}$ = CTL^* without next operator $\bigcirc$

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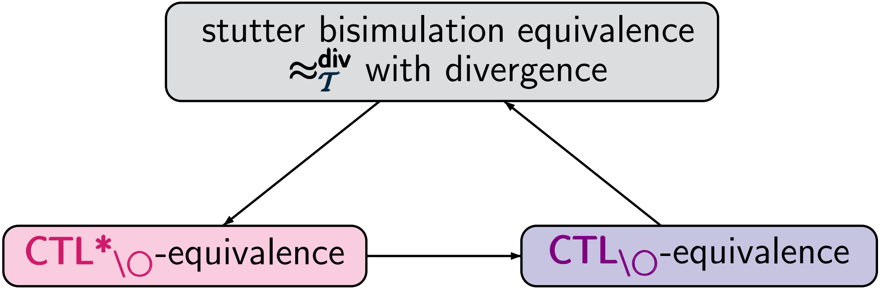

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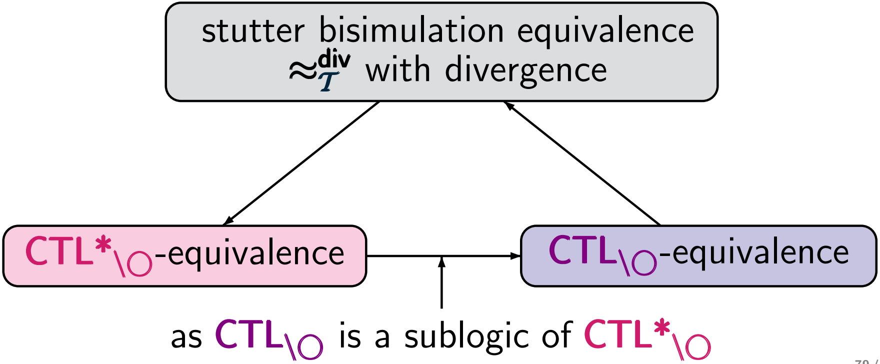

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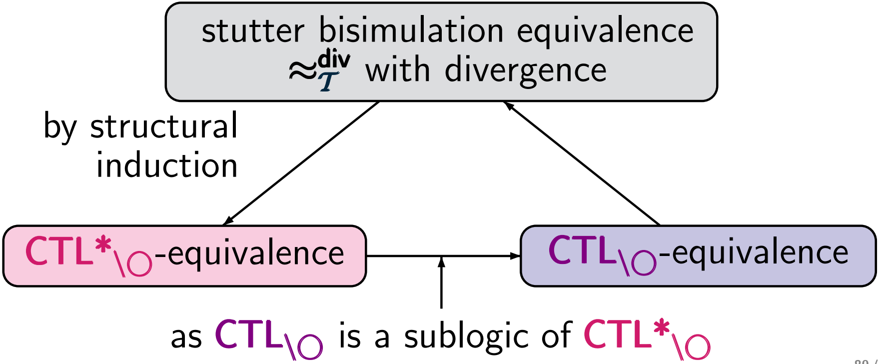

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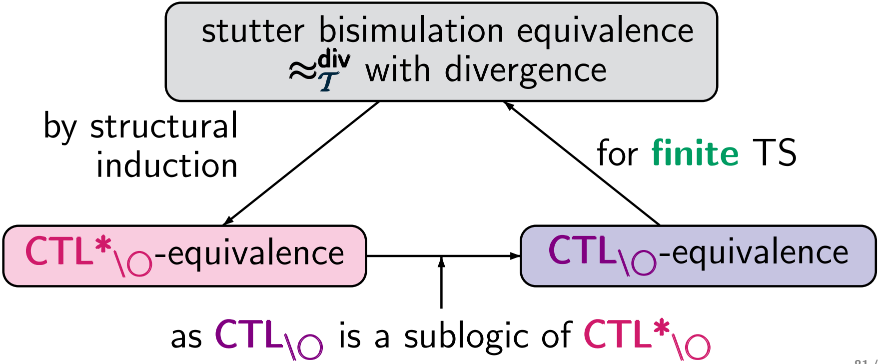

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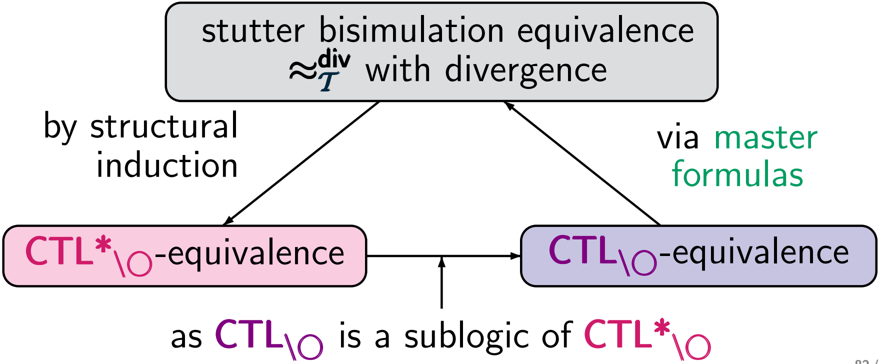

$\mathbf{CTL}_{\setminus\bigcirc}$-equivalence is finer than $\approx_{\mathcal{T}}^{\mathsf{div}}$

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For finite transition system $\mathcal{T}$:

$$\mathcal{R} = \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \mathsf{CTL}_{\setminus\bigcirc} \text{ formulas} \}$$

is a divergence-sensitive stutter bisimulation.

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proof uses $\mathsf{CTL}_{\setminus\bigcirc}$ master formulas for $\approx_{\mathcal{T}}^{\mathsf{div}}$-equivalence classes

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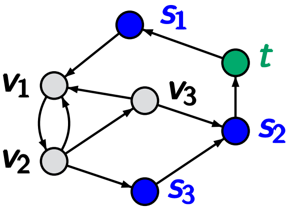

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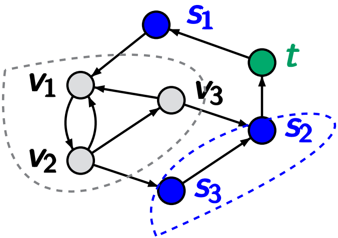

● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$ ● $\widehat{=} \varnothing$

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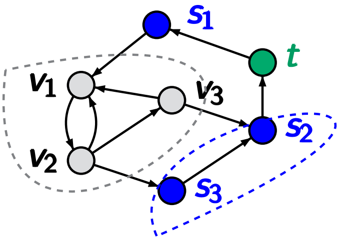


master formulas:

$v_1, v_2, v_3 \models \neg a \wedge \neg b$

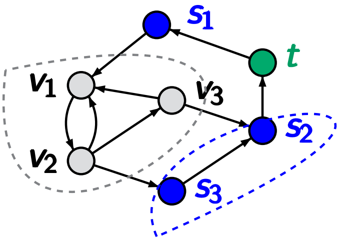
● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$ ● $\widehat{=}\varnothing$

$\text{CTL}_{\setminus\bigcirc}$-equivalence is finer than $\approx_{\mathcal{T}}^{\text{div}}$

For finite transition system $\mathcal{T}$:

$$\mathcal{R} = \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \text{CTL}_{\setminus\bigcirc} \text{ formulas} \}$$

is a divergence-sensitive stutter bisimulation.



master formulas:

$$v_1, v_2, v_3 \models \neg a \wedge \neg b$$
$$t \models \neg a \wedge b$$

● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$ ● $\widehat{=} \varnothing$

$\text{CTL}_{\setminus\bigcirc}$-equivalence is finer than $\approx^{\text{div}}_{\mathcal{T}}$

For finite transition system $\mathcal{T}$:

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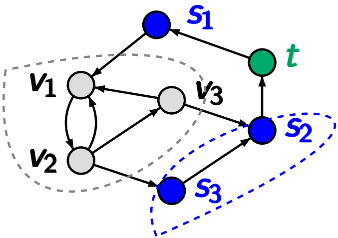


master formulas:

$v_1, v_2, v_3 \models \neg a \wedge \neg b$

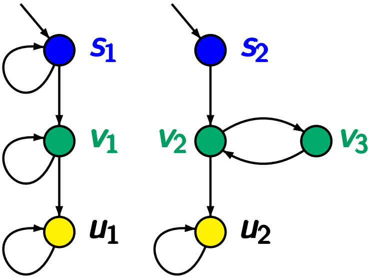
$t \models \neg a \wedge b$

$s_2, s_3 \models a \wedge \exists(a \,\mathsf{U}\, b)$

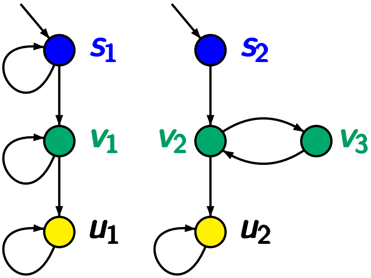
$s_1 \models a \wedge \neg\exists(a \,\mathsf{U}\, b)$

● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$ ● $\widehat{=} \varnothing$

$CTL_{\setminus\bigcirc}$ master formulas



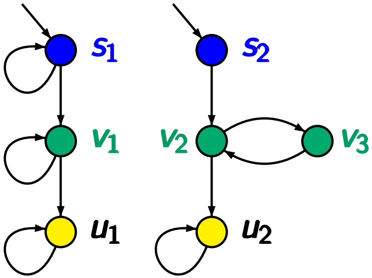
$CTL_{\setminus\bigcirc}$ master formulas



equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\text{div}}$:

$\{u_1, u_2\}$ $\{v_1, v_2, v_3\}$ $\{s_1\}$ $\{s_2\}$

$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas



$u_1, u_2 \models \neg blue \wedge \neg green$

equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\mathbf{div}}$:

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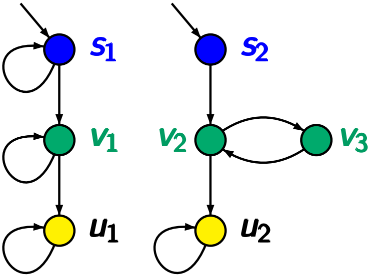
$\text{CTL}_{\setminus\bigcirc}$ master formulas



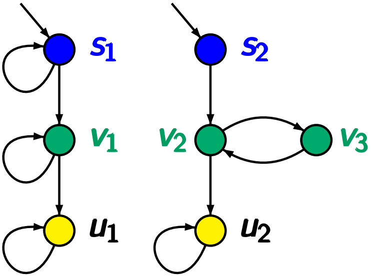
$v_1, v_2, v_3 \models \textit{green}$

$u_1, u_2 \models \neg \textit{blue} \wedge \neg \textit{green}$

equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\text{div}}$:

$\{u_1, u_2\}$ $\{v_1, v_2, v_3\}$ $\{s_1\}$ $\{s_2\}$

$CTL_{\setminus\bigcirc}$ master formulas



$s_1 \models \exists\Box\, blue$

$s_2 \models blue \wedge \neg\exists\Box\, blue$

$v_1, v_2, v_3 \models green$

$u_1, u_2 \models \neg blue \wedge \neg green$

equivalence classes w.r.t. $\approx_{\mathcal{T}}^{\mathsf{div}}$:

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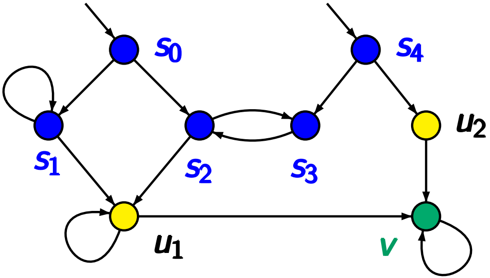
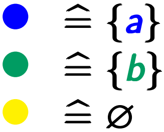
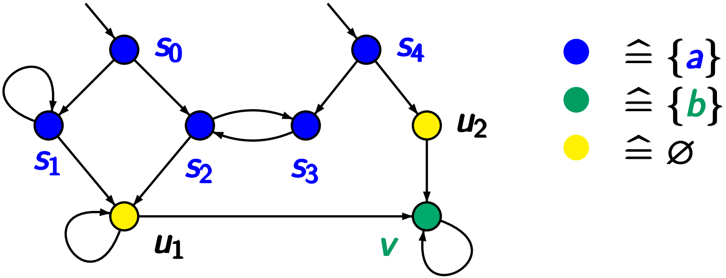
$CTL_{\setminus \bigcirc}$ master formulas

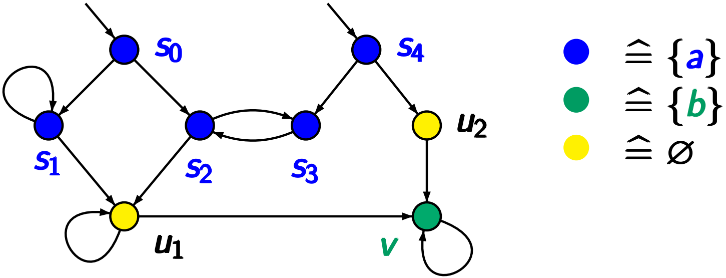




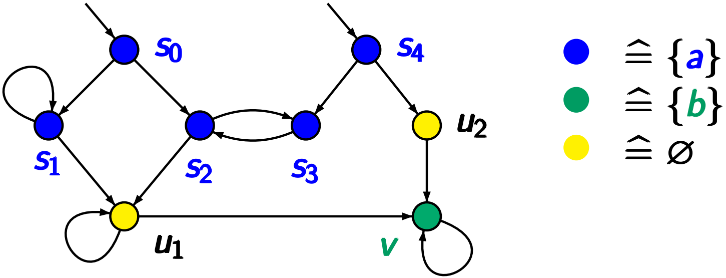
$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas



$\approx^{\mathsf{div}}_{\mathcal{T}}$-equiv. classes	
$\{v\}$	
$\{u_1\}$	
$\{u_2\}$	
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	

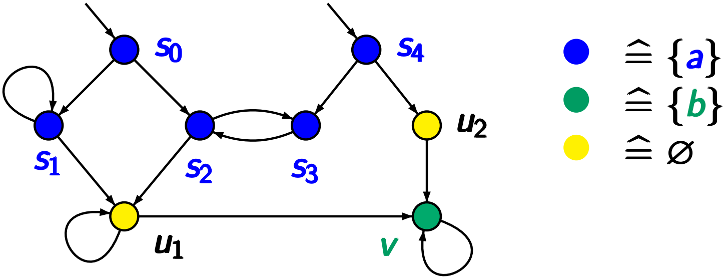


$\approx_{\mathcal{T}}^{\text{div}}$-equiv. classes	$CTL_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	
$\{u_2\}$	
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	



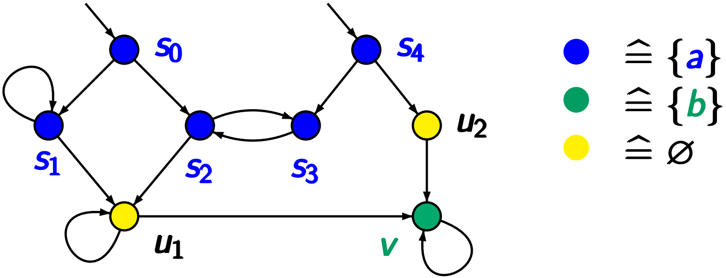
$\approx_{\mathcal{T}}^{\mathrm{div}}$-equiv. classes	$\mathrm{CTL}_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	

$CTL_{\setminus\bigcirc}$ master formulas



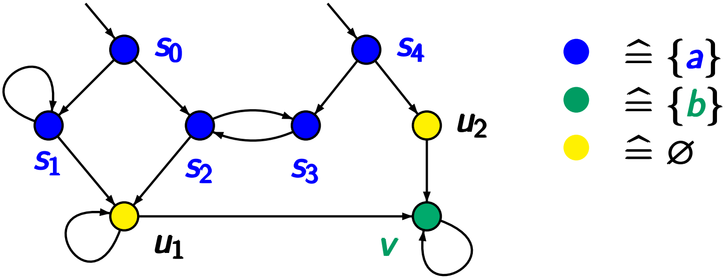
$\approx_{\mathcal{T}}^{\mathsf{div}}$-equiv. classes	$CTL_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	$\neg\exists\Box(\neg a \wedge \neg b) \wedge \neg a \wedge \neg b$
$\{s_0, s_1, s_2, s_3\}$	
$\{s_4\}$	

$\text{CTL}_{\setminus\bigcirc}$ master formulas



$\approx_{\mathcal{T}}^{\text{div}}$-equiv. classes	$\text{CTL}_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	$\neg\exists\Box(\neg a \wedge \neg b) \wedge \neg a \wedge \neg b$
$\{s_0, s_1, s_2, s_3\}$	$a \wedge \quad \forall(a \mathsf{W} \exists\Box(\neg a \wedge \neg b))$
$\{s_4\}$	

$\text{CTL}_{\setminus\bigcirc}$ master formulas



$\approx^{\text{div}}_{\mathcal{T}}$-equiv. classes	$\text{CTL}_{\setminus\bigcirc}$ master formulas
$\{v\}$	b
$\{u_1\}$	$\exists\Box(\neg a \wedge \neg b)$
$\{u_2\}$	$\neg\exists\Box(\neg a \wedge \neg b) \wedge \neg a \wedge \neg b$
$\{s_0, s_1, s_2, s_3\}$	$a \wedge \forall(a \,\mathsf{W}\, \exists\Box(\neg a \wedge \neg b))$
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$\approx^{\mathsf{div}}$ and CTL*$_{\setminus\bigcirc}$/CTL$_{\setminus\bigcirc}$-equivalence

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Let s_1, s_2 be states of a finite TS without terminal states. Then, the following statements are equivalent:

(1) $s_1 \approx^{\textbf{div}}_{\mathcal{T}} s_2$

(2) s_1, s_2 satisfy the same CTL*$_{\setminus\bigcirc}$ formulas

(3) s_1, s_2 satisfy the same CTL$_{\setminus\bigcirc}$ formulas

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(2) $\Longrightarrow$ (3): clear as CTL$_{\setminus\bigcirc}$ is a sublogic of CTL*$_{\setminus\bigcirc}$

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(2) s_1, s_2 satisfy the same $\text{CTL}^*_{\setminus\bigcirc}$ formulas

(3) s_1, s_2 satisfy the same $\text{CTL}_{\setminus\bigcirc}$ formulas

$(2) \Longrightarrow (3)$: clear as $\text{CTL}_{\setminus\bigcirc}$ is a sublogic of $\text{CTL}^*_{\setminus\bigcirc}$

$(3) \Longrightarrow (1)$: $\text{CTL}_{\setminus\bigcirc}$-equivalence is a divergence-sensitive stutter abstract bisimulation

$\approx^{\text{div}}$ and CTL*$_{\setminus\bigcirc}$/CTL$_{\setminus\bigcirc}$-equivalence

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(1) $\Longrightarrow$ (2): proof by structural induction

$\approx^{\text{div}}$ and CTL*$_{\setminus\bigcirc}$/CTL$_{\setminus\bigcirc}$-equivalence

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$(1) \Longrightarrow (2)$: proof by structural induction

$\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths

Recall: $\sim_{\mathcal{T}}$ for paths

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

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$\pi_1 \sim_{\mathcal{T}} \pi_2$ iff π_1 and π_2 are statewise bisimilar

Recall: $\sim_{\mathcal{T}}$ for paths

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

$\pi_1 \sim_{\mathcal{T}} \pi_2$ iff π_1 and π_2 are statewise bisimilar, i.e., if

$$\pi_1 = s_{1,0}\, s_{1,1}\, s_{1,2}\, s_{1,3}\, \ldots$$

$$\pi_2 = s_{2,0}\, s_{2,1}\, s_{2,2}\, s_{2,3}\, \ldots$$

s.t. $s_{1,i} \sim_{\mathcal{T}} s_{2,i}$ for all $i \geq 0$

Recall: $\sim_{\mathcal{T}}$ for paths

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analogous definition for $\approx_{\mathcal{T}}^{\text{div}}$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

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$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ iff there exist infinite sequences

$$C_0, C_1, C_2, \ldots \quad \in (S/\approx_{\mathcal{T}}^{\mathsf{div}})^{\omega}$$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths

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$$C_0, C_1, C_2, \ldots \quad \in (S/\approx_{\mathcal{T}}^{\mathsf{div}})^{\omega}$$

$$n_0, n_1, n_2, \ldots \quad \in \mathbb{N}_{\geq 1}^{\omega}$$

$$m_0, m_1, m_2, \ldots \quad \in \mathbb{N}_{\geq 1}^{\omega}$$

Let π_1, π_2 be infinite path fragments in $\mathcal{T}$.

$\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$ iff there exist infinite sequences

$$C_0, C_1, C_2, \ldots \in (S/\approx_{\mathcal{T}}^{\text{div}})^{\omega}$$
$$n_0, n_1, n_2, \ldots \in \mathbb{N}_{\geq 1}^{\omega}$$
$$m_0, m_1, m_2, \ldots \in \mathbb{N}_{\geq 1}^{\omega}$$

such that

$$\pi_1 = \overbrace{s_{1,1} \ \ldots \ s_{1,n_0}}^{\in C_0} \overbrace{t_{1,1} \ldots t_{1,n_1}}^{\in C_1} \overbrace{u_{1,1} \ldots u_{1,n_2}}^{\in C_2} \ldots$$

$$\pi_2 = \underbrace{s_{2,1} \ldots s_{2,m_0}}_{\in C_0} \underbrace{t_{2,1} \ \ldots \ t_{2,m_1}}_{\in C_1} \underbrace{u_{2,1} \ \ldots \ u_{2,m_2}}_{\in C_2} \ldots$$

Stutter relations for paths

stutter trace equivalence: $\pi_1 \triangleq \pi_2$

π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
labeling	A_0 A_1 A_2 $\ldots$

where $A_0, A_1, A_2, \ldots$ are subsets of AP

stutter trace equivalence: $\pi_1 \triangleq \pi_2$

π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
labeling	$A_0 \qquad A_1 \qquad A_2 \quad \ldots$

where $A_0, A_1, A_2, \ldots$ are subsets of AP

stutter bis. equiv. with divergence: $\pi_1 \approx^{\mathsf{div}}_{\mathcal{T}} \pi_2$

π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
equiv.class	$C_0 \qquad C_1 \qquad C_2 \quad \ldots$

where $C_0, C_1, C_2, \ldots$ are $\approx^{\mathsf{div}}_{\mathcal{T}}$-equivalence classes

stutter trace equivalence: $\pi_1 \triangleq \pi_2$

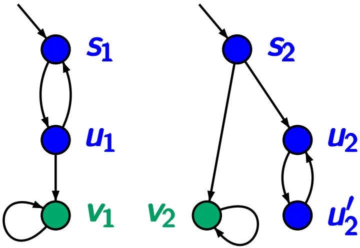
π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
labeling	A_0 A_1 A_2 ...

stutter bis. equiv. with divergence: $\pi_1 \approx^{\text{div}}_{\mathcal{T}} \pi_2$

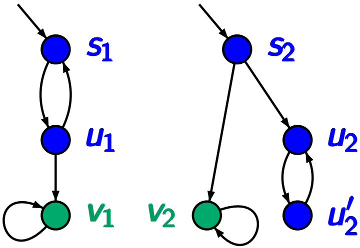
π_1 π_2	$s_1\ s_2 \ldots s_i\ u_1 u_2 u_3 \ldots u_j\ v_1 \ldots v_k\ \ldots$ $s'_1\ s'_2\ \ldots\ s'_r\ u'_1 \ldots u'_t\ v'_1\ v'_2 \ldots v'_q\ \ldots$
equiv.class	C_0 C_1 C_2 ...

If $\pi_1 \approx^{\text{div}}_{\mathcal{T}} \pi_2$ then $\pi_1 \triangleq \pi_2$.

Example: $\approx^{\mathrm{div}}_{\mathcal{T}}$ for paths

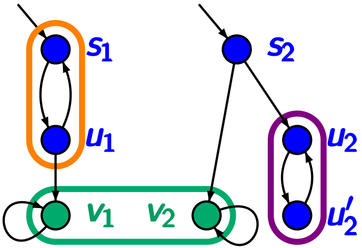


Example: $\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths



$$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$

Example: $\approx_{\mathcal{T}}^{\text{div}}$ for paths



$\approx_{\mathcal{T}}^{\text{div}}$-equiv. classes:

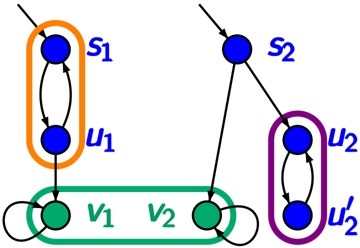
$\{s_1, u_1\}$, $\{s_2\}$

$\{u_2, u_2'\}$, $\{v_1, v_2\}$

$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$

$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$

Example: $\approx_{\mathcal{T}}^{\text{div}}$ for paths



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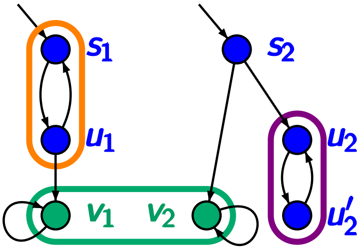
$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$

$\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$

Example: $\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths



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$$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$$

$$\pi_1' = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$
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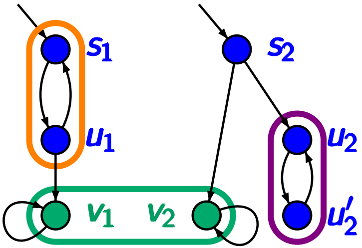
Example: $\approx_{\mathcal{T}}^{\text{div}}$ for paths



$\approx_{\mathcal{T}}^{\text{div}}$-equiv. classes:

$\{s_1, u_1\}$, $\{s_2\}$

$\{u_2, u_2'\}$, $\{v_1, v_2\}$

$\pi_1 = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$

$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$

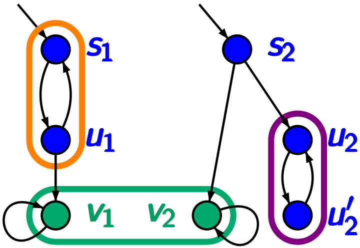
$\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$

$\pi_1' = s_1\ u_1\ s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ \ldots$

$\pi_2' = s_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ \ldots$

$\pi_1' \triangleq \pi_2'$,

Example: $\approx_{\mathcal{T}}^{\mathsf{div}}$ for paths



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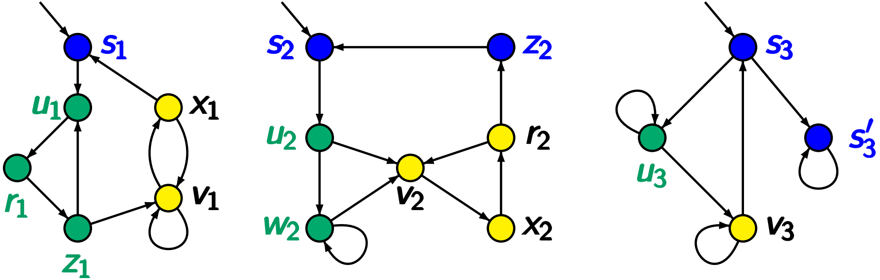
$$\pi_2 = s_1\ u_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ v_1\ \ldots$$

$$\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$$

$$\pi_1' = \boxed{s_1\ u_1\ s_1\ u_1}\ v_1\ v_1\ v_1\ v_1\ \ldots$$
$$\pi_2' = \boxed{s_2}\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ v_2\ \ldots$$

$$\pi_1' \triangleq \pi_2',$$
$$\pi_1' \not\approx_{\mathcal{T}}^{\mathsf{div}} \pi_2'$$

For which indices i, j, does $\pi_i \approx^{\mathbf{div}}_{\mathcal{T}} \pi_j$ hold?

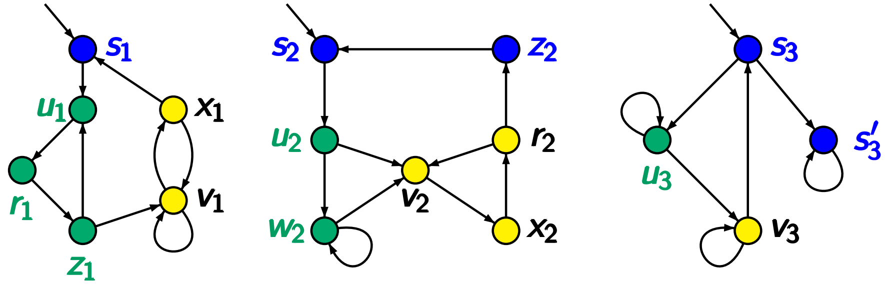


$\pi_1 = s_1\ u_1\ r_1\ z_1\ x_1\ x_1\ x_1\ v_1\ s_1\ u_1\ r_1\ z_1\ x_1\ x_1\ x_1\ x_1 \ldots$

$\pi_2 = s_2\ u_2\ w_2\ w_2\ w_2\ v_2\ x_2\ r_2\ v_2\ x_2\ r_2\ z_2\ s_2\ u_2\ v_2\ x_2 \ldots$

$\pi_3 = s_3\ u_3\ u_3\ u_3\ u_3\ v_3\ v_3\ v_3\ v_3\ v_3\ v_3\ s_3\ u_3\ u_3\ v_3\ v_3 \ldots$

For which indices i, j, does $\pi_i \approx_{\mathcal{T}}^{\text{div}} \pi_j$ hold?



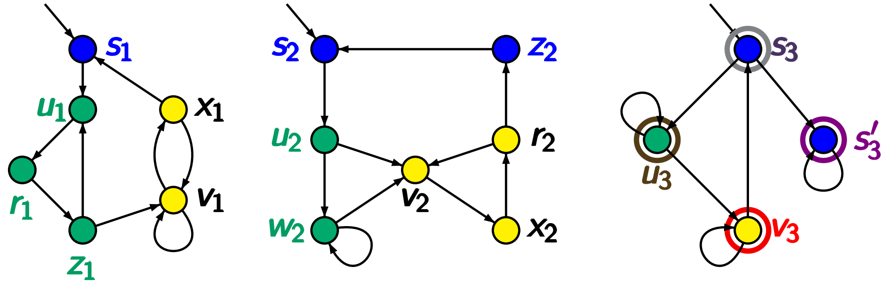
$\pi_1 = s_1\ u_1\ r_1\ z_1\ x_1\ x_1\ x_1\ v_1\ s_1\ u_1\ r_1\ z_1\ x_1\ x_1\ x_1\ x_1 \ldots$

$\pi_2 = s_2\ u_2\ w_2\ w_2\ w_2\ v_2\ x_2\ r_2\ v_2\ x_2\ r_2\ z_2\ s_2\ u_2\ v_2\ x_2 \ldots$

$\pi_3 = s_3\ u_3\ u_3\ u_3\ u_3\ v_3\ v_3\ v_3\ v_3\ v_3\ v_3\ s_3\ u_3\ u_3\ v_3\ v_3 \ldots$

$\approx_{\mathcal{T}}^{\text{div}}$-equivalence classes: ?

For which indices i, j, does $\pi_i \approx^{\text{div}}_{\mathcal{T}} \pi_j$ hold?



$\pi_1 = \; s_1 \; u_1 \; r_1 \; z_1 \; x_1 \; x_1 \; x_1 \; v_1 \; s_1 \; u_1 \; r_1 \; z_1 \; x_1 \; x_1 \; x_1 \; x_1 \ldots$

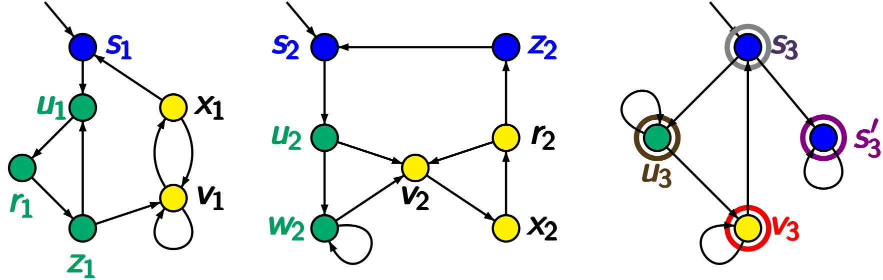
$\pi_2 = \; s_2 \; u_2 \; w_2 \; w_2 \; w_2 \; v_2 \; x_2 \; r_2 \; v_2 \; x_2 \; r_2 \; z_2 \; s_2 \; u_2 \; v_2 \; x_2 \ldots$

$\pi_3 = \; s_3 \; u_3 \; u_3 \; u_3 \; u_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; s_3 \; u_3 \; u_3 \; v_3 \; v_3 \ldots$

$\approx^{\text{div}}_{\mathcal{T}}$-equivalence classes: $\{s_1, s_2, z_2\}$ $\{s_3\}$ $\{s'_3\}$ $\{u_3\}$

$\{u_1, r_1, z_1, u_2, w_2\}$ $\{v_1, x_1, v_2, x_2, r_2\}$ $\{v_3\}$

For which indices i, j, does $\pi_i \approx_{\mathcal{T}}^{\text{div}} \pi_j$ hold?



$\pi_1 =$ s_1 | $u_1\ r_1\ z_1$ | $v_1\ v_1\ v_1\ x_1$ | s_1 | $u_1\ r_1\ z_1$ | $v_1\ v_1\ v_1\ x_1 \ldots$

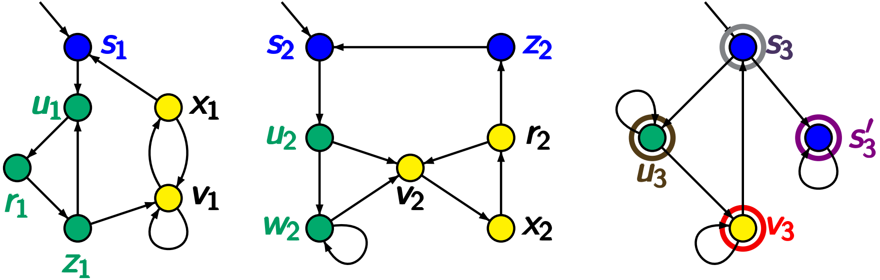
$\pi_2 =$ s_2 | $u_2\ w_2\ w_2\ w_2$ | $v_2\ x_2\ r_2\ v_2\ x_2\ r_2$ | $z_2\ s_2$ | u_2 | $v_2\ x_2 \ldots$

$\pi_3 = s_3\ u_3\ u_3\ u_3\ u_3\ v_3\ v_3\ v_3\ v_3\ v_3\ v_3\ s_3\ u_3\ u_3\ v_3\ v_3 \ldots$

$\approx_{\mathcal{T}}^{\text{div}}$-equivalence classes: $\{s_1, s_2, z_2\}$ $\{s_3\}$ $\{s_3'\}$ $\{u_3\}$

$\{u_1, r_1, z_1, u_2, w_2\}$ $\{v_1, x_1, v_2, x_2, r_2\}$ $\{v_3\}$

For which indices i, j, does $\pi_i \approx_{\mathcal{T}}^{\text{div}} \pi_j$ hold?



$\pi_1 = s_1 \; u_1 \; r_1 \; z_1 \; v_1 \; v_1 \; v_1 \; x_1 \; s_1 \; u_1 \; r_1 \; z_1 \; v_1 \; v_1 \; v_1 \; x_1 \ldots$

$\pi_2 = s_2 \; u_2 \; w_2 \; w_2 \; w_2 \; v_2 \; x_2 \; r_2 \; v_2 \; x_2 \; r_2 \; z_2 \; s_2 \; u_2 \; v_2 \; x_2 \ldots$

$\pi_3 = s_3 \; u_3 \; u_3 \; u_3 \; u_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; v_3 \; s_3 \; u_3 \; u_3 \; v_3 \; v_3 \ldots$

$\approx_{\mathcal{T}}^{\text{div}}$-equivalence classes: $\{s_1, s_2, z_2\}$ $\{s_3\}$ $\{s_3'\}$ $\{u_3\}$

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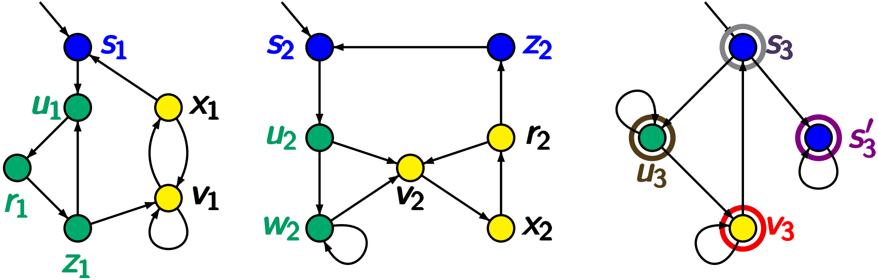
For which indices i, j, does $\pi_i \approx^{\text{div}}_{\mathcal{T}} \pi_j$ hold?



$\pi_1 =$ s_1 | $u_1\ r_1\ z_1$ | $v_1\ v_1\ v_1\ x_1$ | s_1 | $u_1\ r_1\ z_1$ | $v_1\ v_1\ v_1\ x_1 \ldots$

$\pi_2 =$ s_2 | $u_2\ w_2\ w_2\ w_2$ | $v_2\ x_2\ r_2\ v_2\ x_2\ r_2$ | $z_2\ s_2$ | u_2 | $v_2\ x_2 \ldots$

$\pi_3 =$ s_3 | $u_3\ u_3\ u_3\ u_3$ | $v_3\ v_3\ v_3\ v_3\ v_3\ v_3$ | s_3 | $u_3\ u_3$ | $v_3\ v_3 \ldots$

$\pi_1 \approx^{\text{div}}_{\mathcal{T}} \pi_2$, but $\pi_1, \pi_2 \not\approx^{\text{div}}_{\mathcal{T}} \pi_3$

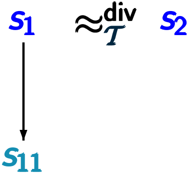
Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

Path lifting for $\approx_{\mathcal{T}}^{\mathrm{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathrm{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathrm{div}} \pi_2$.

Path lifting for $\approx_{\mathcal{T}}^{\text{div}}$

If $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$.



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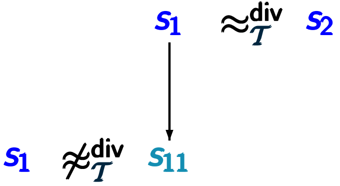
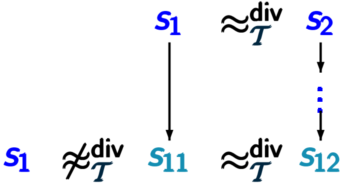


Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

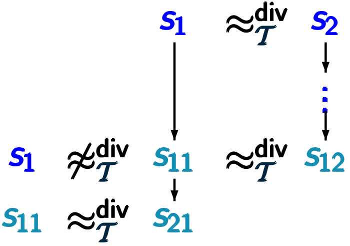


stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathsf{div}} u_i$

Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in Paths(s_1)$ there exists $\pi_2 \in Paths(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.



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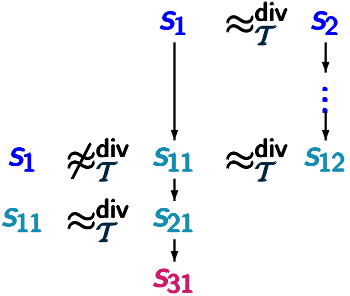
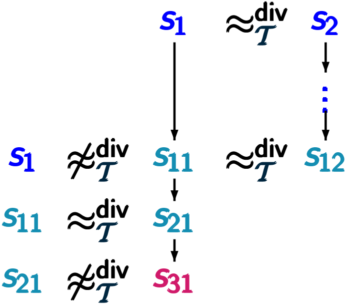


stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathsf{div}} u_i$

Path lifting for $\approx^{div}_{\mathcal{T}}$

If $s_1 \approx^{div}_{\mathcal{T}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx^{div}_{\mathcal{T}} \pi_2$.

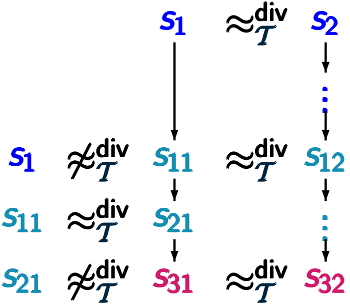


stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx^{div}_{\mathcal{T}} u_i$

Path lifting for $\approx_{\mathcal{T}}^{\mathsf{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.



stutter steps $s_2 u_1 \ldots u_n$

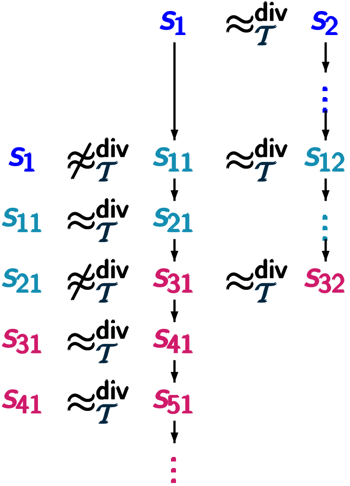
$s_2 \approx_{\mathcal{T}}^{\mathsf{div}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx_{\mathcal{T}}^{\mathsf{div}} v_j$

Path lifting for $\approx_{\mathcal{T}}^{\text{div}}$

> If $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$.



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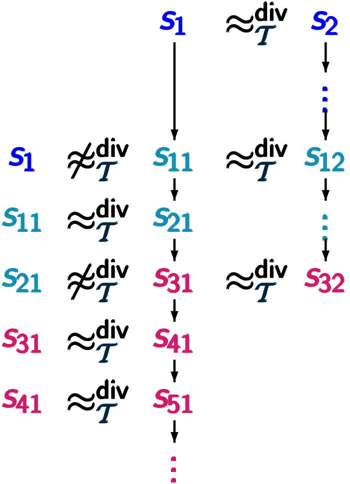

stutter steps $s_2 u_1 \ldots u_n$

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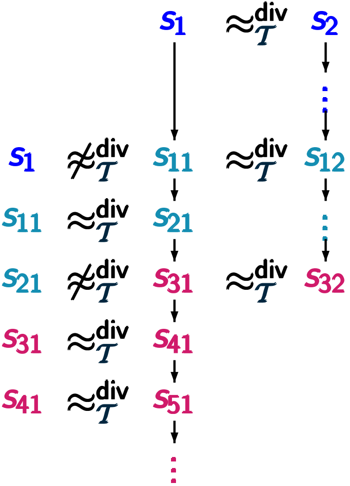
stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx_{\mathcal{T}}^{\text{div}} v_j$

s_{31} divergent

Path lifting for $\approx_{\mathcal{T}}^{\mathrm{div}}$

If $s_1 \approx_{\mathcal{T}}^{\mathrm{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathrm{div}} \pi_2$.



stutter steps $s_2 u_1 \ldots u_n$

$s_2 \approx_{\mathcal{T}}^{\mathrm{div}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$

with $s_{12} \approx_{\mathcal{T}}^{\mathrm{div}} v_j$

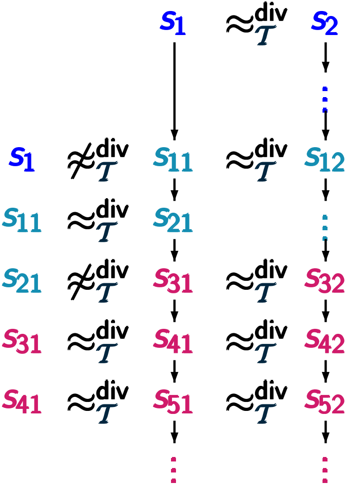
s_{31} divergent

$\Downarrow$

s_{32} divergent

Path lifting for $\approx_{\mathcal{T}}^{\text{div}}$

If $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$.



stutter steps $s_2 u_1 \ldots u_n$
$s_2 \approx_{\mathcal{T}}^{\text{div}} u_i$

stutter steps $s_{12} v_1 \ldots v_m$
with $s_{12} \approx_{\mathcal{T}}^{\text{div}} v_j$

s_{31} divergent
$\Downarrow$
s_{32} divergent

Properties of $\approx_{\mathcal{T}}^{\mathsf{div}}$ on paths

STUTTER5.4-35A

Properties of $\approx_{\mathcal{T}}^{\mathsf{div}}$ on paths

We just saw:

- If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$.

Properties of $\approx_{\mathcal{T}}^{\text{div}}$ on paths

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- If $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ then for all paths $\pi_1 \in Paths(s_1)$ there exists $\pi_2 \in Paths(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$.
- If $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$ then $\pi_1 \triangleq_{\mathcal{T}} \pi_2$.

↑ stutter trace equivalence

We just saw:

- If $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ then for all paths $\pi_1 \in \mathit{Paths}(s_1)$ there exists $\pi_2 \in \mathit{Paths}(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$.
- If $\pi_1 \approx_{\mathcal{T}}^{\text{div}} \pi_2$ then $\pi_1 \triangleq_{\mathcal{T}} \pi_2$.

↑ stutter trace equivalence

Hence we get:

> Stutter bisimulation equivalence with divergence is finer than stutter trace equivalence, i.e.,
>
> $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$ implies $s_1 \triangleq_{\mathcal{T}} s_2$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ is finer than CTL*$_{\setminus\bigcirc}$-equivalence

CTL* $_{\setminus \bigcirc}$ state formulas:

$$\Phi ::= \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg \Phi \mid \exists \varphi$$

CTL* $_{\setminus \bigcirc}$ path formulas:

$$\varphi ::= \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg \varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

$\approx_{\mathcal{T}}^{\mathsf{div}}$ is finer than $\mathrm{CTL}^*_{\setminus \bigcirc}$-equivalence

$\mathrm{CTL}^*_{\setminus \bigcirc}$ state formulas:

$$\Phi ::= \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

$\mathrm{CTL}^*_{\setminus \bigcirc}$ path formulas:

$$\varphi ::= \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \varphi_1 \mathsf{U}\, \varphi_2$$

we show by structural induction:

1. If $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all $\mathrm{CTL}^*_{\setminus \bigcirc}$ state formulas Φ:
   $$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$
2. If $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$ then for all $\mathrm{CTL}^*_{\setminus \bigcirc}$ path formulas φ:
   $$\pi_1 \models \varphi \quad \text{iff} \quad \pi_2 \models \varphi$$

Divergence-sensitive TS

Divergence-sensitive TS

Suppose $\mathcal{T}$ is divergence-sensitive, i.e.,

whenever $s_1 \approx_{\mathcal{T}} s_2$ and s_1 is $\approx_{\mathcal{T}}$-divergent
then s_2 is $\approx_{\mathcal{T}}$-divergent.

Suppose $\mathcal{T}$ is divergence-sensitive, i.e.,

> whenever $s_1 \approx_{\mathcal{T}} s_2$ and s_1 is $\approx_{\mathcal{T}}$-divergent then s_2 is $\approx_{\mathcal{T}}$-divergent.

Hence: $\approx_{\mathcal{T}}$ is a divergence-sensitive stutter bisimulation.

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Hence: $\approx_{\mathcal{T}}$ is a divergence-sensitive stutter bisimulation.

This yields: $s_1 \approx_{\mathcal{T}} s_2$ implies $s_1 \approx_{\mathcal{T}}^{\mathbf{div}} s_2$

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> whenever $s_1 \approx_{\mathcal{T}} s_2$ and s_1 is $\approx_{\mathcal{T}}$-divergent
> then s_2 is $\approx_{\mathcal{T}}$-divergent.

Hence: $\approx_{\mathcal{T}}$ is a divergence-sensitive stutter bisimulation.

This yields: $s_1 \approx_{\mathcal{T}} s_2$ implies $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$

As $\approx_{\mathcal{T}}$ is coarser than $\approx_{\mathcal{T}}^{\mathsf{div}}$ we get: $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\mathsf{div}}$

↑

even holds for any transition system

Divergence-sensitive TS

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then the following statements are equivalent:

(1) $s_1 \approx_{\mathcal{T}} s_2$

(2) $s_1 \approx_{\mathcal{T}}^{\text{div}} s_2$

If s_1, s_2 are states of a divergence-sensitive, finite TS $\mathcal{T}$ then the following statements are equivalent:

(1) $s_1 \approx_{\mathcal{T}} s_2$

(2) $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$

(3) s_1, s_2 satisfy the same $\mathrm{CTL}^*_{\setminus \bigcirc}$ formulas

Divergence-sensitive TS

If s_1, s_2 are states of a divergence-sensitive, finite TS $\mathcal{T}$ then the following statements are equivalent:

(1) $s_1 \approx_{\mathcal{T}} s_2$

(2) $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$

(3) s_1, s_2 satisfy the same $\mathbf{CTL^*}_{\setminus\bigcirc}$ formulas

(4) s_1, s_2 satisfy the same $\mathbf{CTL}_{\setminus\bigcirc}$ formulas

Stutter bisimulation and $CTL_{\setminus\bigcirc}$/$CTL^*_{\setminus\bigcirc}$

For finite divergence-sensitive transition systems:

$s_1 \approx_{\mathcal{T}} s_2$ iff s_1, s_2 satisfy the same $CTL^*_{\setminus\bigcirc}$ formulas
iff s_1, s_2 satisfy the same $CTL_{\setminus\bigcirc}$ formulas

wrong for non-divergence-sensitive TS:

For finite divergence-sensitive transition systems:

$s_1 \approx_{\mathcal{T}} s_2$ iff s_1, s_2 satisfy the same $\text{CTL}^*_{\setminus\bigcirc}$ formulas
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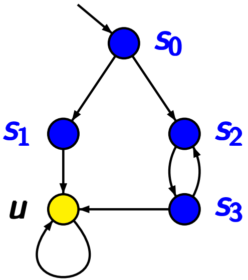
wrong for non-divergence-sensitive TS:



For finite divergence-sensitive transition systems:

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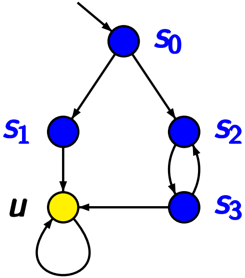


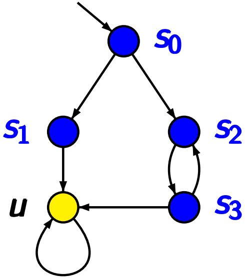
stutter bis. equivalence classes:
$\{s_0, s_1, s_2, s_4\}$ $\{u\}$

Stutter bisimulation and $CTL_{\setminus\bigcirc}$/$CTL^*_{\setminus\bigcirc}$

For finite divergence-sensitive transition systems:

$s_1 \approx_{\mathcal{T}} s_2$ iff s_1, s_2 satisfy the same $CTL^*_{\setminus\bigcirc}$ formulas
iff s_1, s_2 satisfy the same $CTL_{\setminus\bigcirc}$ formulas

wrong for non-divergence-sensitive TS:



stutter bis. equivalence classes:
$\{s_0, s_1, s_2, s_4\}$ $\{u\}$

$s_1 \not\models \exists\Box$ **blue**

$s_2 \models \exists\Box$ **blue**

Correct or wrong?

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then

$$s_1 \approx_{\mathcal{T}} s_2 \quad \text{implies} \quad s_1 \triangleq_{\mathcal{T}} s_2$$

$\approx_{\mathcal{T}}$ stutter bisimulation equivalence (without divergence)

$\triangleq_{\mathcal{T}}$ stutter trace equivalence

Correct or wrong?

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then

$$s_1 \approx_{\mathcal{T}} s_2 \quad \text{implies} \quad s_1 \triangleq_{\mathcal{T}} s_2$$

correct

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then

$$s_1 \approx_{\mathcal{T}} s_2 \quad \text{implies} \quad s_1 \triangleq_{\mathcal{T}} s_2$$

correct

- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\textbf{div}}$ in divergence-sensitive TS

Correct or wrong?

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if $s_1 \approx_{\mathcal{T}}^{\mathsf{div}} s_2$ then for all $\pi_1 \in Paths(s_1)$
there exists $\pi_2 \in Paths(s_2)$ such that $\pi_1 \approx_{\mathcal{T}}^{\mathsf{div}} \pi_2$

Correct or wrong?

If s_1, s_2 are states of a divergence-sensitive TS $\mathcal{T}$ then

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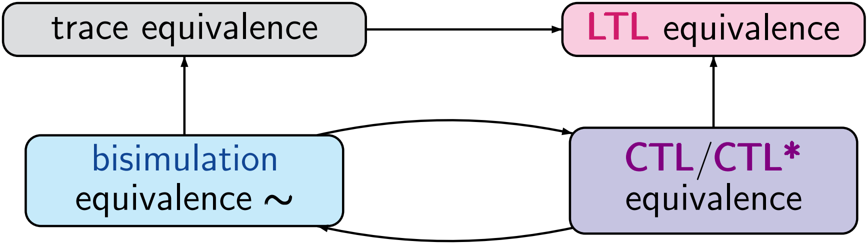
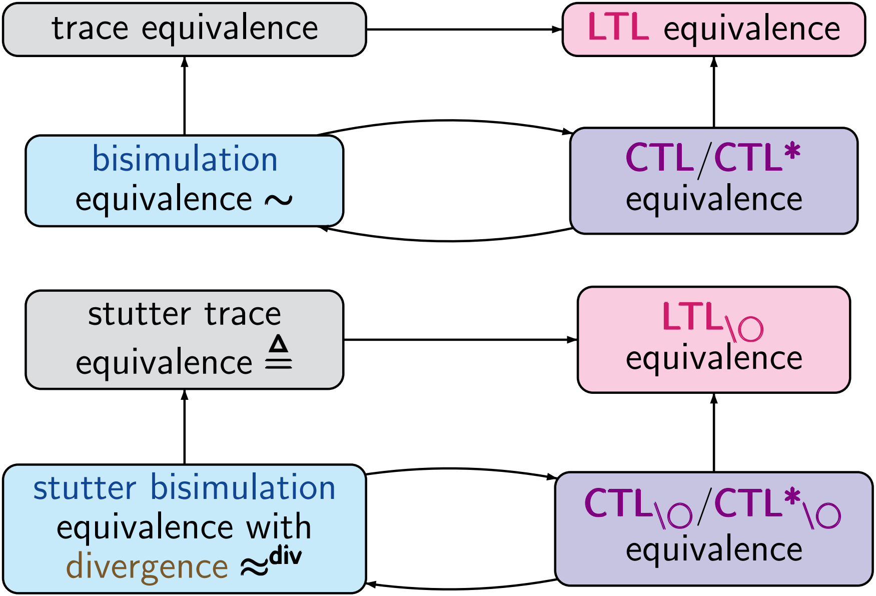
- $\pi_1 \approx_{\mathcal{T}}^{\mathbf{div}} \pi_2$ implies $\pi_1 \triangleq_{\mathcal{T}} \pi_2$

Summary: equivalences on finite TS



Summary: equivalences on finite TS



Let $\mathcal{T}$ be a TS without terminal states, possibly not divergence-sensitive, possibly infinite.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathbf{CTL}_{\setminus \bigcirc}$ formulas of the form $\exists \Diamond a$ where $a \in AP$.

Correct or wrong?

Let $\mathcal{T}$ be a TS without terminal states,
possibly not divergence-sensitive, possibly infinite.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same **CTL**$_{\setminus\bigcirc}$ formulas of the form $\exists\Diamond a$ where $a \in AP$.

correct.

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correct.

Note: path lifting for finite path fragments is possible

Lifting of finite path fragments

$$s_1 = s_{01} \quad \approx_{\mathcal{T}} \quad s_2$$

$$s_{11}$$

$$\vdots$$

$$s_{n1}$$

$$s_{n+1,1}$$

$$\vdots$$

$$s_{m,1}$$

$$s_{m+1,1}$$

$$\vdots$$

$$s_{k,1}$$

$$t$$

Lifting of finite path fragments

$$s_1 = s_{01} \quad \approx_{\mathcal{T}} \quad s_2$$

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$$s_{n+1,1}$$

$$\vdots$$

$$s_{m,1}$$

$$s_{m+1,1}$$

$$\vdots$$

$$s_{k,1}$$

$$t$$

where $t \models a$

Lifting of finite path fragments

$$
\begin{array}{rcl}
s_1 = s_{01} & \approx_{\mathcal{T}} & s_2 \\
s_{11} & & \\
\vdots & & \\
s_{n1} & & \\
s_{n+1,1} & & \\
\vdots & & \\
s_{m,1} & & \\
s_{m+1,1} & & \\
\vdots & & \\
s_{k,1} & & \\
t & &
\end{array}
\quad \rightsquigarrow \quad
\begin{array}{rcl}
s_1 = s_{01} & \approx_{\mathcal{T}} & s_{11} = s_2 \\
s_{11} & & \vdots \\
\vdots & & s_{r2} \\
s_{n1} & & s_{r+1,2} \\
s_{n+1,1} & & \vdots \\
\vdots & & s_{l,2} \\
s_{m,1} & & s_{l+1,2} \\
s_{m+1,1} & & \vdots \\
\vdots & & s_{i,2} \\
s_{k,1} & & u \\
t & &
\end{array}
$$

where $t \models a$

Lifting of finite path fragments

$$
\begin{array}{rcl}
s_1 = s_{01} & \approx_{\mathcal{T}} & s_2 \\
s_{11} & & \\
\vdots & & \\
s_{n1} & & \\
s_{n+1,1} & & \\
\vdots & & \\
s_{m,1} & & \\
s_{m+1,1} & & \\
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s_{k,1} & & \\
t & &
\end{array}
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\vdots & & s_{l,2} \\
s_{m,1} & & s_{l+1,2} \\
s_{m+1,1} & & \vdots \\
\vdots & & s_{i,2} \\
s_{k,1} & & u \\
t & & t \approx_{\mathcal{T}} u
\end{array}
$$

where $t \models a$

Lifting of finite path fragments

$$\begin{array}{rcl} s_1 = s_{01} & \approx_{\mathcal{T}} & s_2 \\ s_{11} & & \\ \vdots & & \\ s_{n1} & & \\ s_{n+1,1} & & \\ \vdots & & \\ s_{m,1} & & \\ s_{m+1,1} & & \\ \vdots & & \\ s_{k,1} & & \\ t & & \end{array}$$

where $t \models a$

$$\rightsquigarrow$$

$$\begin{array}{rcl} s_1 = s_{01} & \approx_{\mathcal{T}} & s_{11} = s_2 \\ s_{11} & & \vdots \\ \vdots & & s_{r2} \\ s_{n1} & & s_{r+1,2} \\ s_{n+1,1} & & \vdots \\ \vdots & & s_{l,2} \\ s_{m,1} & & s_{l+1,2} \\ s_{m+1,1} & & \vdots \\ \vdots & & s_{i,2} \\ s_{k,1} & & u \\ t & & t \approx_{\mathcal{T}} u \end{array}$$

hence: $u \models a$

Correct or wrong?

Let $\mathcal{T}$ be a TS without terminal states,
possibly not divergence-sensitive, possibly infinite.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathbf{CTL}_{\setminus \bigcirc}$ formulas of the form $\forall\Box a$ where $a \in AP$.

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correct.

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correct.

$$s_1 \models \forall\Box a \quad \text{iff} \quad s_1 \not\models \exists\Diamond\neg a$$

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If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathbf{CTL}_{\setminus\bigcirc}$ formulas of the form $\forall\Box a$ where $a \in AP$.

correct.

$$s_1 \models \forall\Box a \quad \text{iff} \quad s_1 \not\models \exists\Diamond\neg a$$
$$\text{iff} \quad s_2 \not\models \exists\Diamond\neg a$$

Let $\mathcal{T}$ be a TS without terminal states,
possibly not divergence-sensitive, possibly infinite.

> If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathrm{CTL}_{\setminus\bigcirc}$ formulas of the form $\forall\Box a$ where $a \in AP$.

correct.

$$
\begin{aligned}
s_1 \models \forall\Box a \quad & \text{iff} \quad s_1 \not\models \exists\Diamond\neg a \\
& \text{iff} \quad s_2 \not\models \exists\Diamond\neg a \\
& \text{iff} \quad s_2 \models \forall\Box a
\end{aligned}
$$

Correct or wrong?

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\text{CTL}_{\setminus \bigcirc}$ formulas of the form $\forall \Box a$ where $a \in AP$.

correct.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\text{CTL}_{\setminus \bigcirc}$ formulas of the form $\forall \Diamond a$ where $a \in AP$.

Correct or wrong?

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $CTL_{\setminus \bigcirc}$ formulas of the form $\forall \Box a$ where $a \in AP$.

correct.

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $CTL_{\setminus \bigcirc}$ formulas of the form $\forall \Diamond a$ where $a \in AP$.

wrong.

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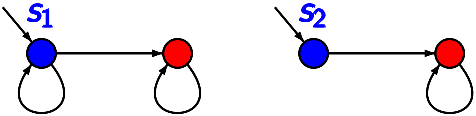
wrong.



Correct or wrong?

If $s_1 \approx_{\mathcal{T}} s_2$ then s_1 and s_2 satisfy the same $\mathrm{CTL}_{\setminus\bigcirc}$ formulas of the form $\forall\Box a$ where $a \in AP$.

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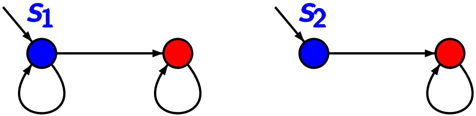

$s_1 \not\models \forall\Diamond a$

$s_2 \models \forall\Diamond a$

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

If $\mathcal{T}_1 \approx \mathcal{T}_2$ then $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL***$_{\setminus \bigcirc}$ formulas

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

If $\mathcal{T}_1 \approx \mathcal{T}_2$ then $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same $\mathbf{CTL^*}_{\setminus \bigcirc}$ formulas

wrong

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

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wrong, as $\mathcal{T}_1 \uplus \mathcal{T}_2$ might be not divergence-sensitive

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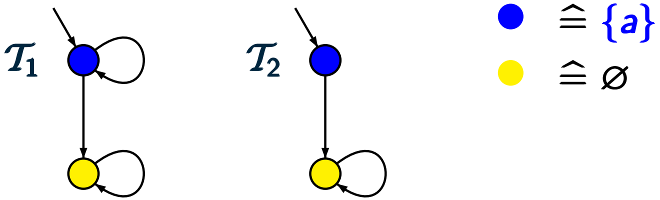

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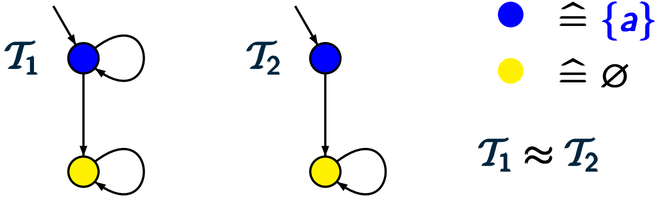

$\mathcal{T}_1 \approx \mathcal{T}_2$

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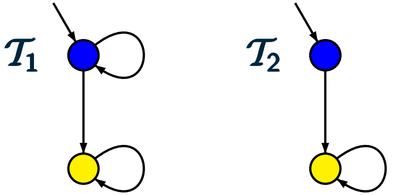

$\mathcal{T}_1 \models \exists \Box a$ $\quad$ $\mathcal{T}_2 \not\models \exists \Box a$

● $\widehat{=} \{a\}$

● $\widehat{=} \varnothing$

$\mathcal{T}_1 \approx \mathcal{T}_2$

Correct or wrong?

If $\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive, finite TS then:

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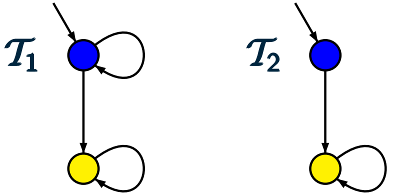

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● $\widehat{=} \varnothing$

$\mathcal{T}_1 \approx \mathcal{T}_2$

$\mathcal{T}_1 \models \exists \Box a$ $\quad$ $\mathcal{T}_2 \not\models \exists \Box a$

$\mathcal{T}_1$, $\mathcal{T}_2$ are divergence-sensitive

$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

$\mathcal{T}$ and $\mathcal{T}/\!\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\!\approx$

$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$ $\longleftarrow$ holds for any TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$
- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\mathsf{div}} =$ $\mathsf{CTL}^*_{\setminus \bigcirc}$-equivalence on $\mathcal{T}$

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$
- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\textsf{div}} =$ $\mathbf{CTL^*}_{\setminus\bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/\approx$ might be not $\mathbf{CTL^*}_{\setminus\bigcirc}$-equivalent

$\mathcal{T}$ and $\mathcal{T}/\!\approx$ for divergence-sensitive TS

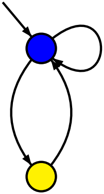
Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\!\approx$
- $\approx_{\mathcal{T}} \; = \; \approx^{\mathsf{div}}_{\mathcal{T}} \; = \;$ **CTL***$_{\setminus \bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/\!\approx$ might be not **CTL***$_{\setminus \bigcirc}$-equivalent
- $\mathcal{T} \uplus \mathcal{T}/\!\approx$ might be not divergence-sensitive

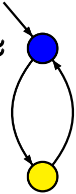
Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\!\approx$
- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\mathsf{div}} =$ **CTL***$_{\setminus\bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/\!\approx$ might be not **CTL***$_{\setminus\bigcirc}$-equivalent
- $\mathcal{T} \uplus \mathcal{T}/\!\approx$ might be not divergence-sensitive

$\mathcal{T}$



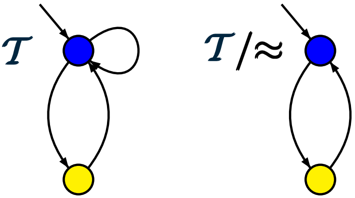
$\mathcal{T}/\!\approx$



$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$
- $\approx_{\mathcal{T}} = \approx_{\mathcal{T}}^{\mathsf{div}} =$ CTL*$_{\setminus \bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/\approx$ might be not CTL*$_{\setminus \bigcirc}$-equivalent
- $\mathcal{T} \uplus \mathcal{T}/\approx$ might be not divergence-sensitive

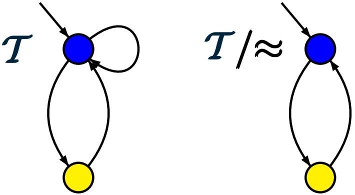


$\mathcal{T}$ (and $\mathcal{T}/\approx$) are divergence-sensitive

$\mathcal{T}$ and $\mathcal{T}/\approx$ for divergence-sensitive TS

Let $\mathcal{T}$ be a divergence-sensitive, finite TS. Then:

- $\mathcal{T} \approx \mathcal{T}/\approx$
- $\approx_{\mathcal{T}} = \approx^{\text{div}}_{\mathcal{T}} =$ **CTL***$_{\setminus \bigcirc}$-equivalence on $\mathcal{T}$
- $\mathcal{T}$ and $\mathcal{T}/\approx$ might be not **CTL***$_{\setminus \bigcirc}$-equivalent
- $\mathcal{T} \uplus \mathcal{T}/\approx$ might be not divergence-sensitive



$\mathcal{T}$ (and $\mathcal{T}/\approx$) are divergence-sensitive

$\mathcal{T} \models \exists\Box a$ $\quad$ $\mathcal{T}/\approx \not\models \exists\Box a$ $\quad$ where $a \,\widehat{=}\,$ ***blue***

Quotient w.r.t. $\approx^{\mathsf{div}}$

Quotient w.r.t. $\approx^{\text{div}}$

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

quotient system w.r.t. $\approx^{\text{div}}$:

$$\mathcal{T}/{\approx^{\text{div}}} \;=\; (S', Act', \rightarrow_{\text{div}}, S'_0, AP, L')$$

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- state space $S' = S/{\approx^{\mathsf{div}}_{\mathcal{T}}}$

Quotient w.r.t. $\approx^{\textsf{div}}$

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

quotient system w.r.t. $\approx^{\textsf{div}}$:

$$\mathcal{T}/{\approx^{\textsf{div}}} \;=\; (S', Act', \rightarrow_{\textsf{div}}, S'_0, AP, L')$$

- state space $S' = S/{\approx^{\textsf{div}}_{\mathcal{T}}}$
- initial states: $S'_0 = \{[s] : s \in S_0\}$

$[s] = {\approx^{\textsf{div}}_{\mathcal{T}}}$-equivalence class of state s

Quotient w.r.t. $\approx^{\text{div}}$

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

quotient system w.r.t. $\approx^{\text{div}}$:

$$\mathcal{T}/{\approx^{\text{div}}} \;=\; (S', Act', \rightarrow_{\text{div}}, S'_0, AP, L')$$

- state space $S' = S/{\approx^{\text{div}}_{\mathcal{T}}}$
- initial states: $S'_0 = \{[s] : s \in S_0\}$
- labeling function $L'([s]) = L(s)$

$[s] = \approx^{\text{div}}_{\mathcal{T}}$-equivalence class of state s

Quotient w.r.t. $\approx^{\mathsf{div}}$

Let $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ be a TS.

quotient system w.r.t. $\approx^{\mathsf{div}}$:

$$\mathcal{T}/\approx^{\mathsf{div}} \;=\; (S', Act', \rightarrow_{\mathsf{div}}, S_0', AP, L')$$

- state space $S' = S/\approx^{\mathsf{div}}_{\mathcal{T}}$
- initial states: $S_0' = \{[s] : s \in S_0\}$
- labeling function $L'([s]) = L(s)$
- transition relation:

$$\frac{s \rightarrow t \;\wedge\; s \not\approx^{\mathsf{div}}_{\mathcal{T}} \; t}{[s] \rightarrow_{\mathsf{div}} [t]}$$

Quotient w.r.t. $\approx^{\mathsf{div}}$

Let $\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$ be a TS.

quotient system w.r.t. $\approx^{\mathsf{div}}$:

$$\mathcal{T}/\!\approx^{\mathsf{div}} \;=\; (S', Act', \longrightarrow_{\mathsf{div}}, S'_0, AP, L')$$

- state space $S' = S/\!\approx^{\mathsf{div}}_{\mathcal{T}}$
- initial states: $S'_0 = \{[s] : s \in S_0\}$
- labeling function $L'([s]) = L(s)$
- transition relation:

$$\frac{s \longrightarrow t \;\wedge\; s \not\approx^{\mathsf{div}}_{\mathcal{T}} t}{[s] \longrightarrow_{\mathsf{div}} [t]} \qquad \frac{s \text{ is } \approx^{\mathsf{div}}_{\mathcal{T}}\text{-divergent}}{[s] \longrightarrow_{\mathsf{div}} [s]}$$

Relation between $\mathcal{T}$ and its quotient

Relation between $\mathcal{T}$ and its quotient

quotient of $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ w.r.t. $\approx^{\text{div}}$:

$$\mathcal{T}/\!\approx^{\text{div}} \;=\; (S/\!\approx^{\text{div}}_{\mathcal{T}}, Act', \rightarrow_{\text{div}}, S'_0, AP, L')$$

- initial states: $S'_0 = \{[s] : s \in S_0\}$
- labeling function $L'([s]) = L(s)$
- transition relation:

$$\frac{s \rightarrow t \;\wedge\; s \not\approx^{\text{div}}_{\mathcal{T}} t}{[s] \rightarrow_{\text{div}} [t]} \qquad \frac{s \text{ is } \approx^{\text{div}}_{\mathcal{T}}\text{-divergent}}{[s] \rightarrow_{\text{div}} [s]}$$

Relation between $\mathcal{T}$ and its quotient

quotient of $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ w.r.t. $\approx^{\mathsf{div}}$:

$$\mathcal{T}/\!\approx^{\mathsf{div}} \;=\; (S/\!\approx^{\mathsf{div}}_{\mathcal{T}}, Act', \rightarrow_{\mathsf{div}}, S_0', AP, L')$$

- initial states: $S_0' = \{[s] : s \in S_0\}$
- labeling function $L'([s]) = L(s)$
- transition relation:

$$\frac{s \rightarrow t \;\wedge\; s \not\approx^{\mathsf{div}}_{\mathcal{T}} t}{[s] \rightarrow_{\mathsf{div}} [t]} \qquad \frac{s \text{ is } \approx^{\mathsf{div}}_{\mathcal{T}}\text{-divergent}}{[s] \rightarrow_{\mathsf{div}} [s]}$$

$$\mathcal{T} \approx^{\mathsf{div}} \mathcal{T}/\!\approx^{\mathsf{div}}$$

Relation between $\mathcal{T}$ and its quotient

quotient of $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ w.r.t. $\approx^{\mathsf{div}}$:

$$\mathcal{T}/{\approx^{\mathsf{div}}} \;=\; (S/{\approx^{\mathsf{div}}_{\mathcal{T}}}, Act', \rightarrow_{\mathsf{div}}, S_0', AP, L')$$

- initial states: $S_0' = \{[s] : s \in S_0\}$
- labeling function $L'([s]) = L(s)$
- transition relation:

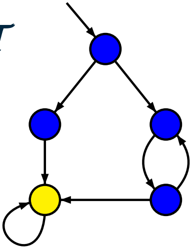
$$\frac{s \rightarrow t \;\wedge\; s \not\approx^{\mathsf{div}}_{\mathcal{T}} t}{[s] \rightarrow_{\mathsf{div}} [t]} \qquad \frac{s \text{ is } \approx^{\mathsf{div}}_{\mathcal{T}}\text{-divergent}}{[s] \rightarrow_{\mathsf{div}} [s]}$$

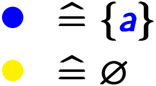
$$\mathcal{T} \;\approx^{\mathsf{div}}\; \mathcal{T}/{\approx^{\mathsf{div}}}$$

as $\{(s,[s]) : s \in S\}$ is a divergence-sensitive stutter bisimulation for $(\mathcal{T}, \mathcal{T}/{\approx^{\mathsf{div}}})$

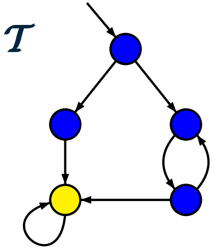
Example: quotient w.r.t. $\approx$ and $\approx^{\text{div}}$

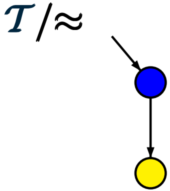
$\mathcal{T}$

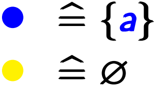




Example: quotient w.r.t. $\approx$ and $\approx^{\text{div}}$

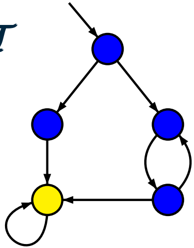






Example: quotient w.r.t. $\approx$ and $\approx^{\text{div}}$

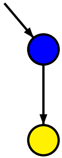
$\mathcal{T}$



● $\widehat{=}\ \{a\}$

● $\widehat{=}\ \varnothing$

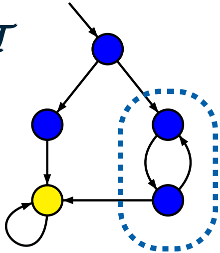
$\mathcal{T}/\!\approx$

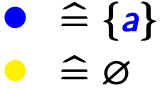


$\mathcal{T} \approx \mathcal{T}/\!\approx$

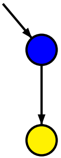
Example: quotient w.r.t. $\approx$ and $\approx^{\text{div}}$

$\mathcal{T}$

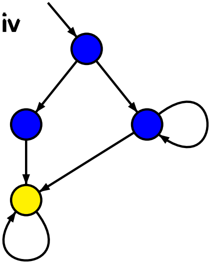




$\mathcal{T}/\approx$



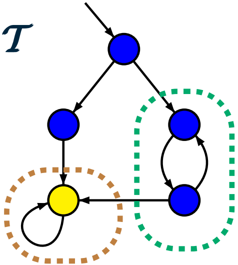
$\mathcal{T}/\approx^{\text{div}}$



$\mathcal{T} \approx \mathcal{T}/\approx$

Example: quotient w.r.t. $\approx$ and $\approx^{\text{div}}$

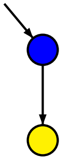
$\mathcal{T}$



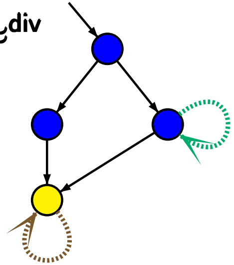
blue ● $\widehat{=}$ $\{a\}$

yellow ● $\widehat{=}$ $\varnothing$

$\mathcal{T}/\approx$

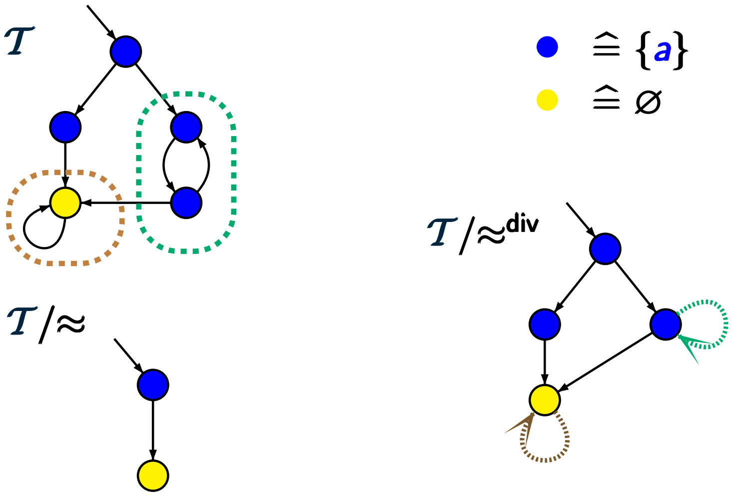


$\mathcal{T}/\approx^{\text{div}}$



$\mathcal{T} \approx \mathcal{T}/\approx$

Example: quotient w.r.t. $\approx$ and $\approx^{\mathsf{div}}$



$\mathcal{T} \approx \mathcal{T}/\approx$

$\mathcal{T} \approx^{\mathsf{div}} \mathcal{T}/\approx^{\mathsf{div}}$